MODELING AND STRESS CONCENTRATION ANALYSIS OF UNCOATED AND FGM COATED INCLUSIONS: A REVIEW

[1]Pushpa Rani[*], [2]Manjeet Singh, [3]Gyander Ghangas

[1]Department of Mathematics, NIILM University, Kaithal, Haryana, India
[2]Department of Mathematics, NIILM University, Kaithal, Haryana, India
[3]Mechanical Engineering Department, SRM Institute of Science and Technology, Delhi NCR Campus, Ghaziabad, India

Email ID: *[1]pushhparani@gmail.com, [2]dr.manjeet.jakhar@gmail.com, [3]ghangas01@gmail.com*

Accepted: 24.03.2023 **Published**: 01.04.2023

Keywords: Functionally Graded Material; Inclusions; Level Set Method; Extended Finite Element Method.

Abstract

The aim of this work is to present a review on modeling and stress concentration analysis around uncoated and the functionally graded material (FGM) coated inclusions subjected to different mechanical and thermal load conditions.

The literature review mainly focus on modeling and analysis of uncoated inclusions, introduction and applications of FGMs, Extended Finite Element Method (XFEM), stress concentration reduction analysis around homogeneous material coated and FGM coated inclusions. It is noticed from the work that FGM is a suitable materiel to reduce stress concentration around different kind of geometrical and material discontinuities such as holes, notches, inclusions material interfaces, etc. The power law function is observed most efficient FGM function in literature review as it has ability to vary gradation by varying its coefficient. The XFEM method is an efficient numerical method which can handle different geometrical and material discontinuities without a conformal mesh.

There is a wide scope of XFEM in modelling and analysis of stress concentration around FGM coated inclusions. Level set method nowadays an integral part of XFEM to handle the discontinuities. Level set function based FGMs are tested in literature for holes and found helpful in modeling a FGM layer around a complicated shape discontinuity as well as multiple discontinuities. The level set function based power law FGMs have a scope to be tested for stress concentration reduction around different inclusions.

Paper Identification

**Corresponding Author*

1. Introduction

Modern engineering and space applications need highly functional materials to fulfill the desired requirements without failure. It is a known fact that pure materials (single phase) such as metals, non-metals, plastics and ceramics have a limited scope in such applications. Therefore, great research effort has been applied by scientists and researchers to develop multiphase materials such as compounds, alloys and composites. Composite materials are the homogeneous/heterogeneous mixture of two or more materials and these can provide the benefit of all their constituent materials. Composite materials have a number of advantages over single-phase or pure materials. One of the advantages is that a composite material can be lightweight as well as strong. One can prepare a composite material as per the functional/desired requirements of a particular application by choosing appropriate constitutional materials (Staab, 1999 [5]). Modern manufacturing techniques can fabricate composite materials in desired shape and size. Composite materials generally have two constituent materials first one is base material known as matrix and second is reinforcement material. The reinforcement can be applied as fiber or particle form. By applying proper reinforcement in the matrix, one can significantly improve the material properties of composite material. The analysis of reinforcement enhanced matrix has become a research trend in recent years. In solid mechanics, the reinforcement i.e. fibers or particles are generally modeled as inclusions. The strength of the composite material is greatly affected by material properties and elastic behavior of fibers/particles. It has been noted from various research articles that the interfacial behavior and material properties mismatch of fiber/particle and matrix plays a vital role in performance of composite material (Yang and Pan, 2002 [1]). The material properties mismatch may develop the stress concentration in the vicinity of the inclusion interface and may lead to material failure near the interface.

One of the approaches used to diminish the interfacial stresses is coating, i.e., the inclusion has been coated with another material that may have suitable material properties. Recently, a new branch of material was introduced known as functionally graded materials (FGMs). These materials are kind of composite materials having varying material phases along the physical dimension(s) and have functional material properties with particular dimension(s) (Ruys and Sun 2002 [2]). The FGM can be a suitable coating material to diminish the interfacial stresses of inclusions Lee, 2013 [6]).

The analytical solution of inclusion problems especially multiple coated inclusions problem is highly cumbersome and one can obtain it for simple cases. Therefore, numerical methods are suitable for such problems. There is a long list of numerical methods available in the literature. However, the Finite Element Method (FEM) is often used in the literature for such kinds of problems due to its simplicity, applicability, and capability to handle and solve complex problems. Conventional FEM has numerous advantages over other numerical methods, but it has some drawbacks also like it needs a conformal mesh with geometry, requirement of significantly refined mesh near the discontinuity and high mesh dependency. The Extended Finite Element Method (XFEM) was introduced by Belytschko and Black (1999) [3] and Moes et al. (1999) [4] to overcome the drawbacks of conventional FEM. XFEM does not require a mesh conformal with geometry and it can model the behavior of material and discontinuity within the element by using special enrichment functions.

2. Modeling and analysis of inclusions

Wilson (1964) [7] theoretically investigated the stresses around rigid circular inclusion in an infinite strip. The strip has finite width and infinite length subjected to far field tension of unit magnitude. Hoop stresses around the rigid inclusion were evaluated using Airy stress functions and plane stress conditions. Shioya (1967) [8] solved the problem of elastic circular in a semi-infinite thin plate using bipolar coordinate system and perturbation technique. The thin plate was assumed infinite under uniaxial tension of 1MPa and inclusion was assumed at a finite distance from the side edge. He found that the stress around the elastic inclusion depends on material properties of inclusion, inclusion size and inclusion distance. Stresses found concentrated near inclusion for soft inclusions. Shioya (1971) [9] theoretically investigated the problem of stress concentration around a pair of circular inclusions. The inclusions were considered along x-axis in an infinite plate subjected to uniaxial tension in x and y-direction. It was observed that the stresses in y-direction loading are more than in x-direction loading. The distance between inclusions, Young's modulus ratio and loading condition significantly affect the stress concentration. Mizushima et al. (1979) [10] analyzed the stress behavior of an infinite plate with a hole having elastic inclusion fitted in the hole. The inclusion and hole interface was considered frictionless and non-cemented. The uniaxial tensile and compressive far field loads were considered for analysis. The stress concentration was observed to be slighter less than the hole without inclusion case. Shioya et al. (1981) [11] calculated the thermal stresses near a row of circular inclusions. The inclusions were assumed in an infinite plate subjected to steady state temperature. It was observed that the maximum stress depends on temperature, inclusion size and distance between inclusions. Shioya and Tsuruno (1986) [12] numerically analyzed the stress concentration around a pair of elastic inclusions in an infinite plate subjected to steady state temperature. They utilized the perturbation method to calculate the stresses. The distance between heat source and inclusion was found most significant to the stress concentration.

