SUPREMACY IN BIPOLAR FUZZY GRAPHS: STRONG & WEAK

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Abstract

The purpose of this paper is to examine a number of consequences relating to a novel class of parameters known as strong (weak) supremacy in bipolar fuzzy graphs. A few new parametric situations are established, and the terms clique strong (weak) supremacy, regular clique strong (weak) supremacy, non-split clique strong (weak) supremacy, split strong (weak) supremacy, global strong (weak) supremacy, inverse clique strong (weak) supremacy, triple connected perfect strong (weak) supremacy, accurate strong (weak) supremacy, equitable strong (weak) supremacy, and double strong. A few new parametric conditions are also introduced, and each of these is introduced using strong edge.

Paper Identification

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Clique strong (weak) supremacy characteristics in a bipolar fuzzy graph

This section defines the novel idea of clique strong (weak) dominance in bipolar fuzzy graphs and discusses associated characteristics.

Definition

The clique dominating set $D_{CSW}(G)$ is said to be clique strong (weak)dominating set of a BFG, G if for every

$v \in V\text{-}D_{CSW}(G)$ there exist $u \in D_{CSW}(G)$ such that $\deg_N(u) \geq deg_N(v)$ andit is denoted by $D_{CSW}(G)$

The clique strong (weak) dominating set $D_{CSW}(G)$ is said to be minimal clique strong (weak) dominating set if no proper subset of $D_{CSW}(G)$ is a strong (weak) dominating set of G.

The minimum fuzzy cardinality of a minimal clique strong (weak) dominating set in G is called the clique strong (weak) supremacy number is denoted by $\gamma_{CSW}(G)$.

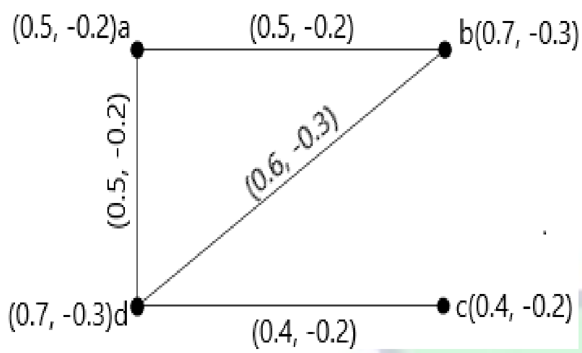
Example

From this example,

The clique strong (weak) dominating set = {d}, {a, d},{b, d} with cardinalities 0.7 and 1.35, 1.4 respectively.

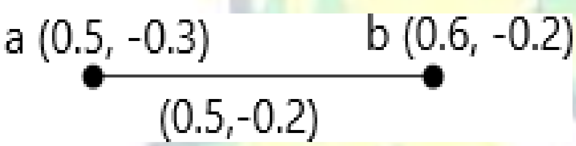
Hence the minimal cardinality = 0.7

Therefore, the clique strong (weak) supremacy number = $\gamma_{CSW}(G) = 0.7$.



Let G = (A, B) be any BFG. If n = 2 then the minimum clique dominating set $u \in D_C$ and V-D_C is dominated by u also a dominating set. Which implies the bipolarfuzzy vertex cardinality of v is maximum and satisfies the condition that for every $v \in$ V-D_C there exist $u \in D_C$ such that $deg(u) \geq deg_N(v)$. Hence $\gamma(G) = \gamma_{CD}(G) = \gamma_{CSW}(G)$.

Example



Theorem

Let G = (A, B) be any BFG, then $\gamma(G) \leq \gamma_{CD}(G) \leq \gamma_{CSW}(G) \leq$ p $\leq$ q.

Proof

Let G = (A, B) be any BFG. From the theorem 2.2.4 we have,

$\gamma(G) \leq \gamma_{CD}(G)$ ----(1). Let$D_C \subseteq M$ be a clique dominating set and $D_{CSW} \subseteq M$be a Clique strong (weak) dominating set of G. If DC = DCSW then $\gamma_{DC}(G) = \gamma_{CSW}(G)$. If DC $\neq$ DCSW then DCSW has at least one vertex more than DC and hence $\gamma_{DC}(G) \leq$

$\gamma_{CSW}(G)$ ---------------------(2). p, sum of the fuzzy cardinality of all the vertices. Therefore $\gamma(G) \leq \gamma_{CD}(G) \leq$ p ----------(3). q, sum of the fuzzy cardinality of all the edges. Therefore, the fuzzy cardinality of all the edges is more than the fuzzy

