THERMODYNAMICAL VIBRATIONS IN AN INITIALLY STRESSED MICROPOLAR MAGNETO-THERMOELASTIC THREE PHASE LAG MODEL WITH LASER PULSE AND TEMPERATURE DEPENDENT PROPERTIES

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Abstract

The present research is pertained to study the dynamical interactions in a micropolar magneto-thermoelastic medium under the effect of temperature dependent properties, laser pulse and initial stress. Three phase lag theory is employed for addressing the mathematical analysis. The analytical solution of the displacement components, temperature and stress components is obtained by applying the normal mode analysis technique. Some particular cases are also discussed in the context of the problem. The numerical evaluation of the field quantities is carried out for magnesium crystal like material in the physical domain. The effects of magnetic field and initial stress are observed on the physical quantities and depicted graphically. Some attempts have also been made to explore the three-dimensional responses of the thermoelastic medium. Numerical results predict finite speed of propagation for thermoelastic waves.

Paper Identification

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1 Introduction

The conventional thermoelastic theory is based on the usual Fourier's law of heat conduction and hence acknowledging thermal signals propagating with infinite speed. Several alternative theories of heat conduction have been developed to overcome this physically unacceptable situation and all these theories are admitting wave-like thermal signals propagating with finite speed. The first theory was developed by Lord and Shulman (L-S) (1967), which involves the concept of thermal relaxation time into the usual Fourier's law of heat conduction. The second among such modelings is the temperature rate dependent thermoelasticity theory known as Green and Lindsay (G-L) (1972) theory with two relaxation times, which modified the stress-strain relationship and the entropy relation that relates the stress and entropy to the temperature.

After that, providing sufficient basic modifications in governing equations, Green and Naghdi (1991, 1992, 1993) produced an alternative theory which was further divided into three different parts, referred to as GN theories of type I, III, II. A remarkable generalization of the coupled theory of thermoelasticity is referred to as dual phase lag thermoelasticity proposed by Tzou (1995) and Chandrasekharaiah (1998). Tzou (1995) introduced two phase lags, one for heat flux vector and the other for temperature gradient. Another generalization, known as three phase lag thermoelasticity, was developed by Roy Choudhuri (2007). In this generalization, the Fourier's law of heat conduction is replaced by an approximation to a modification of the Fourier's law with introduction of three different phase lags for the heat flux vector, the temperature gradient and thermal displacement gradient. The stability of the three-phase lag heat conduction equation and relation among the three phase lag parameters are discussed in detail by Quintanilla and Racke (2008). Othman et al. (2019) investigated the effects of gravity, two-temperature parameter, fiber-reinforcement and time on the various thermo physical quantities in the purview of the three-phase-lag model.

During heating of a metal film by laser pulse, a thermoelastic wave is generated due to thermal expansion in the near surface region and propagates into the target. Several research works have been devoted to problems involving a laser pulse heat source due to the numerous applications in engineering, such as pulsed-laser cutting and welding, drilling, surface hardening, high speed machining etc. The coupled thermoelastic vibrations of a microscale beam resonator induced by laser pulse heating were studied by Sun *et al*. (2008). Othman and Tantawi (2016) employed normal mode technique to investigate the effect of gravitational field on a two-dimensional thermoelastic medium under the influence of laser pulse. Othman *et al*. (2020) used the normal mode analysis to study the effect of heat laser pulse on wave propagation in a generalized thermoelastic micropolar medium in the context of GN-III theory. Bayones *et al*. (2021) scrutinized the influence of gravity on the transient waves in a homogeneous, isotropic thermoelastic medium subjected to laser pulse. They adopted the normal mode technique to obtain the analytic expressions for displacement components, stresses and temperature.

Recent years have seen an ever growing interest in investigation of the problems related to initially stressed elastic medium. Such problems have numerous applications in various fields, such as earthquake engineering, seismology and geophysics. The earth is assumed to be under high initial stresses. The dynamic problem of an elastic medium under initial stress was solved by Biot (1965). The linear theory of thermoelasticity with hydrostatic initial stress for an isotropic medium was developed by Montanaro (1999). Othman and Song (2007) studied the reflection phenomenon of plane waves in the context of Green-Naghdi theory of Types II and III under hydrostatic initial stress. In the context of GN-III theory, Othman *et al*. (2015) scrutinized the impacts of rotation and time on an initially stressed thermoelastic medium with voids due to laser pulse. By adopting the normal mode method, they obtained the expressions of different field variables and represented them graphically. Lotfy (2021) established a novel mathematical model of the magneto-thermoelastic initially stressed medium in the context of the photothermal transport process.

The present paper is devoted to study the interactions in an initially stressed generalized micropolar magneto-thermoelastic medium with three phase lags, temperature dependent property and laser pulse, subjected to thermal load. Normal mode analysis is adopted to solve the governing equations. Analytical and numerical results for temperature, displacement and stress distributions have been obtained. Some comparisons have been shown in figures to estimate the effects of initial stress parameter and magnetic field. Some attempts have also been made to explore the three-dimensional responses of the thermoelastic medium.

