SUPREMACY IN BIPOLAR FUZZY GRAPH

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Abstract

Some brand-new kind of supremacy parameters in bipolar fuzzy graph viz. Clique supremacy, Inverse clique, Non – split clique supremacy, Regular clique supremacy, Perfect supremacy, Perfect connected supremacy, Regular perfect supremacy, Split perfect supremacy, Non – split perfect supremacy, Global perfect supremacy, Equitable supremacy, Double supremacy and Regular double supremacy of a bipolar fuzzy graph are defined and few new parametric situations are established. These are all introduced using strong edge and a few new parametric conditions are introduced.

Paper Identification

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INTRODUCTION:- A clique dominating set DC of a bipolar fuzzy graph G = (A, B) is said to be non – split clique dominating set if the induced bipolar fuzzy subgraph ⟨V-D$_C$⟩ is connected. A non – split clique dominating set D$_{NSC}$ (G) is said to be minimal clique dominating set if no proper subset of D$_{NSC}$ (G) is a dominating set. The minimum fuzzy cardinality of a minimal non – split clique dominating set in G is called the non – split clique supremacy number is denoted by γ_{NSC}(G). A regular clique dominating set D$_{RC}$ (G) of M is said to be minimal regular clique dominating set if no proper subset of D$_{RC}$ (G) is a dominating set of G. The minimum fuzzy cardinality of a minimal regular clique dominating set in G is called the regular clique supremacy number of G and is denoted by γ_{RC}(G).

Properties of Clique supremacy in bipolar fuzzy graph

In this section, the concept of clique supremacy in bipolar fuzzy graph is defined, minimal clique dominating set and the clique supremacy number also defined.

Definition [50]

A dominating set D_C of a BFG, G = (A, B) is a clique dominating set if the induced subgraph $\langle D_C \rangle$ is complete.

A clique dominating set D_C (G) is said to be minimal clique dominating set ifno proper subset of D_C (G) is a dominating set.

The minimum fuzzy cardinality of a minimal clique dominating set in G is called the clique supremacy number is denoted by γ_{CD}(G).

Example

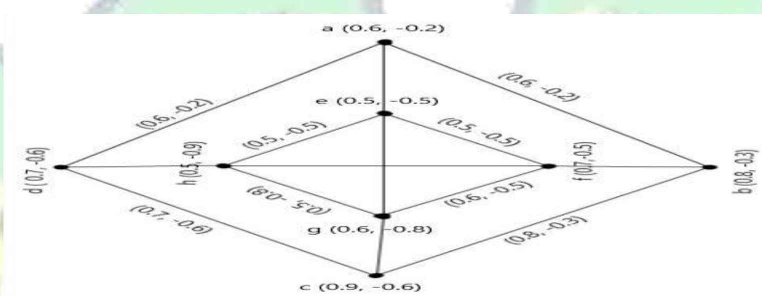


Fig. 2.1

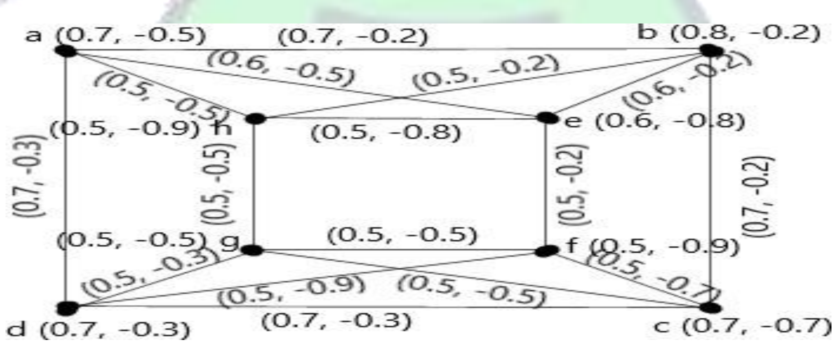
From this example

The dominating set D = {b, h}

The supremacy number γ_D(G) = 1.05

The Clique dominating set D_C(G) = {e, f, g, h}. The Clique supremacy number γ_{CD}(G) = 1.8

Example



From this example,

The clique dominating set $D_C(G) = \{e, f\}$ The clique supremacy number $\gamma_{CD}(G) = 0.6$

For any complete BFG, $G = (A, B)$ every vertex dominates all the other vertices. By choosing the dominating set D_C with minimum supremacy number $\gamma_{CD}(G)$ the induced subgraph of the dominating set, $\langle D_C \rangle$ is complete. By the definition of Clique dominating set of a bipolar fuzzy graph G, this proves the theorem.

Example

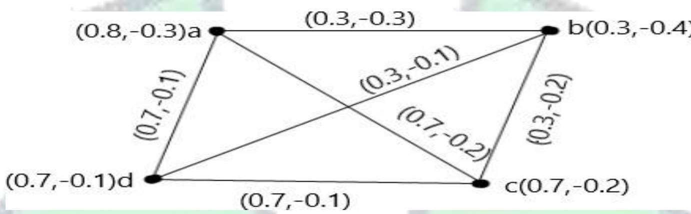


Fig. 2.3

From this example,

The clique dominating set $D_C = \{b\}$

The clique supremacy number $\gamma_C(G) = 0.45$.