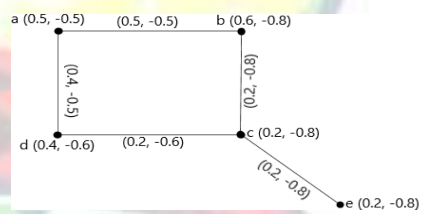
Properties of Regular clique strong (weak) supremacy in bipolar fuzzy graph

This section defines the novel idea of regular clique strong (weak) dominance in a bipolar fuzzy graph and discusses associated features.

Definition

The regular clique dominating set DRC of a BFG, G is said to be a regular clique strong (weak) dominating set in a bipolar fuzzy graph G if for every $v \in$ V-D_{RC} there exist $u \in D_{RC}$ such that $deg_N(u) \geq deg_N(v)$ and the regular clique strong (weak) dominating set is denoted by DRCSW (G). A typical clique that is strong (weak) or dominating If no suitable subset of DRCSW is a dominating set of, then DRCSW is said to be the minimal regular clique strong dominating set G. The minimum fuzzy cardinality of a minimal regular clique strong (weak) dominating set in G is called the regular clique supremacy number of G and is denoted by $\gamma_{RCSW}(G)$.

Example



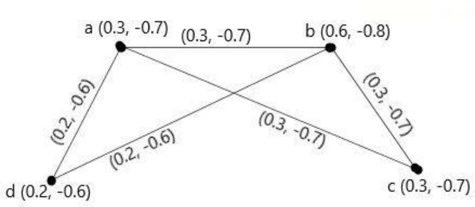
Theorem

The regular clique strong (weak) dominating set DRCSW exist for any regular bipolar fuzzy graph G.

Proof

Let G = (A, B) be a regular bipolar BFG, clearly every $v \in V$ has the same bipolar fuzzy vertex degree. Since every vertex is a regular and complete implies the given BFG is regular clique bipolar fuzzy graph and let $\{u\} \subseteq V$ be the regular clique dominating set and it satisfies the condition $deg(u) \geq deg_N(v)$. Hence by the definition of strong (weak) supremacy on BFG, the regular clique strong (weak) dominating set exists. Hence the theorem.

Example



The regular clique strong (weak) dominating set DRCSW = {a, b}

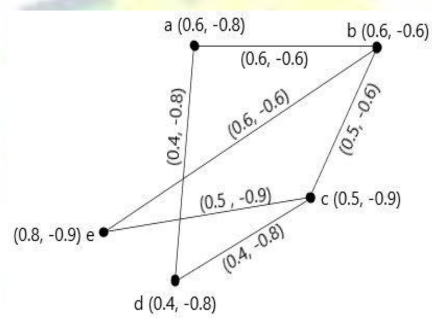
The regular clique strong (weak) supremacy number $\gamma_{RCSW}(G)$ = 0.7.

Theorem

Let G = (A, B) be any BFG, then $\gamma(G) \leq \gamma_{CD}(G) \leq \gamma_{RC}(G) \leq \gamma_{RCSW}(G)$.

Proof

Let G = (A, B) be any BFG, by the previous theorems, we have



$\gamma(G) \leq \gamma_{CD}(G) \leq \gamma_{RC}(G)$ -------------(1) and there exist more than one vertex in DRCSW in G, then $\gamma_{RC}(G) \leq \gamma_{RCSW}(G)$--(2).

Hence $\gamma(G) \leq \gamma_{CD}(G) \leq \gamma_{RC}(G) \leq \gamma_{RCSW}(G)$.

Theorem

For any BFG, G = (A, B) with no isolated vertices and let DRCSW be the regular clique strong (weak) dominating set. Then $\gamma(G) + \gamma_{RCSW}(G) \leq p$.

Properties of Non – Split clique strong (weak) supremacy in bipolar fuzzygraph

The idea of non-split clique strong (or weak) supremacy in a bipolar fuzzy network is defined in this section, along with the minimal non-split clique strong (or weak) dominating set and the non-split clique strong (or weak) supremacy number.

