

Comparative Analysis of 2-out-of-3 Cold Standby Reliability Systems with Perfect and Imperfect Automatic Repair Mechanisms

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Abstract: This paper presents a comparative study of two reliability models for a 2-out-of-3 cold standby system consisting of three identical units, of which two units are required for the successful operation of the system. In the first model, the Automatic Repair Machine (ARM) is assumed to be perfect and does not fail during the repair process. In the second model, the assumption of a perfect ARM is relaxed and the ARM itself is allowed to fail; an expert repairman is then employed to repair the failed ARM.

Both models are analysed using the Regenerative Point Technique (RPT) and Markovian methods. The principal reliability measures considered are Mean Time to System Failure (MTSF), steady-state availability, busy period of the ARM, and profit function. The first model involves four states, whereas the second model requires seven states because additional states are introduced to represent ARM failure and its repair. The comparative analysis demonstrates that incorporating ARM failure provides a more comprehensive representation of the practical behaviour of the system. The second model therefore extends the first model by considering an additional source of system degradation and the corresponding repair mechanism.

Keywords: 2-out-of-3 system, Cold standby, Automatic Repair Machine, ARM failure, Regenerative Point Technique, MTSF, Availability, Busy Period, Reliability.

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Introduction

Reliability analysis of redundant systems has received considerable attention because redundancy can improve the continuity and dependability of engineering and industrial systems. In a redundant system, additional units are provided so that the system can continue functioning even when one or more units fail.

The paper, entitled “Behavioural Analysis of an m-out-of-n Units Cold Standby System with Perfect ARM,”[2] considers a 2-out-of-3 cold standby system in which two units are initially operative and the third unit is maintained in cold standby. An Automatic Repair Machine is incorporated to repair failed units. A major assumption of this model is that the ARM is **perfect**, i.e. it never fails.

The paper, entitled “Stochastic Behaviour of an 2-out-of-3 Cold Standby System Supported by an Automatic Repair Mechanism,”[1] considers essentially the same basic redundant system but removes the assumption of a perfect ARM. In this model, the ARM may itself fail and an expert repairman is called to repair it.

Thus, the second model can be regarded as an extension of the first model, in which the reliability of the repair facility itself is taken into consideration.

Objectives of the Comparative Study

The main objectives are:

1. To compare the assumptions of the two reliability models.
2. To identify the structural differences between the two models.
3. To compare their state-space and transition structures.
4. To examine the effect of ARM failure on the reliability model.
5. To compare the treatment of MTSF and system availability.
6. To compare the busy-period analysis of the ARM.
7. To identify the additional contribution of the second model.
8. To suggest possible future extensions of the models.

Basic System Considered in Both Models

Both Models consider a 2-out-of-3 cold standby system consisting of three identical units.

- Two units are initially operating.
- One unit remains in cold standby.

- The system is in the up-state when two units are operating.
- The system is in a partial up-state when only one unit is operating.
- The system is in the down-state when no unit is operating.
- A failed unit is detected automatically.
- The standby unit is activated when an operative unit fails.
- Switching is assumed to be perfect and instantaneous.
- A repaired unit is considered as good as new.
- Regenerative Point Technique is used for analysis.

Therefore, the basic physical system is common to both Models.

Notations:

E : Set of regenerative states

$\bar{E}$: Set of non-regenerative states

λ : Constant failure rate of a unit

$g(t), G(t)$: probability density function and cumulative density function of repair time of a failed unit

O: Unit is in operative state

C_S : Unit is in cold standby state

F_r : Failed unit under repair

F_R : Repair of failed unit continuous from previous state

F_{wr}, F_{WR} : A failed unit waiting for repair, a failed unit waiting for repair from previous state

ARM_O: Automatic Repair Machine in operative state

Θ : Symbol for Stieltjes Convolution

$\odot$: Symbol for Convolution

Major Differences Between the Two Models

Parameter	First Model	Second Model
System	2-out-of-3 cold standby	2-out-of-3 cold standby
Number of units	3	3
Initial operative units	2	2
Standby units	1 cold standby	1 cold standby