Qiu and Weng (1991) [13] investigated the effect of inclusion shape on tensile strength of the composite. They observed that the disc shape inclusions with higher aspect ratio show higher tensile strength. Wang and Wang (1992) [14] estimated the effect of inclusion shape on viscoelastic behavior of composite. They utilized Eshelby-Mori-Tanak method to calculate the tensile strength and maximum strain. Fibers were found superior in terms of axial tensile strength however, for transverse tensile strength disk reinforcement was observed as superior. Zhang and Katsube (1995) [15] proposed a hybrid FEM for stress analysis around randomly placed inclusions. They observed that the FEM is suitable for stress concentration analysis and hybrid FEM was found more efficient with less number of elements. Chao and Shen (1997) [16] presented a thermoelastic solution for stresses around circular inclusion in an infinite plate. A point heat source was assumed inside the inclusion. The stresses were calculated by Laurent series and complex potential. Chao and Young (1998) [17] provided a solution for an infinite plate containing multiple circular inclusions. The plate was considered in anti-plane shear loading. They used method of successive approximations to calculate the stress distribution around the inclusions. Noda and Matsuo (1998) [18] investigated the interaction effect of two elliptical inclusions in an infinite plate under tension and shear. The body force method was used to calculate the stresses and stress concentration in plate. Results for different situations have been presented such as: single hole, single inclusion, two holes and two inclusions under tensile and shear loading.

You and Long (1998) [19] presented a stress analysis of coated inclusion under thermal load conditions. The elastoplastic behavior of coated inclusion was explained. The coating shows a significant effect on elastoplastic behavior of inclusion. Yang and Pan (2002) [20] analyzed the elastic behavior of quantum dot in multi-layered semiconductor. The quantum dot was assumed as inclusion and the boundary element method was used to analyze strain variation around quantum dot. Lubarda (2003) [21] derived the eigen-strain around the circular inclusion subjected to anti-plane shear loading. The stress behavior around inclusion was analyzed in couple stress elasticity. They found that the couple stresses affect the stress concentration significantly. The stress concentration was found reduced for softer inclusions and higher for stiffer inclusions. Legros et al. (2004) [22] investigated the stress behavior of randomly plated multiple circular inclusions in elastic half-plane under uniaxial tension. The complex Fourier series method was used. The solution of the inclusion touching each other and boundary was also presented. Pan (2004) [23] solved the Eshelby problem of polygonal inclusion using the body force method. The solution was presented for anisotropic piezoelectric half as well as full planes. They found inclusion shape and inclusion material significantly affect the piezoelectric field. Horibe and Tsuchida (2007) [24] presented an analytical solution of circular inclusion in a strip subjected to external pressure from both sides. They analyzed both perfectly bounded and slipping inclusions. The stiffness of inclusion affects the stresses around it. Higher stiffness of inclusion shows lower stress concentration. The distance between inclusion and strip edge also affects stresses significantly.

Allazadeh et al. (2009) [25] analyzed the stress concentration around Al_2O_3 inclusion in a steel matrix. Finite Element analysis has been performed to evaluate the stresses due to thermally induced strain. The relation of stress concentration with inclusion rigidity was presented. The inclusion rigidity and the inclusion size factors substantially affect stress concentration. Luo and Gao (2009) [26] presented Faber series method for different shape inclusions in an infinite plate. The stress distribution around triangular, square, circular and elliptical inclusions was reported for uniaxial tensile load condition. The stress found concentrated near the corners of triangle and square. Further, soft inclusions show high stress concentration and stiffer inclusion has lower stress concentration. Janeiro and Einstein (2010) [27] experimentally investigated the stresses and fracture behavior around single and two inclusions. Multiple configurations of inclusions were studied in compressive load condition. For single inclusion, four shapes namely hexagon, square, circle and diamond were investigated. For double inclusions, a pair of circular and square inclusions were analyzed for stress concentration and crack generation. Sudak (2013) [28] applied a complex variable technique to solve the stress behavior around triangular and diamond shape inclusions. The shear load in anti-plane was considered for analysis. Soft and hard inclusions were tested for stress distribution and stress concentration. They observed that the bonding condition has a meaningful effect on stress concentration. Misseroni et al. (2014) [29] investigated the stiff rectangular and rhombus inclusions in a soft infinite plate using photoelasticity. They observed that the singularity due to inclusion is stronger in linear elastic mode-II fracture than in mode-I fracture. The singularity is more for acute inclusion angles than obtuse inclusion angles. The shape of inclusion greatly influences the stress field in plate. Kang et al. (2015) [30] analyzed the stress field around closely located inclusions. When inclusions are closely located, the stresses between these become significantly high. The distance between inclusions affects stress field and stress concentration significantly.

Zhang et al. (2016) [31] applied distributed dislocation method to solve the problem of circular inclusion in a finite plate subjected to tensile load in both axial directions. Effects of stiffness of inclusion and finiteness of plate on stress concentration were reported. The stress concentration first reduces with increase in stiffness of inclusion, one at equal stiffness of inclusion with plate and starts increasing with further increase in inclusion stiffness. Bigger inclusion shows higher stress concentration and size effect vanishes when the plate size is significantly higher than inclusion size. Lee et al. (2016) [32] investigated the problem of circular, elliptical and pentagram inclusion in an elastic half plane using complex variable method and principle of superposition. The stress fields were presented for different inclusion and different load conditions. Chen (2019) [33] investigated Eshelby's elliptical inclusion. The elliptical inclusion was considered in an infinite plate under remote load conditions. The remote load conditions were assumed as uniaxial tensile, biaxial tensile and shear. Displacement and stress fields near and within the inclusion were computed using complex potentials. Miao and Zhao (2021) [34] described the situations of asymptotic nature of the stress concentration when the distance between two interfacial boundaries of inclusions was assumed as zero. Results of the study provided the equations to solve the problem of stress concentration of above-mentioned conditions i.e. two touching inclusions. Hutsaylyuk et al. (2022) [35] proposed a numerical method for analyzing the problem to determine the mechanical field in the composite structure under the combination of force and dislocation. A multilayer inhomogeneous ribbon type deformable inclusion was utilized for the analysis. The method was found effective to provide clarification for different problems related to elastic behavior of inclusions within the finite dimensions. Salasiya and Sundaram (2022) [36] considered different inclusion shapes in an infinite plate for analysis such as elliptical, super elliptical, hexagonal and square. It was observed that the sharp corner inclusions have higher stress concentration than smooth corner inclusions. Zhou et al. (2022) [37] investigated the brittle material's mechanical behavior for the effects of infilling composition under uniaxial compression. The results of the study revealed that the infilling and the size of inclusion affect the process of crack and failure significantly in brittle materials.