Definition

The Non – split clique dominating set DNSC of a BFG, G = (A, B) is said to be Non – split clique strong (weak) dominating set if for every

$v \in V$-D_{NSC} there exist $u \in D_{NSC}$ such that $deg(u) \geq deg_N(v)$ and it is denotedby DNSCSW (G)

4 The Non – split clique strong (weak) dominating set DNSCSW (G) is said to be minimal Non – split clique strong (weak) dominating set if no proper subset of DNSCSW (G) is a Non – split clique strong (weak) dominating set of G.

5 The minimum fuzzy cardinality of a minimal Non – split clique strong (weak) dominating set in G is called the non – split clique strong (weak) supremacy number is denoted by $\gamma_{NSCSW}(G)$.

Example

The non – split clique strong (weak) dominating set DNSCSW = {b, c}

The non – split clique strong (weak) supremacy number $\gamma_{NSCSW}(G)$ = 0.8

Theorem

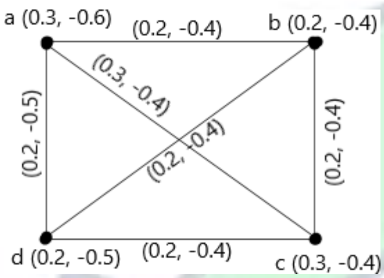
Every complete BFG, G = (A, B) has a non – split clique strong (weak)dominating set.

Proof

Let G = (A, B) be a complete BFG, then every bipolar fuzzy vertex is clique and dominates every other vertices of G. By choosing the minimum strong (weak) dominating set then, for every $v \in V$ - D_{NSCSW} there exist $u \in D_{NSCSW}$ such that

$deg(u) \geq deg_N(v)$ then the removal of u produces the connected BFG. Therefore, by the definition of non – split supremacy $\langle D_{NSCSW} \rangle$ is connected. Hence the Theorem.

Example



Properties of Split strong (weak) supremacy in bipolar fuzzy graph

The concept of split strong (weak) supremacy in bipolar fuzzy graph is defined; minimal split strong (weak) dominating set and split strong (weak) supremacy number also defined.

Definition

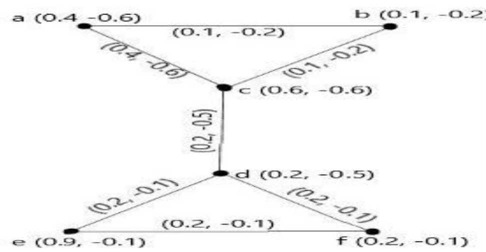
A split dominating set D_S of a bipolar fuzzy graph G = (A, B) is said to be split strong (weak) dominating set D_{SSW} if for every $v \in V$-D_S there exist $u \in D_S$ such that $deg(u) \geq deg_N(v)$ and it is denoted by D_{SSW} (G)

A split strong (weak) dominating set D_{SSW} (G) is said to be minimal split strong (weak) dominating set if no proper subset of D_{SSW} (G) is split strong (weak) dominating set of G.

The minimum fuzzy cardinality of a minimal split strong (weak) dominating set in G is called the split strong (weak) supremacy number is denoted by γ_{SSW}(G).

The split strong (weak) dominating set D_{SSW} = {a, d}

The split strong (weak) supremacy number SSW (G) = 0.75



Theorem

Let G = (A, B) be any BFG, then $\gamma(G) \leq \gamma_{SSW}(G) \leq p \leq q$

Let G = (A, B) be any BFG. Let $D \subseteq M$ be a dominating set and $D_{SSW} \subseteq M$ be a split strong (weak) dominating set of G. If D = D_{SSW} then $\gamma(G) = \gamma_{SSW}(G)$. If D ≠ D_{SSW} then D_{SSW} has at least one vertex more than D and hence $\gamma(G) \leq \gamma_{SSW}(G)$ --- (1).

p, the sum of the fuzzy cardinality of all the vertices. Therefore

$\gamma(G) \leq \gamma_{SSW}(G) \leq p$ (2). q, the sum of the fuzzy cardinality of all the edges. If the given bipolar fuzzy graph is having the split strong (weak) dominating set then the number of edges is at least one edge is more than the number of vertices. Therefore, the fuzzy cardinality of all the edges is more than the fuzzy cardinality of all the vertices. Hence p ≤ q --------------------- (3). From (1), (2) and (3) we have, $_D$(G) ≤ $\gamma_{SSW}(G) \leq p \leq q$. Hence the theorem.

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