2 Governing equations

In accordance with Roy Choudhuri (2007), Aouadi (2006) and Montanaro (1999), the constitutive relations and field equations for a homogeneous isotropic micropolar magneto-thermoelastic medium with laser pulse and initial stress in the presence of body force are given as:

(i) Constitutive relations:

$$\sigma_{ij} = -p\left(\delta_{ij} + \omega_{ij}\right) + 2\mu e_{ij} + \lambda e \delta_{ij} - \beta_1 \Theta \delta_{ij} + K\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right), \quad (1)$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ \omega_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right), \qquad (2)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \quad (3)$$

where p is the initial pressure, ω_{ij} are the components of rotation tensor, $\beta_1 = \left(3\lambda + 2\mu + K\right)\alpha_t, \alpha_t$ is the coefficient of linear thermal expansion, ϕ_i are the components of microrotation vector, ε_{ijr} is the permutation tensor, m_{ij} are the components of couple stress tensor, α, β, γ and K are micropolar material constants.

(ii) Equation of motion:

$$\sigma_{ji,j} + F_i = \rho\ddot{u}_i, \qquad (4)$$

$$m_{ik,i} + \varepsilon_{kij}\sigma_{ij} = \rho j\ddot{\phi}_k, \quad (5)$$

where j is the microinertia and F_i are the components of the Lorentz force induced by the magnetic field, given as

$$F_i = \mu_0\left(\vec{J} \times \vec{H}\right)_i. \qquad (6)$$

The linearized equations of electrodynamics of slowly moving medium (Maxwell's electro-magnetic field equations) are:

$$\vec{J} = \nabla \times \vec{h} - \varepsilon_0\dot{\vec{E}},\ \nabla \times \vec{E} = -\mu_0\dot{\vec{h}},\ \vec{h} = \nabla \times \left(\vec{u} \times \vec{H}\right),$$

$$\vec{E} = -\mu_0\left(\dot{\vec{u}} \times \vec{H}\right),\ \operatorname{div}\vec{h} = 0,\ \operatorname{div}\vec{E} = 0, \qquad (7)$$

where $\vec{J}$ is the current density vector, μ_0 is the magnetic permeability, ε_0 is electric permittivity, $\vec{E}$ is an induced electric vector field, $\vec{H}$ is applied magnetic field and $\vec{h}$ is an induced magnetic field.

(iii) Heat conduction equation:

$$\left[K_1^*\left(1 + \tau_v\frac{\partial}{\partial t}\right) + K^*\frac{\partial}{\partial t}\left(1 + \tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2\Theta$$

$$= \left(1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E\ddot{\Theta} + \beta_1 T_0\ddot{e}\right) - \dot{Q}, \quad (8)$$

where K^* is thermal conductivity, $K_1^* = \frac{C_E\left(\lambda + 2\mu\right)}{4}$ is material constant characteristic of the theory, τ_T is phase lag for the temperature gradient, τ_q is phase lag for heat flux, τ_v is phase lag for the thermal displacement gradient and Q is the heat source. A comma followed by suffix denotes material derivative and a superposed dot denotes the derivative with respect to time t. The inequality among the three different phase lags to be satisfied is $0 \le \tau_v \le \tau_T \le \tau_q$.

Moreover, our aim is to investigate the influence of temperature dependence of elastic and thermal moduli. Therefore, we may assume that

$$(\lambda, \mu, \alpha, \beta, \gamma, K, \beta_1) = (\lambda', \mu', \alpha', \beta', \gamma', K', \beta_1') f(T_0), \qquad (9)$$

where $\lambda', \mu', \alpha', \beta', \gamma', K'$ and β_1' are constants and $f(T_0)$ is a given non-dimensional function of reference temperature such that $f(T_0) = 1 - \alpha^* T_0$, where α^* is an empirical material constant. In case of temperature independent properties, we have $f(T_0) = 1$.

3 Formulation of the problem

We consider a homogeneous, isotropic, micropolar magneto-thermoelastic medium with laser pulse heat source, initial stress and temperature dependent properties. Origin of the cartesian co-ordinate system (x, y, z) is taken at any point on the plane surface and the z-axis points vertically downwards into the considered medium which is represented by $z \ge 0$ (Figure 1). Orientation of the primary magnetic field $\vec{H} = (0, H_0, 0)$ is towards the positive direction of y-axis. Due to application of this magnetic field, there arises in the medium an induced magnetic field $\vec{h}$ and an induced electric field $\vec{E}$. We assume that both $\vec{h}$ and $\vec{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermoelasticity. The xz-plane is chosen to confine the deformation and propagation of disturbance. The half space is heated uniformly by laser pulse with non Gaussian temporal profile, Sun *et al*. (2008).

$$L(t) = \frac{L_0 t}{t_p^2} \exp\left(-\frac{t}{t_p}\right), (10)$$

where t_p is the time duration of a laser pulse and L_0 is the laser intensity, which is defined as the total energy carried by a laser pulse per unit cross section of the laser beam. In the present problem, we take $t_p = -2ps$ as the time duration. The thermal conduction in the beam can be modelled as a one-dimensional problem with energy source $Q(z,t)$ as

$$Q(z,t) = \frac{R_a}{\delta} \exp\left(\frac{z - h/2}{\delta}\right) L(t), \qquad (11)$$

where δ is the absorption depth of the heating energy and R_a is the absorptivity of the irradiated surface.