3. Functionally Graded Material (FGM)

Modern space and engineering applications need high functional materials that can fulfill the desired requirement without failure. It is a known fact that pure materials (single-phase) such as metals, non-metals, plastics and ceramics have a limited scope in such applications. Therefore, scientists and researchers have applied great research effort to develop multiphase materials such as compounds, alloys and composites [1]. Composite materials are the homogeneous/heterogeneous mixture of two or more materials and can benefit all their constituent materials. Composite materials have several advantages over single-phase or pure materials. One of the advantages is that a composite material can be lightweight as well as strong. One can prepare a composite material as per the functional/desired requirements of a particular application by choosing appropriate constitutional materials [2]. Modern manufacturing techniques can fabricate composite material for desired shape and size.

In 1980s, Japanese material scientists invented a new kind of composite material known as Functionally Graded Material (FGM) (Yamanoushi et al. 1990 [38]). FGM is a material that has varying material properties with its dimensions. It can be achieved by varying its constituents which causes smooth variation in material properties. The varying material properties make FGM a potential material candidate for many engineering and space applications where conventional pure materials and composites fail to offer the desired requirements (Koizumi and Niino 1995 [39]). Initially, FGM was designed for a thermal barrier in a fusion reactor as well as an aerospace outer structure (Hirai and Chen 1999 [40]). FGM was a replacement of conventional metal ceramic laminated composite thermal barrier. At very high temperature, metal and ceramic behave differently due to their different thermal properties and prone to fail at interface of laminated composite. This mismatch in material properties can be omitted by applying FGM. It can be a mixture of same metal and ceramic with varying volume fractions, which has no instant property mismatch (Uemura et al. 2003 [41]). Apart from thermal barrier, FGM can be applied in many other applications as it has smooth stress distribution, high fracture resistance, low residual stresses, etc. (Birman and Byrd 2007 [42]). Niino and Meda

(1990) [43] reported the fabrication of SiC/C FGM by chemical vapour decomposition technique. The fabricated FGM found smooth stress distribution wen subjected to sever thermal environment. The life of prepared specimen also observed higher than the conventional material.

Koizumi and Niino (1995) [39] revealed that FGM could be suitable material for energy conversion due to its varying properties. Hirai and Chen (1999) [40] presented a review on developments of FGMs. They reported people trying to develop new/modified techniques to produce bulk FGMs. Powder metallurgy, depositions techniques, spray coating and centrifugal casting have been explored in pursuit of producing bulk FGM. Birman and Byrad (2007) [42] reported a review on theoretical and application aspects of FGMs. They summarized that the Self-consistent and MoriTanaka are more popularly used to model the behavior of FGMs. People were considering FGM as locally homogeneous material but heterogeneous on a long dimension scale. FGM has varying material properties and these properties affect the stress field significantly. Improper tailoring of material may cause more stress concentration instead of reducing it. Further, they also suggested that the computer-assisted additive techniques were more convenient to produce bulk FGM. Jha et al. (2013) [44] discussed the mathematical and mechanics aspects of FGMs. The common gradation function people used are linear, exponential and power low functions. Gupta and Talha (2015) [45] reviewed the modeling of FGMs. They reported their review on the following aspects (i) applications (ii) processing techniques (iii) gradation functions (iv) kinematics (v) stability analysis (vi) bending (vii) vibration and (viii) homogenization schemes. They observed that powder metallurgy was the most frequently used technique to prepare bulk FGM.

Kim and Paulino (2002) [46] propose a FEM for FGMs. Their graded finite elements produce more accurate results than conventional finite elements. In graded finite elements, the material properties also vary with the standard partition of unity finite element function. In a numerical example having geometric cutouts, they found FGM can have ability to change the stress field around the cutouts. A properly tailored FGM can offer lower stress concentration, whereas an ill-tailored material can increase the stress concentration. Kubair and Bhanu-chander (2008) [47] analyzed the stress concentration in a FGM plate with hole. The hole was considered of circular shape in the center of infinite plate. The load condition was assumed uniaxial tensile at far field. The FGM was assumed to vary with exponential and power law functions in different directions such as x, y and radial direction. They observed that if Young's modulus $E(x,y)$ of FGM was assumed to increase away from the circular hole in the radial direction, the stress concentration was significantly reduced irrespective to geometric parameters. However, vice versa may happen if Young's modulus decreases with radial direction. Yang et al. (2010) [48] investigated the stresses in an exponential FGM infinite plate weakened by the circular hole. They assumed different variations of Young's modulus in the plate. The stress analysis was presented for different tensile and shear loads. The complex variable method was used for analysis and FGM was modeled by piecewise homogeneous material layers. The stress concentration was significantly reduced by increasing E(*x*,*y*) away from boundary of the hole. Further, a very less effect of varying Poisson's ratio has been noted on stresses and which can be neglected. Mohammadi et al. (2011) [49] formulated the relationship between stiffness and stress concentration. They studied the effect of exponential FGM stiffness on stresses near circular hole. The stress field was analyzed for biaxial tension and shear loads

Yang et al. (2012) [50] studied the effect of hole size in finite FGM plates subjected to different load conditions. Stresses were analyzed using complex variable method. The size effect of hole can significantly affect the stress concentration, and FGM can greatly help to reduce the stress concentration. Kubair (2013) [51] reported a solution for stress field in FGM plate subjected to anti-plane shear stresses. The FGM plate was weakened by the circular hole. Radially graded FGM has lower stresses than non FGM plate. Sburlati et al. (2013) [52] tested the FGM layer around circular hole and noticed a significant reduction in SCF. They found that the stress field near the cutouts can be tailored by adding a FGM layer near the cutout. Enab (2014) [53] studied the effect of grading direction and grading function on stresses around elliptical hole using FEM. The elliptical hole was assumed in an infinite FGM plate subjected to biaxial tension. He observed angular direction graded FGM has least effect on the stress concentration and radial direction gradation has highest effect on stress concentration.