ARM	Perfect ARM	Imperfect ARM
ARM failure	Not considered	Considered
ARM repair	Not required	Expert repairman repairs ARM
Unit repair	By ARM	By ARM
Number of model states	4 states	7 states
Regenerative analysis	RPT	RPT
MTSF	Analysed	Analysed
Availability	Analysed	Analysed
ARM busy period	Analysed	Analysed
Repairman busy period	Not separately required	Analysed
Profit function	Analysed	Analysed
Model complexity	Comparatively simpler	More complex
Practical representation	Assumes repair facility is always available	Includes failure of repair facility

Comparison of State Structures

First Model

The first model has four states:

- S_0, S_1, S_2, S_3

Among these, S_0, S_1 and S_2 are regenerative states, while S_3 is treated as a non-regenerative state.

The model therefore has a relatively simple transition structure because the ARM is assumed never to fail.

Second Model

The second model contains seven states:

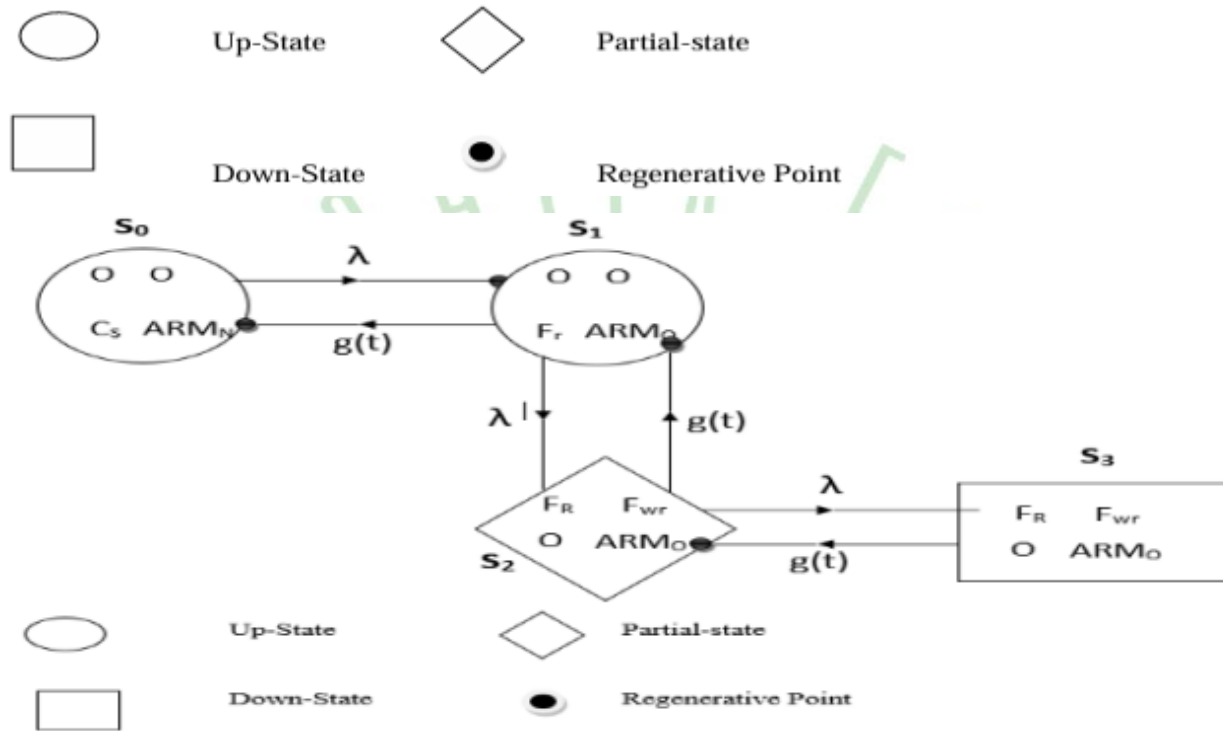
- $S_0, S_1, S_2, S_3, S_4, S_5, S_6,$

The additional states are required to represent situations associated with ARM failure and its repair.