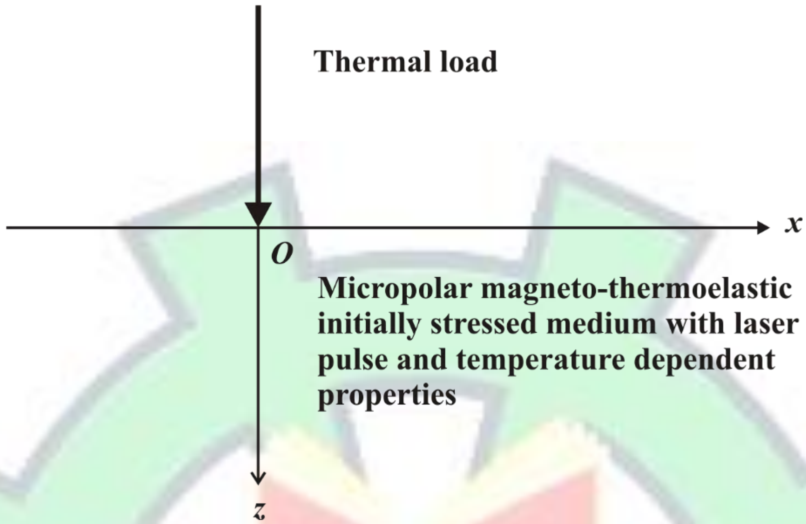

Figure 1: Geometry of the problem

For a two-dimensional problem in *xz*-plane, we can write the displacement vector and microrotation vector as

$$\vec{u} = (u, 0, w),\ u = u(x,z,t),\ v = 0,\ w = w(x,z,t) \text{ and } \vec{\phi} = (0, \phi_2, 0). \quad (12)$$

The components of initial magnetic field vector $\vec{H}$ are

$$H_x = 0,\ H_y = H_0,\ H_z = 0. \quad (13)$$

The induced electric intensity vector is normal to both the magnetic intensity and the displacement vectors. Thus, $\vec{E}$ have the components

$$E_x = E_1,\ E_y = 0,\ E_z = E_3. \quad (14)$$

The current density vector $\vec{J}$ must be parallel to the electric intensity vector $\vec{E}$, thus

$$J_x = J_1,\ J_y = 0,\ J_z = J_3. \quad (15)$$

Expression (6) with the help of (7) and (12) - (15) takes the form

$$F_x = \mu_0 H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} - \varepsilon_0 \mu_0 \frac{\partial^2 u}{\partial t^2} \right), \quad (16)$$

$$F_z = \mu_0 H_0^2 \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} - \varepsilon_0 \mu_0 \frac{\partial^2 w}{\partial t^2} \right). \quad (17)$$

With the aid of expressions (2), (9) and (12), the stresses arising from Eqs. (1) and (3), take the form

$$\sigma_{xx}=-p+\left[\left(2\mu'+\lambda'+K'\right)\frac{\partial u}{\partial x}+\lambda'\frac{\partial w}{\partial z}-\beta_1'\Theta\right]\alpha_0, \qquad (18)$$

$$\sigma_{zz}=-p+\left[\left(2\mu'+\lambda'+K'\right)\frac{\partial w}{\partial z}+\lambda'\frac{\partial u}{\partial x}-\beta_1'\Theta\right]\alpha_0, \qquad (19)$$

$$\sigma_{xz}=\left[\left(\mu'+K'-\frac{p}{2\alpha_0}\right)\frac{\partial w}{\partial x}+\left(\mu'+\frac{p}{2\alpha_0}\right)\frac{\partial u}{\partial z}+K'\phi_2\right]\alpha_0, \qquad (20)$$

$$\sigma_{zx}=\left[\left(\mu'+K'-\frac{p}{2\alpha_0}\right)\frac{\partial u}{\partial z}+\left(\mu'+\frac{p}{2\alpha_0}\right)\frac{\partial w}{\partial x}-K'\phi_2\right]\alpha_0, \qquad (21)$$

$$m_{xy}=\alpha_0\gamma'\frac{\partial\phi_2}{\partial x}, \qquad (22)$$

$$m_{zy}=\alpha_0\gamma'\frac{\partial\phi_2}{\partial z}, \qquad (23)$$

where $\alpha_0=f\left(T_0\right)=1-\alpha^*T_0$.

With the help of expressions (9) - (12), (16) and (17), the equations of motion and heat conduction, defined in (4), (5) and (8), take the forms:

$$\left[\left(\mu'+K'\right)\nabla^2u+\left(\mu'+\lambda'+\frac{\mu_0H_0^2}{\alpha_0}\right)\frac{\partial^2u}{\partial x^2}+\left(\mu'+\lambda'+\mu_0H_0^2+\frac{p}{2\alpha_0}\right)\frac{\partial^2w}{\partial z\partial x}\right.$$
$$\left.-\frac{p}{2\alpha_0}\frac{\partial^2u}{\partial z^2}-\beta_1'\frac{\partial\Theta}{\partial x}-K'\frac{\partial\phi_2}{\partial z}\right]\alpha_0=\left(\mu_0^2H_0^2\varepsilon_0+\rho\right)\frac{\partial^2u}{\partial t^2}, \qquad (24)$$

$$\left[\left(\mu'+K'\right)\nabla^2w+\left(\mu'+\lambda'+\frac{\mu_0H_0^2}{\alpha_0}\right)\frac{\partial^2w}{\partial z^2}+\left(\mu'+\lambda'+\mu_0H_0^2+\frac{p}{2\alpha_0}\right)\frac{\partial^2u}{\partial x\partial z}\right.$$
$$\left.-\frac{p}{2\alpha_0}\frac{\partial^2w}{\partial x^2}-\beta_1'\frac{\partial\Theta}{\partial z}+K'\frac{\partial\phi_2}{\partial x}\right]\alpha_0=\left(\mu_0^2H_0^2\varepsilon_0+\rho\right)\frac{\partial^2w}{\partial t^2}, \qquad (25)$$

$$\left[\gamma'\,\nabla^2\phi_2+\left(K'-\frac{p}{\alpha_0}\right)\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2K'\phi_2\right]\alpha_0=\rho j\frac{\partial^2\phi_2}{\partial t^2}, \qquad (26)$$

$$\left[K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right)+K^*\frac{\partial}{\partial t}\left(1+\tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2\Theta=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\rho\left(C_E\ddot{\Theta}+\beta_1'T_0\ddot{e}\right)$$
$$-\frac{\partial}{\partial t}\left(\frac{R_aL_0t}{\delta t_p^2}\exp\left(\frac{2zt_p^2-ht_p^2-2t\delta}{2t_p^2\delta}\right)\right). \qquad (27)$$