Gouasmi et al. (2015) [54] investigated the stress field around semicircular notch in a FGM panel. The load was considered uniaxial tensile in *y*-direction and the notch was assumed along *x*-axis. The stresses were observed to be reduced by power law FGM. Yang and Gao (2016) [55] analyzed the behavior of varying Young's modulus on stress concentration around elliptical hole. They assumed elliptical hole enclosed by a FGM layer in a plate loaded by different load conditions. Goyat et al. (2017, 2018) [56-57] reported stress concentration analysis around rounded rectangular hole and pair of circular holes. The holes were assumed reinforced by FGM layer. The different mechanical load conditions were assumed for analysis. They used XFEM for analysis. A more than 50% reduction in Stress concentration was noticed in both cases. Goyat et al. (2018) [58] tested different FGM functions in pursuit of getting optimal FGM function to reduce stress concentration. They tested linear, power law, exponential and sigmoidal functions for circular hole. The power law function graded in radial direction was observed as optimal function. Goyat et al. (2021) [59] proposed a level set function based power law FGM for pair of holes. They stated that level set function could be used to grade FGM around multiple discontinuities with a single function. Jaiswal et al. (2021) [60] studied the stress field near the rounded rectangular hole. The hole was reinforced by FGM layer and far field tension was applied for analysis. They designed FGM layer to get least stress concentration. Yang et al. (2021) [61] experimentally examined and validated the stress concentration of FGM coated circular hole. Goyat et al. (2022) [62] proposed an inverse distance weighted function based FGM for circular hole in a finite plate.

4. Stress Concentration Reduction around Inclusions by Coating

Shen et al. (2010) [63] modeled piezoelectric material with inclusion for kind of electrical and mechanical loads. The analysis was presented for the three phase piezoelectric composite material having an arbitrary shape coated inclusion embedded in an infinite size matrix. The stresses were presented for circular, elliptical, square, rectangle and diamond shapes. Luo and Gao (2011) [64] investigated the stress near arbitrary shape inclusions coated by a homogeneous material using Faber and Laurent series. Wang and Chen (2013) [65] investigated the induced thermal stresses around a three-phase arbitrarily shaped inclusion bonded to an infinite size matrix with an interphase layer subjected to uniform internal hydrostatic type thermal stress. They noted that the thermal stresses are induced by the change in temperature can be uniform around the elliptical or hypotrochoidal shape inclusion if the thickness of coating/interphase layer designed properly. Fang et al. (2015) [66] investigated the effect of interface energy on the electro-mechanical properties of a piezoelectric composite having arbitrary shape nano-inclusion subjected to anti-plane shear loading using a combination of complex variable method, Laurent series expansion technique and conformal mapping method. The inclusion was coated with a third phase and it was observed that the effect of electro-mechanical properties is more on stress field at coating interface rather than inclusion interface.

5. Stress Concentration Reduction around Inclusions by FGM Coating

Ru (2000) [67] suggested grading FGM stepwise in complex variable theory. He solved the problem of stress field near FGM coated circular material inhomogeneity subjected to thermo-mechanical loading. A significant effect of coating was observed on the stress concentration. Li

(2000) [68] analyzed the thermo-elastic behavior of composite material using multi-inclusions model for inclusions with FGM interface. The intermetallic matrix and SiC reinforcement based composited have been studied for thermal load conditions and it has been noted that the FGM interface significantly affects the stress field. You and You (2004) [69] present a thermal stress analysis for composite having varying material properties. Plane strain results were obtained by applying the theory of elasticity. Different variations of volume fraction were proposed to vary material properties with different functions. Zhang et al. (2006) [70] proposed a numerical model in order to compute the residual stresses due to thermal loading near cylindrical geometry with and without multilayer coating. The magnitude of residual stress can be controlled by gradation function and its variables such as thickness, elastic modulus, etc.

Marbini and Shodja (2007) [71] evaluated the stress behavior near spherical as well as cylindrical inclusion having FGM coated interface in an infinite matrix. The result of continuous FGM coating has been obtained by assuming it as a piece wises continuous functions with a finite number of jumps or steps. For accurate solution, these number of steps are to be large enough. They examined the effect of interface conditions, inclusion stiffness, gradation of shear and thermal expansion coefficient. Wang et al. (2008) [72] analyzed the effect of circular inclusion in a FGM plane when subjected to anti-plane eigen-strain. The exponential variation in shear modulus was considered for analysis. Displacement and strain fields around and inside the inclusion were obtained and it was observed that these fields are intrinsically nonuniform due to inhomogeneity. Abbasion et al. (2009) [73] presented a micromechanics model to evaluate the stresses around a FGM coated fiber in an infinite matrix subjected to body force. Various interface conditions have been analyzed with fiber loading as a distributed body force. The power law based FGM gradation function has been used to grade the coating material in radial direction. The optimal FGM settings can reduce the stress concentration near the fiber effectively. Wang and Schiavone (2012) [74] studied the effect of multi coating of displacement and stress field near the inclusion. The inclusions of triangle, square and pentagon shapes were assumed for analysis. The multi coating was considered in such a way that the properties of each coat changed with a specific function. Different configuration of multi coating was tested to obtain a uniform stress field around inclusion.

Artioli and Bisegna (2013) [75] calculated the shear modulus when matrix contains the fibers coated with FGM. Different variations in FGM material properties were considered for analysis. They observed that the stress concentration of shear stress near fiber could be lowered by choosing the right grading parameter. Yang and Gao (2013) [76] evaluated the stress concentration of thermal stresses around uncoated and FGM coated circular inclusion for 2D panel in uniform heat flux. They found that by controlling the FGM parameters one can reduce the stress concentration around inclusion. Yang and Gao (2014) [77] analyzed the thermal stresses in an infinite 2D matrix with a radially graded FGM coated inclusion. The shape of inclusion was considered as circular for simplicity and the load condition was assumed as uniform heat flow. They used complex variable method in combination with piecewise homogeneous layers approach for analysis. The thermal stress concentration was found to be significantly affected by the matrix size. Infinite matrix has lesser stress than finite matrix. FGM interface can control the stresses if graded appropriately. Sabiston et al. (2016) [78] proposed a micromechanics model for a FGM coated fiber in a matrix. The fiber was assumed of circular in shape with annulus FGM coating. The material property variation was considered with linear, quadratic and exponential functions. They analyzed the effect of fiber coating on the material properties of composite and also calculated its effect on stress concentration around the fiber. Yang et al. (2018) [79] explored the theory of complex variables to evaluate the stresses in an infinite matrix having arbitrary shape inclusion coated with normal direction graded FGM subjected to different load conditions. The piecewise homogeneous layers have been used to model the FGM and conformal mapping technique has been used to model the shape of inclusion within complex variable theory.