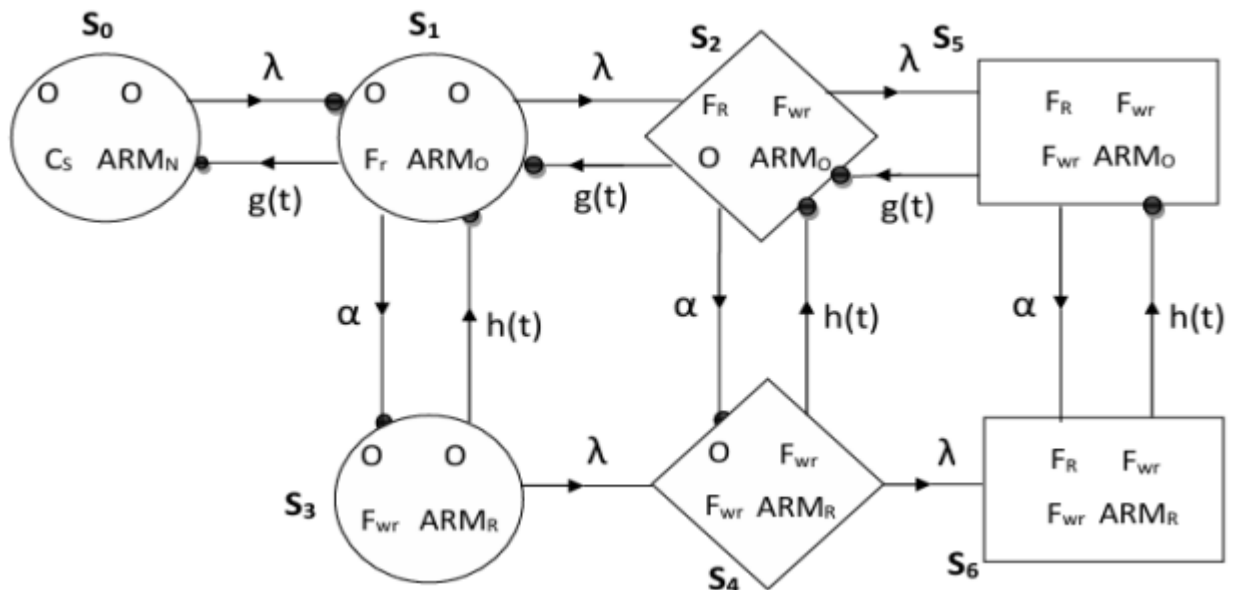
Thus, the increase from four states to seven states is a direct consequence of relaxing the perfect-ARM assumption.

Structural Diagram of Both Models

State Transition Diagram:



MODEL-01



MODEL-02

Comparison of ARM Assumptions

This is the **main difference** between the two Models.

Model I: Perfect ARM

The first Model assumes:

The Automatic Repair Machine will never fail.

Consequently, whenever a unit fails, the ARM is available to repair it. There is no separate state for ARM failure.

Model II: Imperfect ARM

The second Model assumes:

The Automatic Repair Machine may also fail.

When the ARM fails:

1. The repair facility becomes unavailable.
2. The ARM enters a failed state.
3. An expert repairman is called.
4. The repairman repairs the ARM.
5. The ARM returns to operation.

This introduces an additional source of unreliability into the system.

Comparison of MTSF Analysis

In both Models, **Mean Time to System Failure (MTSF)** is obtained using integral equations based on regenerative states and their transition probabilities.

In the first Model, the MTSF equations involve fewer states and consequently have a comparatively simpler mathematical form.

In the second Model, additional transitions associated with ARM failure and repair are introduced. Therefore, the expression for MTSF becomes considerably more complicated.

The second model provides a broader reliability description because system failure can be influenced not only by unit failures but also by the failure and subsequent repair of the ARM.

Comparison of Availability

Availability is evaluated in both models as the probability that the system is operational at a specified time.

In the first Model, availability is determined mainly by:

- unit failure,
- unit repair,
- Standby activation.

In the second Model, availability is additionally influenced by:

- ARM failure,
- ARM repair,
- The time for which the ARM remains unavailable.

Therefore, the second model incorporates the reliability of the **repair facility itself** into the availability analysis.