To eliminate the numerical difficulties in solution procedure, we introduce the following non-dimensional variables:

$$\left(\hat{x},\hat{z}\right)=\frac{\omega^*}{c_1}\left(x,z\right),\ \left(\hat{u},\hat{w}\right)=\frac{\rho\omega^* c_1}{\beta_1' T_0}\left(u,w\right),\ \hat{\phi}_2=\frac{\rho c_1^2}{\beta_1' T_0}\phi_2,$$

$$\hat{\sigma}_{ij}=\frac{\sigma_{ij}}{\beta_1' T_0},\ \left(\hat{t},\hat{t}_p,\hat{\tau}_T,\hat{\tau}_q,\hat{\tau}_v\right)=\omega^*\left(t,t_p,\tau_T,\tau_q,\tau_v\right),\ \hat{m}_{ij}=\frac{\omega^*}{c_1\beta_1' T_0}m_{ij},$$

$$\hat{\Theta}=\frac{\Theta}{T_0},\ \hat{p}=\frac{p}{\beta_1' T_0},\ \left(\hat{\delta},\hat{h}\right)=\frac{\omega^*}{c_1}\left(\delta,h\right), \qquad (28)$$

where

$$\omega^*=\frac{\rho C_E c_1^2}{K^*},\ c_1^2=\frac{\lambda'+2\mu'+K'}{\rho}.$$

Using Helmholtz decomposition, the displacement components can be written as

$$u=\frac{\partial q}{\partial x}+\frac{\partial\psi}{\partial z},\ w=\frac{\partial q}{\partial z}-\frac{\partial\psi}{\partial x},\ \psi=\left(-\vec{U}\right)_y, \qquad (29)$$

where $q\left(x,z,t\right)$ and $\psi\left(x,z,t\right)$ are scalar potential functions and $\vec{U}\left(x,z,t\right)$ is the vector potential function.

In view of above considerations (dimensionless parameters and potential functions) described in (28) and (29), Eqs. (18) - (27) along with some simplifications, assume the forms (after dropping the hat notation)

$$\sigma_{xx}=-p+a_{11}\frac{\partial u}{\partial x}+a_{12}\frac{\partial w}{\partial z}-\alpha_0\Theta, \qquad (30)$$

$$\sigma_{zz}=-p+a_{11}\frac{\partial w}{\partial z}+a_{12}\frac{\partial u}{\partial x}-\alpha_0\Theta, \qquad (31)$$

$$\sigma_{xz}=a_{13}\frac{\partial w}{\partial x}+a_{14}\frac{\partial u}{\partial z}+a_{15}\phi_2, \quad (32)$$

$$\sigma_{zx}=a_{13}\frac{\partial u}{\partial z}+a_{14}\frac{\partial w}{\partial x}-a_{15}\phi_2, \quad (33)$$

$$m_{xy}=a_{16}\frac{\partial\phi_2}{\partial x}, \ (34)$$

$$m_{zy}=a_{16}\frac{\partial\phi_2}{\partial z}, \ (35)$$

$$\left(\nabla^2-a_{17}\frac{\partial^2}{\partial t^2}\right)q-a_{18}\Theta=0, \quad (36)$$

$$\left(\nabla^2 - a_{19}\frac{\partial^2}{\partial t^2}\right)\psi - a_{20}\phi_2 = 0, \quad (37)$$

$$\left(\nabla^2 - a_{21} - a_{22}\frac{\partial^2}{\partial t^2}\right)\phi_2 + a_{23}\nabla^2\psi = 0, \quad (38)$$

$$\left[K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K^*\omega^*\frac{\partial}{\partial t}\left(1+\tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2\Theta = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)$$

$$\times\frac{\partial^2}{\partial t^2}\left(a_{24}\Theta + a_{25}\nabla^2 q\right) - a_{26}\left(1-\frac{t}{t_p}\right)\exp\left(\frac{2zt_p - ht_p - 2t\delta}{2t_p\delta}\right), \quad (39)$$

where all the constants used are defined in **Appendix 1**.

4 Solution of the problem

In this section, we employ the normal mode technique, which provides exact solutions without any assumed condition on the actual physical variables that originate in the governing equations of the considered problem. So, the solution of the physical variables under consideration can be decomposed in terms of normal modes in the following form:

$$\left(u, w, q, \psi, \phi_2, \Theta, m_{ij,}\sigma_{ij}\right)(x,z,t) = \left(u^*, w^*, q^*, \psi^*, \phi_2^*, \Theta^*, m_{ij}^*, \sigma_{ij}^*\right)(z)\, e^{(\omega t + \iota kx)}, \quad (40)$$

where $u^*(z), w^*(z), q^*(z), \psi^*(z), \phi_2^*(z), \Theta^*(z), m_{ij}^*(z), \sigma_{ij}^*(z)$ are the amplitudes of the functions, ω is the angular frequency, ι is an imaginary unit and k is the wave number in x-direction.