6. Extended Finite Element Method

The applicability of analytical methods is limited to simple problems. Complexity in terms of geometry and variables makes analytical methods cumbersome. Numerical methods prove their applicability in such cases. Numerical methods can handle complexity to a great extent. Different engineering application involves complex geometry which requires numerical methods to model it. There are several numerical methods available for engineering problems such as Finite Element Method (FEM), Boundary Element Method, Finite Difference Method, etc. But, FEM proves itself as a prominent method as it can handle a variety of engineering problems and can handle geometrical complexity. Its applicability, simplicity and efficiency make it more popular in solving engineering problems. FEM can handle problems of Civil Engineering, Automobile Engineering, Aerospace Engineering Mechanical Engineering, Geomechanics, Biomechanics, Electromagnetics and many more.

In this method, the geometry can be divided into small finite geometries known as finite elements. The quality of result depends on the element size and numbers. A higher number of elements i.e. fine element mesh shows a good quality results and vice versa. However, FEM has a variety of advantages over other numerical methods it has a few drawbacks also. These are as follows: FEM required conformal meshing integrally to model the discontinuities and interfaces accurately. If some problem has multiple discontinuities, it will require highly refined mesh near each discontinuity. Further, moving discontinuities required frequent re-meshing at each stage. These will make computation cumbersome and time consuming. The singularity behavior of discontinuity needs to be model accurately by some approximation function. However, in finite elements singularity must be a part of the mesh (Yazid et al. (2009) [80]). The Extended Finite Element Method (XFEM) was coined by Belytschko et al. (1999) [3] and Moes et al. (1999) [4] to model the discontinuities without conformal meshing. The XFEM is capable to overcome the limitations of FEM such as conformal mesh, frequent re-meshing, highly denser mesh near discontinuity, discontinuity propagation, etc. It handles the discontinuity by enriching the nodes locally. The enrichment function defines the behavior of discontinuity to the mesh. These features allow XFEM to model the following problems efficiently over conventional FEM: (i) crack analysis and propagation (ii) moving discontinuities (iii) phase transformations (iv) inclusions and material interfaces (v) holes and notches (vi) surface flow (vii) interaction of fluid and structure (viii) contact modeling (ix) grain boundaries and (x) boundary layers.

There are several Advantages of XFEM over FEM, these are: (i) the mesh is prepared to describe outer boundaries of domain and the discontinuities are independent of mesh. (ii) Same mesh can be used to model a different kinds of discontinuities (iii) Discontinuities can be modeled independently and accurately by using enrichment functions (iv) Movement of discontinuities is independent of mesh and there is no requirement of re-meshing after movement of discontinuities (v) Difficult shape discontinuities can be modeled efficiently (vi) Simple mesh elements can be used with fast convergence rate. Dolbow et al. (2000) [81] presented XFEM implementation for different types of cracks in a plate. They presented the crack independent to the finite element mesh. They discussed the numerical integration scheme for enriched and non-enriched elements. Daux et al. (2000) [82] discussed the implementation of XFEM for cracks and holes. They solved the problem of multi branch cracks and hole with crack under different load conditions.

Stolarska et al. (2001) [83] utilized the level set method to handle the crack inside the finite element mesh. Level set method was coined by Osher and Sethian (1988) [84]. Level set method offers a single higher order function to define the boundaries of different discontinuities with the mesh. Different enrichment functions can also be expressed with the help of level set function. Sukumar et al. (2001) [85] proposed the level set method to handle the hole, notches and inclusions inside finite element mesh. They calculated a sign distance function from the boundary of discontinuity. To define the boundary of hole or inclusion, they used zero sign distance functions. The hole region or the inclusion region can be described by the negative sign distance function. The matrix region can be represented by the positive sign distance function. Stazi et al. (2003) [86] reported the application of higher order elements in modeling of curved interfaces such as curved cracks. Yazid et al. (2009) [80] reviewed different works on XFEM and found that the XFEM can handle a variety of discontinuities without conformal meshing. The material inhomogeneity and material interfaces can also be handled by the XFEM efficiently. Level set method is one of the prominent tools to represent the geometrical and material discontinuities independent of mesh. Goyat et al. (2017) [56] utilized XFEM to model the FGM plate. They used graded finite elements to model the FGM around the hole. The level set method was used to handle hole as well as FGM Layer around the hole. Goyat et al. (2022) [87] presented an implementation of XFEM in a FGM plate having hole as well as cracks. The far field load was considered in different directions. The summarized that XFEM can handle geometrical and material discontinuities efficiently.

7. Conclusions

FGM is a material that has varying material properties with its dimensions. It can be achieved by varying its constituents which causes smooth variation in material properties. The varying material properties make FGM a potential material candidate for many engineering and space applications where conventional pure materials and composites fail to offer the desired requirements. From literature survey it has been observed that FGM is a suitable materiel to reduce stress concentration around different geometrical and material discontinuities. The work found in the literature is manly associated with FGM application in reduction of stress concentration near holes and notches and very less work has been observed on FGM coating around inclusions to reduce stress concentration. There is a wide scope of research in modeling and stress concentration analysis of FGM coated inclusions.

The gradation function has a significant effect on stress concentration around different geometrical and material discontinuities. In literature following grading function have been used most often (i) linear function, (ii) power law function, (iii) sigmoid function and (iv) exponential function. The power law function is noticed most efficient in literature survey as it has ability to vary gradation by varying its coefficient.

The analytical solution of composite shapes such as rounded rectangular shape and multiple inclusion seems cumbersome and time consuming. Further, FGM is non-homogeneous in nature which add more difficulty if FGM coated inclusions have been analyzed with analytical methods. However, numerical methods can handle such problems efficiently. The XFEM method is one proven numerical method to handle different geometrical and material discontinuities. A vast literature has been available on XFEM implementation for inclusions however literature on FGM coated single and multiple inclusions is scantly available. There is a scope to apply XFEM for modelling and analysis of stress concentration around FGM coated inclusions.

Level set method nowadays an integral part of XFEM to handle the discontinuities without conformal mesh. It produces sign distance function from the discontinuity boundary. The boundary can be traced within mesh by zero value level set sign distance function. As a single level set function can handle multiple inclusions and it is obtained in normal direction to the inclusion boundaries therefore, it is suitable to generate FGM function. Level set function based FGM have been tested in literature for holes and have scope to be tested for different inclusions.