Busy Period Analysis

Both Models investigate the busy period of the Automatic Repair Machine.

In the first Model, the ARM busy period is associated primarily with repairing failed system units.

In the second Model, the analysis is extended because the ARM itself can fail. The second paper additionally considers the **busy period of the repairman** who repairs the failed ARM.

This is an important extension because it provides information not only about the system's repair facility but also about the utilisation of the human repair resource.

Similarities

The two Models have several important similarities:

1. Both consider a **2-out-of-3 cold standby system**.
2. Both use three identical units.
3. Two units are required for normal operation.
4. One unit remains in cold standby.
5. Failed units are repaired automatically.
6. Repaired units are considered as good as new.
7. Switching time is assumed negligible.
8. Unit failure is assumed to follow an exponential distribution.

9. Repair times are treated using general distributions.
10. Both use **Markov/renewal concepts and Regenerative Point Technique**.
11. Both evaluate MTSF.
12. Both evaluate steady-state availability.
13. Both analyse the ARM busy period.
14. Both consider a profit function.

Both use particular cases to study reliability parameters.

Industrial Significance

The industrial benefit of this comparative reliability model is that it helps industries understand how reliable their systems will be when both the operating units and the automatic repair mechanism (ARM) can fail.

Benefits to Industry

1. **Higher system reliability**
 - The 2-out-of-3 configuration provides redundancy.
 - If one unit fails, the standby unit can automatically take over, allowing operation to continue.
2. **Better maintenance planning**
 - The model calculates MTSF (Mean Time to System Failure).
 - Industries can use this to decide when preventive maintenance or replacement should be carried out.
3. **Realistic assessment of automatic repair systems**
 - The first model assumes the ARM never fails.
 - The second model considers ARM failure and its repair, which is more realistic for industrial equipment.
4. **Reduced production downtime**
 - Availability analysis tells the industry how much of the time the system is operational.
 - This can help reduce unexpected shutdowns.
5. **Repair-resource planning**
 - The second model includes an expert repairman for repairing a failed ARM.

- The repairman's busy period can help management plan manpower and maintenance resources.

6. Cost and profit analysis

- The profit function combines system availability with repair-related costs.
- Management can use this framework to compare different maintenance strategies.

7. Useful for critical industries

This type of model can be applied to systems where failure is costly, such as:

- Power-generation systems
- Manufacturing/production lines
- Telecommunication systems
- Computer/server systems
- Transportation and control systems
- Industrial automation

Main industrial significance

The first Model gives an idealised model assuming a perfect repair facility, while the second Model gives a more practical model by considering that the repair facility itself can fail.

Therefore, the comparison helps an industry answer questions such as:

“If my equipment has redundancy but the automatic repair facility can also fail, how will that affect system availability, failure time, maintenance requirements and operating cost?”

Conclusion

A comparative study of the two Models shows that both models deal with the reliability behaviour of a 2-out-of-3 cold standby redundant system consisting of three identical units. The fundamental system configuration, standby policy, unit failure mechanism, repair of failed units, instantaneous switching, and use of the Regenerative Point Technique are common to both models.

The principal distinction lies in the treatment of the Automatic Repair Machine (ARM). In the first Model, the ARM is assumed to be perfect and therefore its failure is not considered. This assumption results in a relatively simple four-state model. In the second Model, this assumption is relaxed by allowing the ARM itself to fail. An expert repairman is introduced to repair the failed ARM, resulting in a seven-state model and a more extensive set of transition equations.

The second model therefore extends the first model by incorporating an additional source of system unavailability—the failure of the repair facility itself. This extension also permits the analysis of the repairman's busy period in addition to the ARM's busy period. Consequently, the second model provides a more detailed framework for studying systems in which both the operating units and the automatic repair facility are subject to failure.

Overall, the comparison establishes that the second Model is an extension/generalisation of the first Model, obtained by replacing the perfect-ARM assumption with an imperfect-ARM assumption and incorporating ARM repair. The framework can be further extended by considering time-dependent failure and repair rates, multiple repairmen, inspection time, imperfect switching, waiting time of failed units, repair priorities, and more detailed cost and profit functions.

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