Using expression (40) in Eqs. (36) - (39), we get the following differential equations

$$\left(D^4 - A_1D^2 + A_2\right)\left(\phi_2^*(z), \psi^*(z)\right) = 0, \quad (41)$$

$$\left(D^4 - A_3D^2 + A_4\right)\left(q^*(z), \Theta^*(z)\right) = b_{23}\exp\left(\frac{z-h/2}{\delta} - \frac{t}{t_p} - \omega t - \iota kx\right), \quad (42)$$

where $D = \frac{d}{dz}$. Since the intent is that the solutions vanish at infinity so as to satisfy the regularity condition at infinity (which are assumed to be bounded as $z \to \infty$), we can express ϕ_2, ψ, q and Θ in the following forms:

$$[\phi_2, \psi](x,z,t) = \left(\sum_{i=1}^{2}[1, H_{1i}]R_i(k,\omega)\, e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)}, \quad (43)$$

$$q(x,z,t) = \left(\sum_{i=3}^{4}R_i(k,\omega)e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)} + \varepsilon_1 f_1(z,t), \quad (44)$$

$$\Theta(x,z,t)=\left(\sum_{i=3}^{4}H_{1i}R_i(k,\omega)e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}+\varepsilon_2 f_1(z,t) \text{ for } \mathrm{Re}(\lambda_i)>0, \tag{45}$$

where $\lambda_1^2, \lambda_2^2, \lambda_3^2$ and λ_4^2 are roots of the characteristic equations of (41) and (42) respectively and $R_i(k,\omega), (i=1,2,3,4)$ are the parameters depending upon k and ω.

Application of normal mode analysis to the expressions for stress components (31), (33) and (35) and displacement components (29) yields, (in non-dimensional form)

$$u(x,z,t)=\left(\sum_{i=1}^{4}H_{2i}R_i e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}+\varepsilon_3 f_1(z,t), \tag{46}$$

$$w(x,z,t)=\left(\sum_{i=1}^{4}H_{3i}R_i e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}+\varepsilon_4 f_1(z,t), \tag{47}$$

$$\sigma_{zx}(x,z,t)=\left(\sum_{i=1}^{4}H_{4i}R_i e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}+\varepsilon_5 f_1(z,t), \tag{48}$$

$$\sigma_{zz}(x,z,t)=\left(\sum_{i=1}^{4}H'_{5i}R_i e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}+\varepsilon_6 f_1(z,t)-p, \tag{49}$$

$$m_{zy}(x,z,t)=\left(\sum_{i=1}^{2}H_{6i}R_i e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}, \tag{50}$$

where all the constants used are defined in **Appendix 2**.

5 Application: Thermal load acting on the surface

In this section, we will determine the parameters $R_i\,(i=1,2,3,4)$ by postulating the boundary conditions at the surface of the half space. The bounding plane of the surface $z=0$ is subjected to time dependent heat source. Therefore, at surface of the half space, the sum of normal stresses and initial pressure must be equal to zero and shear stress and couple stress must also vanish. Mathematically, boundary conditions of the problem can be expressed as

(i) Mechanical boundary conditions: The mechanical boundary conditions at the surface z = 0 are given as follows:

$$\left.\begin{aligned}&\sigma_{zz}(x,0,t)+\bar{\sigma}_{zz}(x,0,t)+p=0\\&\sigma_{zx}(x,0,t)+\bar{\sigma}_{zx}(x,0,t)=0\\&m_{zy}(x,0,t)=0\end{aligned}\right\}, \tag{51}$$

where Maxwell's stress is given in the form

$$\bar{\sigma}_{zj}=\mu_0\left(H_z h_j+H_j h_z-H_k h_k\delta_{ij}\right).$$

(ii) Thermal boundary condition: The thermal boundary condition is

$$\Theta(x,0,t) = f(x,t), \quad (52)$$

where $f(x,t) = f^{*}e^{(\omega t + \iota kx)}$. Here f^{*} is the intensity of the load applied.

Applying normal mode analysis to (51) and (52) and using Eqs. (45) and (48) - (50) in the resulting equations, we arrive at a non-homogeneous system of four linear equations which can be written in the matrix form as

$$\begin{bmatrix} 0 & 0 & H_{13} & H_{14} \\ H_{41} & H_{42} & H_{43} & H_{44} \\ H_{51} & H_{52} & H_{53} & H_{54} \\ H_{61} & H_{62} & 0 & 0 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ 0 \end{bmatrix}, \qquad (53)$$

where

$$H_{5i} = H'_{5i} + H''_{5i},\ H''_{5i} = \mu_0 H_0^2\left(\lambda_i^2 - k^2\right),\ (i = 1,2,3,4).$$

Solution of system (4.53) provides us the values of $R_i\ (i = 1, 2, 3, 4)$ as:

$$R_1 = \frac{\Delta_1}{\Delta},\ R_2 = \frac{\Delta_2}{\Delta},\ R_3 = \frac{\Delta_3}{\Delta},\ R_4 = \frac{\Delta_4}{\Delta}, (54)$$

where all the parameters used are defined in **Appendix 3**.