References:

[1] Matthews, F. L., Davies, G. A. O., Hitchings, D., & Soutis, C. "*Finite element modelling of composite materials and structures"* Elsevier, 2000.

[2] Ruys, A. and Sun, D. "*Functionally graded materials (FGM) and their production methods*" 2002. URL: http://www.azom.com/article.aspx.

[3] Belytschko, T. and Black, T. "*Elastic crack growth in finite elements with minimal remeshing*", International journal for numerical methods in engineering, 1999, vol. 45(5), pp. 601-620.

[4] Moes, N., Dolbow, J. and Belytschko, T. "*A finite element method for crack growth without remeshing*", International journal for numerical methods in engineering, 1999 vol. 46(1), pp.131-150.

[5] Staab, G. "*Laminar composite*'', London: Butterworth-Heinemann, 1999.

[6] Lee, P.H. "*Fabrication, characterization and modeling of functionally graded materials*", (Doctoral dissertation, Columbia University), 2013.

[7] Wilson, R. G. "*The stresses in an infinite strip containing a circular inclusion*", Mathematika, 1964, vol. 11(2), pp. 155-170.

[8] Shioya, S. "*On a semi-infinite thin plate with a circular inclusion under uniform tension*", Bulletin of JSME, 1967, vol. 10(37), pp.1-9.

[9] Shioya, S. "*On the tension of an infinite tin plate containing a pair of circular inclusions*", Bulletin of JSME, 1971, vol. 14(68), pp.117-126.

[10] Mizushima, I., Hamada, M. and Kusano, N. "*Tensile and compressive stress problems for a circular disk in an infinite plate*", Bulletin of JSME, 1979, vol. 22(171), pp.1175-1181.

[11] Shioya, S., Matsuda, M. and Takahashi, K. "*On the Thermal Stresses of an Infinite Plate with an Infinite Row of Circular Inclusions unber the Steady State of Temperature: Case of an Infinite Plate Having an Infinite Row of Pairs of Heat Sources*", Bulletin of JSME, 1981, vol. 24(188), pp.305-313.

[12] Shioya, Shunsuke, and Syozo Tsuruno. "*Thermal Stresses in an Infinite Plate With a Pair of Circular Inclusions Under Steady-State Temperature (Case of Infinite Plate Having a Pair of Heat Sources)*", Journal Of Thermal Stresses, 1986, vol. 9(1), pp. 1-18.

[13] Qiu, Y.P. and Weng, G.J. "*The influence of inclusion shape on the overall elastoplastic behavior of a two-phase isotropic composite*", International Journal of Solids and Structures, 1991, vol. 27(12), pp.1537-1550.

[14] Wang, Y. M., and Weng G. J. "*The influence of inclusion shape on the overall viscoelastic behavior of composites*", Journal of Applied Mechanics, 1992, vol. 59(3), pp. 510-518.

[15] Zhang, J., and Katsube N. "*A hybrid finite element method for heterogeneous materials with randomly dispersed rigid inclusions*", International journal for numerical methods in engineering, 1995, vol. 38(10), pp. 1635-1653.

[16] Chao, C. K., and Shen, M. H. "*On bonded circular inclusions in plane thermoelasticity*", Journal of Applied Mechanics, 1997, Vol. 64, pp. 1000-1004.

[17] Chao, C. K., and Young, C. W. "*On the general treatment of multiple inclusions in antiplane elastostatics*", International journal of solids and structures, 1998, vol. 35(26-27), pp. 3573-3593.

[18] Noda, Nao-Aki, and Tadatoshi Matsuo. "*Singular integral equation method for interaction between elliptical inclusions*", ASME Journal of Applied Mechanics, 1998 Vol. 65, pp. 310-319.

[19] You, L. and Long, S. "*Effects of material properties of interfacial layer on stresses in fibrous composites subjected to thermal loading*", Composites Part A: Applied Science and Manufacturing, 1998, vol. 29(9-10), pp.1185-1192.

[20] Yang, B. and Pan, E. "*Elastic analysis of an inhomogeneous quantum dot in multilayered semiconductors using a boundary element method*", Journal of Applied Physics, 2002, vol. 92(6), pp.3084-3088.

[21] Lubarda, V. A. "*Circular inclusions in anti-plane strain couple stress elasticity*", International journal of solids and structures, 2003, vol. 40(15), pp. 3827-3851.

[22] Legros, B., Mogilevskaya, S.G. and Crouch, S.L. "*A boundary integral method for multiple circular inclusions in an elastic half-plane*", Engineering analysis with boundary elements, 2004, vol. 28(9), pp.1083-1098.

[23] Pan, E. "*Eshelby problem of polygonal inclusions in anisotropic piezoelectric full-and half-planes*", Journal of the Mechanics and Physics of Solids, 2004, vol. 52(3), pp.567-589.

[24] Horibe, T. and Tsuchida, E. "*Stresses in an elastic strip having a circular inclusion subjected to side pressure*", Journal of Solid Mechanics and Materials Engineering, 2007, vol. 1(1), pp.47-57.

[25] Allazadeh, M. R., Garcia, C. I., DeArdo, A. J. and Lovell, M. R. "*Analysis of stress concentration around inclusions due to thermally induced strain to the steel matrix*", ASTM International, 2009.

[26] Luo, J. C., & Gao, C. F. "*Faber series method for plane problems of an arbitrarily shaped inclusion*", Acta mechanica, 2009, vol. 208(3), pp. 133-145.

[27] Janeiro, Raymond P., and Herbert H. Einstein "*Experimental study of the cracking behavior of specimens containing inclusions (under uniaxial compression)*", International Journal of Fracture, 2010, vol. 164, pp. 83-102.

[28] Sudak, L. J. "*An irregular-shaped inclusion with imperfect interface in antiplane elasticity*", Acta Mechanica, 2013, vol. 224(9), pp. 2009-2023.

[29] Misseroni, D., Dal Corso, F., Shahzad, S. and Bigoni, D. "*Stress concentration near stiff inclusions: validation of rigid inclusion model and boundary layers by means of photoelasticity*", Engineering Fracture Mechanics, 2014, vol. 121, pp.87-97.

[30] Kang, H., Lee, H. and Yun, K. "*Optimal estimates and asymptotics for the stress concentration between closely located stiff inclusions*", Mathematische Annalen, 2015, vol. 363(3), pp.1281-1306.

[31] Zhang, J., Qu, Z., & Huang, Q. "*Elastic fields of a finite plate containing a circular inclusion by the distributed dislocation method*", Archive of Applied Mechanics, 2016, vol. 86(4), pp. 701-712.