Substitution of (54) into (43) and (45) - (50), leads to the expressions of field variables as:

$$\phi_2(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=1}^{2}\Delta_i e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)}, \quad (55)$$

$$\Theta(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=3}^{4}\Delta_i H_{1i} e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)} + \varepsilon_2 f_1(z,t), \quad (56)$$

$$u(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=1}^{4}\Delta_i H_{2i} e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)} + \varepsilon_3 f_1(z,t), \quad (57)$$

$$w(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=1}^{4}\Delta_i H_{3i} e^{-\lambda_i z}\right)e^{(\omega t + \iota mx)} + \varepsilon_4 f_1(z,t), \quad (58)$$

$$\sigma_{zx}(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=1}^{4}\Delta_i H_{4i} e^{-\lambda_i z}\right)e^{(\omega t + \iota kz)} + \varepsilon_5 f_1(z,t), \quad (59)$$

$$\sigma_{zz}(x,z,t) = \left(\frac{1}{\Delta}\sum_{i=1}^{4}\Delta_i H_{5i} e^{-\lambda_i z}\right)e^{(\omega t + \iota kx)} + \varepsilon_6 f_1(z,t) - p, \qquad (60)$$

$$m_{zy}\left(x,z,t\right)=\left(\frac{1}{\Delta}\sum_{i=1}^{2}\Delta_i H_{6i}e^{-\lambda_i z}\right)e^{(\omega t+\iota kx)}. \qquad (61)$$

6 Special cases

6.1 Neglecting magnetic field

If we neglect the magnetic field from the considered half space, then we shall be left with the corresponding problem in a micropolar initially stressed thermoelastic medium with laser pulse and temperature dependent properties. For this purpose, we set $H_0 = 0$. If we further neglect the effect of temperature dependent property and initial stress from the considered medium, then the results of present research coincide with those of Othman *et al*. (2020) with appropriate changes in the theory and boundary conditions.

6.2 Neglecting initial stress

The transient disturbances in a micropolar magneto-thermoelastic medium with laser pulse and temperature dependent properties can be obtained if we remove the initial stress from the governing equations. For this purpose, we shall set $p = 0$. Taking into consideration the above mentioned modification, the corresponding expressions of the physical fields can be obtained from the expressions (55) - (61). If we further neglect the effect of laser pulse, temperature dependent properties and magnetic field from the medium, then the results coincide with those of Othman and Singh (2007) (in the absence the rotation) with appropriate changes in theory.

7 Numerical results and discussions

The dynamical interactions between thermal and mechanical fields in solids have many applications in aeronautics, nuclear reactors and high energy particle accelerators. To understand the interaction phenomena, we have evaluated the numerical results for non dimensional displacement component *w*, normal stress σ_{zz}, couple stress m_{zy} and temperature distribution Θ and displayed graphically. For numerical computations, we take the following values of relevant parameters for magnesium crystal like material [Deswal and Kalkal (2014)]:

$\rho = 1.74\times10^{3}\ kg\ m^{-3}$, $\lambda' = 9.4\times10^{10}\ kg\ m^{-1}s^{-2}$, $\mu' = 4.0\times10^{10}\ kg\ m^{-1}s^{-2}$,

$K' = 1.0\times10^{10}\ kg\ m^{-1}s^{-2}$, $\gamma' = 0.779\times10^{-9}\ kg\ ms^{-2}$, $j = 0.2\times10^{-19}\ m^{2}$,

$K^{*} = 2.510\, Wm^{-1}k^{-1}$, $C_E = 9.623\times10^{2}\ J\ kg^{-1}k^{-1}$, $R_a = 0.5$, $\delta = 0.01$,

$L_0 = 1\times10^{11}\ j\ m^{-1}$, $t_p = 2\,ps$, $h = 0.01$, $\alpha_t = 2.36\times10^{-5}\ K^{-1}$, $p = 1$,

$T_0 = 293\,K$, $\tau_q = 0.2s$, $\tau_T = 0.15s$, $\tau_v = 0.1s$, $\alpha^{*} = 0.005$.

Aim of considering the numerical example is not only to investigate the effect of physical properties of materials on the field variables but also to ensure the accuracy of mathematical derivations during the entire process. We have analyzed the effects of magnetic field and initial stress on the fields by dividing the graphical representations into different categories. In Category-I (Figures 2 - 5), the effects of initial stress and magnetic field are depicted on field variables for the cases: (i) micropolar, initial stress and temperature dependent property with magnetic field (MITM, solid line), (ii) micropolar, initial stress and temperature dependent property (MIT, long-dashed line) and (iii) micropolar with temperature dependent property and magnetic field (MTM, small-dashed line). Some 3D plots of field variable are represented in Category-II (Figures 6 - 9).

Category-I: Effects of initial stress and magnetic field

Figure 2 elucidates the space variation of normal displacement component w with location z for TPL theory. Absence of magnetic field (MIT) and initial stress (MTM) acts to decrease the magnitude of displacement field. The figure shows that normal displacement continuously decreases and converges to zero as z increases. Figure 3 illustrates the variation of normal stress against distance z for the three different media. The curves of the normal stress σ_{zz} experience a similar pattern of variation for all the three i.e., MITM, MIT and MTM media. However, dissimilarity lies on the ground of magnitude. Also, the curves for media MITM and MIT do not converge to zero, as the media are already stressed, whereas in the absence of initial stress (MTM media) the curve converges to zero, as z increases.

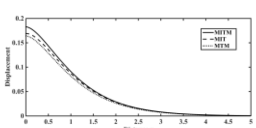

Figure 2: Effects of initial stress and magnetic field on displacement distribution

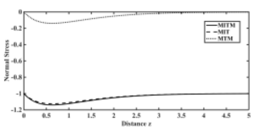

Figure 3: Effects of initial stress and magnetic field on normal stress distribution

Effects of magnetic field and initial stress on couple stress are presented in figure 4. As expected, couple stress distribution is having a coincident starting point of zero magnitude for all the cases, which is in good agreement with the boundary condition. Presence of magnetic field causes an increasing effect on the profile of couple stress. On the other hand, presence of initial stress has a decreasing effect on couple stress. Figure 5 is plotted to analyze the impacts of magnetic field and initial stress on the behaviour of temperature distribution with distance *z*. It is clear from the figure that presence of magnetic field and initial stress has an increasing effect on the profile of temperature field. It is also observed that all the curves have different values near the boundary of half space. Also, this difference becomes indistinct along with the passage of time.

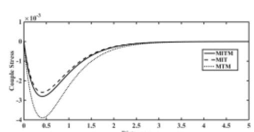

Figure 4: Effects of initial stress and magnetic field on couple stress distribution

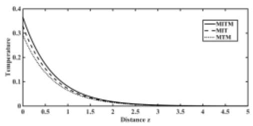

Figure 5: Effects of initial stress and magnetic field on temperature distribution

Category-II: Three-dimensional description

The 3D plots representing normal displacement distribution, normal stress distribution, couple stress distribution and temperature distribution are explained in figures (6-9) for a wide range of $z(0 \le z \le 5)$ and for a wide range of dimensionless time $t(0 \le t \le 0.1)$. Variations of normal displacement with distance *z* and time *t* have been pictured in figure 6. Figure 7 depicts the variations of normal stress against distance *z* and time *t*. It can be noticed from the figure that the normal stress does not converge to zero. This is due to the fact that medium is already stressed.

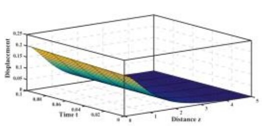

Figure 6: Profile of displacement field

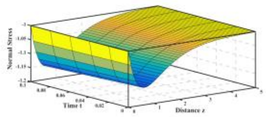

Figure 7: Profile of normal stress

Figure 8 outlines the behaviour of couple stress distribution with distance z and time t. From the profile of couple stress, we observe that the curve has value zero in the vicinity of source, which agrees with the boundary condition. The values of couple stress increase as the time t increases, while it increases and decreases with increasing distance z. Figure 9 depicts the distribution of Θ with distance z and time t. The temperature distribution is acting like a decreasing function of distance z. It is clear from figure 9 that the temperature distribution has maximum value (in magnitude) in the vicinity of source.

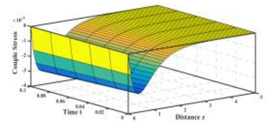

Figure 8: Profile of couple stress

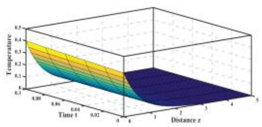

Figure 9: Profile of temperature field

8 Concluding remarks

In this paper, a mathematical treatment has been presented to obtain the solutions of displacement, temperature and stresses within the framework of micropolar magneto-thermoelasticity with three phase lags, initial stress, temperature dependent property and laser pulse. Normal mode analysis technique is used which has the advantages of findings the exact solutions without any assumed restrictions on the field variables. The numerical work has been carried out with the help of computer programming using the software MATLAB. From the analysis of the illustrations, we can arrive at the following conclusions:

- All the field variables satisfy the boundary conditions and hence deformation of a solid depends on the type of the applied force as well as on the type of boundary conditions.
- All the field variables have non zero values in a bounded region of space except normal stress. Outside the region, values vanish identically and this means that the region has not felt thermal disturbance yet.
- The initial stress has an increasing effect (in magnitude) on field variables w, σ_{zz} and Θ and a decreasing effect is observed on the magnitude of field variable m_{zy}.
- The magnetic field plays an important role in the variations of all the field quantities. In the absence of magnetic field, the magnitude of all the fields decreases.

Appendix 1

$$a_{11} = \frac{(\lambda' + 2\mu' + K')\alpha_0}{\rho c_1^2},\ a_{12} = \frac{\lambda'\alpha_0}{\rho c_1^2},\ a_{13} = \frac{(2\mu' + 2K')\alpha_0 - \beta_1' T_0 p}{2\rho c_1^2},$$

$$a_{14}=\frac{2\mu'\alpha_0+\beta_1'T_0p}{2\rho c_1^2},\ a_{15}=\frac{K'\alpha_0}{\rho c_1^2},\ a_{16}=\frac{\gamma'\omega^{*2}\alpha_0}{\rho c_1^4},\ a_{17}=\frac{c_1^2\left(\rho+\mu_0^2H_0^2\varepsilon_0\right)}{\left(\lambda'+2\mu'+K'\right)\alpha_0+\mu_0H_0^2},$$

$$a_{18}=\frac{\rho c_1^2}{\left(\lambda'+2\mu'+K'\right)\alpha_0+\mu_0H_0^2},\ a_{19}=\frac{2\rho c_1^2\left(\rho+\mu_0^2H_0^2\varepsilon_0\right)}{\left(2\mu'+2K'\right)\alpha_0-\beta_1'T_0p},$$

$$a_{20}=\frac{2K'\alpha_0}{\left(2\mu'+2K'\right)\alpha_0-\beta_1'T_0p},\ a_{21}=\frac{2K'c_1^2}{\gamma'\omega^{*2}},\ a_{22}=\frac{\rho jc_1^2}{\gamma'\alpha_0},\ a_{23}=\frac{\left(K'\alpha_0-\beta_1'T_0p\right)c_1^2}{\gamma'\omega^{*2}\alpha_0},$$

$$a_{24}=\rho c_1^2C_E,\ a_{25}=\frac{\beta_1'^2T_0\alpha_0}{\rho},\ a_{26}=\frac{R_aL_0\omega^*c_1}{T_0\delta t_p^2}.$$