[32] Lee, Y.G., Zou, W.N. and Ren, H.H. "*Eshelby's problem of inclusion with arbitrary shape in an isotropic elastic half-plane*", International Journal of Solids and Structures, 2016, vol. 81, pp.399-410.

[33] Chen, Y. Z. "*Complex variable method for equivalence of the elliptical inhomogeneity to Eshelby's elliptical inclusion under remote loading*", Zeitschrift für angewandte Mathematik und Physik, 2019, vol. 70(6), 1-16.

[34] Miao, C. and Zhao, Z. "*Singular analysis of the stress concentration in the narrow regions between the inclusions and the matrix boundary*", arXiv preprint arXiv:2109.04394, 2021.

[35] Hutsaylyuk, V., Piskozub, Y., Piskozub, L., & Sulym, H. "*Deformation and Strength Parameters of a Composite Structure with a Thin Multilayer Ribbon-like Inclusion*", Materials, 2022, vol. 15(4), pp. 1435.

[36] Salasiya, P., & Sundaram, N. K. "*Receding contact between an inclusion of generalized shape and a remotely-stressed plate*", International Journal of Solids and Structures, 2022, vol. 252, pp. 111826.

[37] Zhou, Changtai, Heping Xie, Jianbo Zhu, Zhihe Wang, Cunbao Li, and Fei Wang. "*Mechanical and fracture behaviors of brittle material with a circular inclusion: insight from infilling composition*", Rock Mechanics and Rock Engineering, 2022, vol. 55(6), pp 3331-3352.

[38] Yamanouchi, M., et al. "*Functionally Graded Materials Forum*", Proceedings of the First International Symposium on Functionally Graded Materials, 1990.

[39] Koizumi, M. and Niino, M. "*Overview of FGM research in Japan*", Mrs Bulletin, 1995, vol. 20(1), pp. 19-21.

[40] Hirai, T. and Chen, L. "*Recent and prospective development of functionally graded materials in Japan*", Materials Science Forum, 1999, vol. 308, pp. 509-514. Trans Tech Publications.

[41] Uemura, S., Noda, Y., Shinohara, Y. and Watanabe, Y. "*Development and application of functionally graded materials*", CMC Publishing, Tokyo, Japan, 2003.

[42] Birman, V. and Byrd, L.W. "*Modeling and analysis of functionally graded materials and structures*", Applied mechanics reviews, 2007, vol. *60*(5), pp.195-216.

[43] Niino, M. and Maeda, S. "*Recent development status of functionally gradient materials*" ISIJ International, 1990, vol. 30(9), pp.699-703.

[44] Jha, D.K., Kant, T. and Singh, R.K. "*A critical review of recent research on functionally graded plates*", Composite Structures, 2013, vol. 96, pp.833-849.

[45] Gupta, A. and Talha, M. "*Recent development in modeling and analysis of functionally graded materials and structures*", Progress in Aerospace Sciences, 2015, vol. 79, pp.1-14.

[46] Kim, J.H. and Paulino, G.H. "*Isoparametric graded finite elements for nonhomogeneous isotropic and orthotropic materials*", Journal of Applied Mechanics, 2002, vol. 69(4), pp.502-514.

[47] Kubair, D. V., & Bhanu-Chandar, B. "*Stress concentration factor due to a circular hole in functionally graded panels under uniaxial tension*", International journal of mechanical sciences, 2008, vol. 50(4), 732-742.

[48] Yang, Q., Gao, C.F. and Chen, W. "*Stress analysis of a functional graded material plate with a circular hole*", Archive of Applied Mechanics, 2010, vol. 80(8), pp. 895-907.

[49] Mohammadi, M., Dryden, J.R. and Jiang, L. "*Stress concentration around a hole in a radially inhomogeneous plate*", International Journal of Solids and Structures, 2011, vol. 48(3-4), pp.483-491.

[50] Yang, Q., Gao, C., & Chen, W. "*Stress concentration in a finite functionally graded material plate*", Science China Physics, Mechanics and Astronomy, 2012, vol. 55(7), 1263-1271.

[51] Kubair, D.V. "*Stress concentration factors and stress-gradients due to circular holes in radially functionally graded panels subjected to antiplane shear loading*", Acta Mechanica, 2013, vol. 224(11), pp.2845-2862.

[52] Sburlati, R. "*Stress concentration factor due to a functionally graded ring around a hole in an isotropic plate*", International Journal of Solids and Structures, 2013, vol. 50(22-23), pp.3649-3658.

[53] Enab, T.A. "*Stress concentration analysis in functionally graded plates with elliptic holes under biaxial loadings*", Ain Shams Engineering Journal, 2014, vol. 5(3), pp. 839-850.

[54] Gouasmi, S., Megueni, A., Bouchikhi, A.S., Zouggar, K. and Sahli, A. "*On the reduction of stress concentration factor around a notch using a functionally graded layer*", Materials Research, 2015, vol. 18(5), pp. 971-977.

[55] Yang, Q. and Gao, C.F. "*Reduction of the stress concentration around an elliptic hole by using a functionally graded layer*", Acta Mechanica, 2016, vol. 227(9), pp.2427-2437.

[56] Goyat, V., Suresh V. and Garg R. K. "*Reduction of stress concentration for a rounded rectangular hole by using a functionally graded material layer*", Acta Mechanica, 2017, vol. 228, pp. 3695-3707.

[57] Goyat, V., Suresh V. and Garg R. K. "*Reduction in stress concentration around a pair of circular holes with functionally graded material layer*", Acta Mechanica, 2018, vol. 229, pp. 1045-1060.

[58] Goyat, V., Verma, S., & Garg, R. K. "*On the reduction of stress concentration factor in an infinite panel using different radial functionally graded materials*", International Journal of Materials and Product Technology, 2018, vol. 57(1-3), pp. 109-131.

[59] Goyat, V., Verma, S., & Garg, R. K. "*Level Set function-based Functionally Graded Material for the reduction of maximum stresses around a pair of inclined unequal circular holes*", Strength, Fracture and Complexity, 2021, vol. 14(1), pp. 27-43.

[60] Jaiswal, P., Makin, S., Dubey, A. D., Ghangas, G., & Goyat, V. "*Analysis of stress concentration reduction around rounded rectangular slot with FGM ring*", Materials Today: Proceedings, 2022, vol. 50, pp. 1953-1957.