Appendix 2

$$A_1=b_{12}+b_{13}-a_{20}a_{23},\ A_2=b_{12}b_{13}-a_{20}a_{23}k^2,\ A_3=\frac{b_{20}}{b_{14}},\ A_4=\frac{b_{21}}{b_{14}},\ b_{11}=k^2+a_{17}\omega^2,$$

$$b_{12}=k^2+a_{19}\omega^2,\ b_{13}=k^2+a_{21}+a_{22}\omega^2,\ b_{14}=K_1^*\left(1+\tau_v\omega\right)+K^*\omega^*\omega\left(1+\tau_T\omega\right),$$

$$b_{15}=\left(1+\tau_q\omega+\frac{\tau_q^2\omega^2}{2}\right)\omega^2a_{24},\ b_{16}=\left(1+\tau_q\omega+\frac{\tau_q^2\omega^2}{2}\right)\omega^2a_{25},\ b_{17}=a_{26}\left(1-\frac{t}{t_p}\right),$$

$$b_{18}=k^2b_{14}+b_{15},\ b_{19}=b_{12}b_{13}-a_{20}a_{23},\ b_{20}=b_{12}b_{13}-a_{20}a_{23}k^2,$$

$$b_{21}=b_{14}b_{11}+b_{18}+a_{17}b_{16},\ b_{22}=b_{18}b_{11}+a_{17}b_{16}k^2,\ b_{23}=\frac{b_{17}a_1}{b_{14}},$$

$$H_{1i}=-\left(\frac{a_{20}}{\lambda_i^2-b_{12}}\right)(i=1,2),\ H_{1i}=-\left(\frac{\lambda_i^2-b_{11}}{a_{17}}\right)(i=3,4),\ H_{2i}=-\lambda_iH_{1i}\ (i=1,2),$$

$$H_{2i}=-\iota k\ (i=3,4),\ H_{3i}=-\iota kH_{1i}\ (i=1,2),\ H_{3i}=-\lambda_i\ (i=3,4),$$

$$H_{4i}=\left(a_{13}\lambda_i^2+a_{14}k^2\right)H_{1i}-a_{15}\ (i=1,2),\ H_{4i}=-\left(a_{13}+a_{14}\right)k\iota\lambda_i\ (i=3,4),$$

$$H'_{5i}=\left(a_{11}-a_{12}\right)\iota k\lambda_iH_{1i}\ (i=1,2),\ H'_{5i}=\lambda_i^2a_{11}-a_{12}k^2-\alpha_0H_{1i}\ (i=3,4),$$

$$H_{6i}=-\lambda_ia_{16}\ (i=1,2),\ \varepsilon_1=\left(\frac{b_{23}\delta^4}{1-A_3\delta^2+A_4\delta^4}\right),\ \varepsilon_2=\frac{\varepsilon_1}{a_{18}}\left(\frac{1-b_{11}\delta^2}{\delta^2}\right),$$

$$f_1(z,t)=\exp\left(\frac{z-h/2}{\delta}-\frac{t}{t_p}\right),\ \varepsilon_3=\iota k\varepsilon_1,\ \varepsilon_4=\frac{\varepsilon_1}{\delta},\ \varepsilon_5=\frac{\left(a_{13}+a_{14}\right)\iota k\varepsilon_1}{\delta},$$

$$\varepsilon_6=\frac{\left(1-a_{11}k^2\delta^2\right)\varepsilon_1}{\delta^2}-\varepsilon_2.$$

Appendix 3

$$\Delta = H_{62}\left(H_{51}L_2 - H_{41}L_1\right) + H_{61}\left(H_{52}L_2 - H_{42}L_1\right),\ \Delta_1 = H_{62}\left(M_1L_3 + M_2L_1 + M_3L_2\right),$$

$$\Delta_2 = H_{61}\left(L_1M_2 - L_2M_3 - L_3M_1\right),\ \Delta_3 = H_{14}\left(L_7M_3 - L_6M_2\right) + M_1\left(L_4H_{62} + L_5H_{61}\right),$$

$$\Delta_4 = H_{13}\left(L_6M_2 - L_7M_3\right) - M_1\left(L_8H_{62} + L_9H_{61}\right),\ L_1 = H_{13}H_{54} - H_{14}H_{53},$$

$$L_2 = H_{13}H_{44} - H_{14}H_{43},\ L_3 = H_{43}H_{54} - H_{44}H_{53},\ L_4 = H_{44}H_{51} - H_{41}H_{54},$$

$$L_5 = H_{42}H_{54} - H_{44}H_{52},\ L_6 = H_{51}H_{62} - H_{52}H_{61},\ L_7 = H_{41}H_{62} - H_{42}H_{61},$$

$$L_8 = H_{43}H_{51} - H_{41}H_{53},\ L_9 = H_{42}H_{53} - H_{43}H_{52},\ M_1 = \left(f^* - \varepsilon_2 f_1(z,t)\right)e^{-(\omega t + \iota kx)},$$

$$M_2 = -\varepsilon_5 f_1(z,t)e^{-(\omega t + \iota kx)},\ M_3 = -\varepsilon_6 f_1(z,t)e^{-(\omega t + \iota kx)},$$

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