[61] Yang, Q., Cao, H., Tang, Y., Li, Y., & Chen, X. "*Experimental Investigation of Stress Distributions in 3D Printed Graded Plates with a Circular Hole*", Materials, 2021, vol. 14, pp. 7845.

[62] Goyat, V., Enab, T. A., Ghangas, G., Kadiyan, S., & Kumar, A. "*On stress concentration analysis of inverse distance weighted function based finite FGM panel with circular hole under biaxial loading*", Multidiscipline Modeling in Materials and Structures, 2022, vol. 18(4), pp. 708-733.

[63] Shen, M.H., Chen, F.M. and Hung, S.Y. "*Piezoelectric study for a three-phase composite containing arbitrary inclusion*", International Journal of Mechanical Sciences, 2010, vol. 52(4), pp.561-571.

[64] Luo, J.C. and Gao, C.F. "*Stress field of a coated arbitrary shape inclusion*", Meccanica, 2011, vol. 46(5), pp.1055-1071.

[65] Wang, X. and Chen, W. "*Three-phase inclusions of arbitrary shape with internal uniform hydrostatic thermal stresses*", Zeitschrift für angewandte Mathematik und Physik, 2013, vol. 64(4), pp.1399-1411.

[66] Fang, X.Q., Liu, H.W., Liu, J.X. and Nie, G.Q. "*Interface energy effect on electromechanical response of piezoelectric composites with an arbitrary nano-inclusion under anti-plane shear*", Acta Mechanica, 2015, vol. 226(7), pp.2323-2333.

[67] Ru, C.Q. "*A new method for an inhomogeneity with stepwise graded interphase under thermomechanical loadings*", Journal of elasticity, 2000, vol. 56(2), pp.107-127.

[68] Li, J.Y. "*Thermoelastic behavior of composites with functionally graded interphase: a multi-inclusion model*", International Journal of Solids and Structures, 2000, vol. 37(39), pp. 5579-5597.

[69] You, L. H., and X. Y. You. "*A unified numerical approach for thermal analysis of transversely isotropic fiber-reinforced composites containing inhomogeneous interphase*", Composites Part A: Applied Science and Manufacturing, 2005, vol. 36(6), pp. 728-738.

[70] Zhang, X.C., Xu, B.S., Wang, H.D., Jiang, Y. and Wu, Y.X. "*Prediction of three-dimensional residual stresses in the multilayer coating-based systems with cylindrical geometry*", Composites science and technology, 2006, vol. 66(13), pp. 2249-2256.

[71] Hatami-Marbini, H. and Shodja, H.M. "*Thermoelastic fields of a functionally graded coated inhomogeneity with sliding/perfect interfaces*", Journal of applied mechanics, 2007, vol. 74(3), pp.389-398.

[72] Wang, X., Pan, E. and Roy, A. K. "*A functionally graded plane with a circular inclusion under uniform antiplane eigenstrain*", Journal of Applied Mechanics, 2008, vol. 75, pp. 014501.

[73] Abbasion, S., Rafsanjani, A., Irani, N. and Farshidianfar, A. "*Stress analysis for a coated fiber embedded in an infinite matrix subjected to body force*", European Journal of Mechanics-A/Solids, 2009, vol. 28(4), pp.777-785.

[74] Wang, Xu, and Peter Schiavone, "*Multi-coating an inclusion of arbitrary shape to achieve uniformity of interior stresses*", Mathematics and Mechanics of Solids, 2013, vol. 18(2), pp. 218-227.

[75] Artioli, Edoardo, and Paolo Bisegna, "*Effective longitudinal shear moduli of periodic fibre-reinforced composites with functionally-graded fibre coatings",* International Journal of Solids and Structures, 2013, vol. 50(7-8), pp. 1154-1163.

[76] Yang, Q. and Gao, C.F. "*Non-axisymmetric thermal stress of a functionally graded coated circular inclusion in an infinite matrix*" Mechanics Research Communications, 2013, vol. 50, pp.27-32.

[77] Yang, Q. and Gao, C. "*Thermal stresses around a circular inclusion with functionally graded interphase in a finite matrix*", SCIENCE CHINA Physics, Mechanics & Astronomy, 2014, vol. 57(10), pp.1927-1933.

[78] Sabiston, T., Mohammadi, M., Cherkaoui, M., Lévesque, J., & Inal, K. "*Micromechanics for a long fibre reinforced composite model with a functionally graded interphase*", Composites Part B: Engineering, 2016, vol. 84, pp. 188-199.

[79] Yang, Q., Zhu, W., Li, Y. and Zhang, H. "*Stress field of a functionally graded coated inclusion of arbitrary shape*", Acta Mechanica, 2018, vol. 229(4), pp.1687-1701.

[80] Yazid, A., Abdelkader, N. and Abdelmadjid, H. "*A state-of-the-art review of the X-FEM for computational fracture mechanics*", Applied Mathematical Modelling, 2009, 33(12), pp.4269-4282.

[81] Dolbow, J., Moës, N. and Belytschko, T. "*Modeling fracture in Mindlin–Reissner plates with the extended finite element method*", International journal of solids and structures, 2000, vol. 37(48-50), pp.7161-7183.

[82] Daux, C., Moës, N., Dolbow, J., Sukumar, N. and Belytschko, T. "*Arbitrary branched and intersecting cracks with the extended finite element method*", International journal for numerical methods in engineering, 2000, vol. 48(12), pp.1741-1760.

[83] Stolarska, M., Chopp, D.L., Moës, N. and Belytschko, T. "*Modelling crack growth by level sets in the extended finite element method*", International journal for numerical methods in Engineering, 2001, vol. 51(8), pp.943-960.

[84] Osher, S. and Sethian, J.A. "*Fronts propagating with curvaturedependent speed: algorithms based on Hamilton-Jacobi formulations*", Journal of computational physics, 1988, vol. 79(1), pp.12-49.

[85] Sukumar, N., Chopp, D. L., Moës, N., & Belytschko, T. "*Modeling holes and inclusions by level sets in the extended finite-element method*", Computer methods in applied mechanics and engineering, 2001, vol. 190(46-47), pp. 6183-6200.

[86] Stazi, F.L., Budyn, E., Chessa, J. and Belytschko, T. "*An extended finite element method with higher-order elements for curved cracks*", Computational Mechanics, 2003, 31(1-2), pp.38-48.

[87] Goyat, V., Verma, S., & Garg, R. "*Effect of an edge crack on stress concentration around hole surrounded by functionally graded material layer*", Engineering Solid Mechanics, 2022, vol. 10(4), pp. 325-340.