Theorem

For any complete BFG, $G = (A, B)$ with $n \geq 2$, then $\gamma_D(G) = \gamma_{CD}(G)$.

Proof

Let $G = (A, B)$ be a complete BFG with $n \geq 2$, every vertex dominates every other vertex. Let D be the minimal dominating set with minimum supremacy number $\gamma_D(G)$ and hence, $\langle D \rangle$ is complete. Therefore, D is also clique dominating set. Hence, $\gamma_D(G) = \gamma_{CD}(G)$.

Theorem

Let $G = (A, B)$ be a complete BFG and let D_C is the clique dominating set in G, then $(V - D_C)$ is complete.

Proof Let $G = (A, B)$ be a complete BFG with vertex set $v_i \in V$, $i = 1,2,3\ldots\ldots n$. Let D_C is the clique dominating set in G. Hence, the resultant vertices in $(V - D_C)$ are dominates every other vertex. Therefore, it is complete. Hence the proof.

Properties of Inverse clique supremacy in bipolar fuzzy graph

The authors Saqr H. AL-Emrany , Mahioub M. Q. Shubatah **[81]** introduced the concept of inverse supremacy on bipolar fuzzy graph. In this section, these concept is extended to the new kind of parameter inverse clique dominating set of bipolar fuzzy graph and established the parametric conditions.

Definition

Let G = (A, B) be a BFG without isolated vertices. A subset $D_C \subseteq V$ is a minimum dominating set of a BFG, G. If $D_C' \subseteq V - D_C$ is a clique dominating set of G, then D_C' (G) is called the inverse clique dominating set with respect to D_C.

The inverse clique dominating set D_C of a bipolar fuzzy graph G is called minimal inverse clique dominating set of G if there does not exist any inverse clique dominating set of G whose cardinality is less than the cardinality of D_C.

The minimum fuzzy cardinality among all minimal inverse clique dominating set of G is called inverse clique dominating set and its supremacy number is denotedby γ_{CD}' (G).

Example

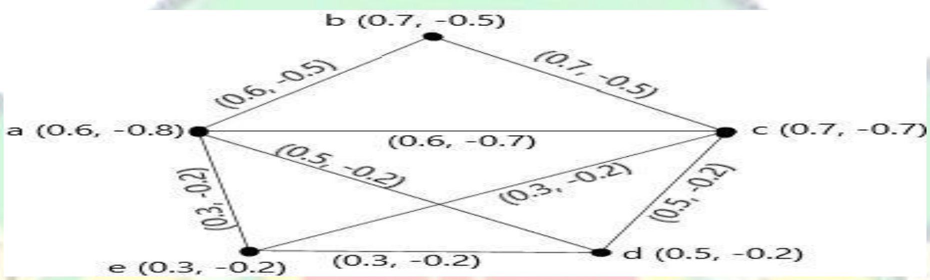


Fig. 2.4

From the this example,

The clique dominating set D_C (G) = {a} & V- D_C (G) = {b, c, d, e} The clique supremacy number γ_{CD}(G) = 0.4

The inverse clique supremacy number γ'_{CD} (G) = 0.5.

Theorem

For any BFG, G = (A, B) with minimum dominating set Dc, then $\gamma_{CD}(G) + \gamma_{CD}'(G) \leq p$.

Proof

Let DC be the minimum dominating set of a BFG, G. If D C is an inverse clique dominating set of G with respect to D_C then γ' (G) ≤ p - γ_{CD}(G).

Hence, γ_{CD}' (G) ≤p.

Theorem

Let G = (A, B) be a BFG, with at least one isolated vertex. Then γ_{CD}' (G) = 0

Proof

Let G = (A, B) be bipolar fuzzy graph. Let D_C be the minimum dominating set of a bipolar fuzzy graph G and {u} □ DC be the isolated vertex of G. Then, for allv □ V – DC, there is no vertex in V – DC dominates u. Hence, there is no dominating set of G in V – DC. Hence, γ'_{CD} (G) = 0.

Theorem

For any BFG, G = (A, B) then γ'_{CD} (G) < p.

Proof

Let G = (A, B) be a bipolar fuzzy graph. Then by Theorem 2.3.4 γ'_{CD} (G) < p.

Hence, γ'_{CD} (G) < p.

Properties of Non – Split clique supremacy in bipolar fuzzy graph

In this section, the concept of Non – split clique supremacy in bipolar fuzzy graph is defined, minimal non – split clique dominating set and the non – split clique supremacy number also defined.

Definition

A clique dominating set DC of a bipolar fuzzy graph G = (A, B) is said to be non – split clique dominating set if the induced bipolar fuzzy subgraph ⟨V-D_C⟩ is connected.

A non – split clique dominating set DNSC (G) is said to be minimal clique dominating set if no proper subset of DNSC (G) is a dominating set.

The minimum fuzzy cardinality of a minimal non – split clique dominating set in G is called the non – split clique supremacy number is denoted by γ_{NSC}(G).

Example

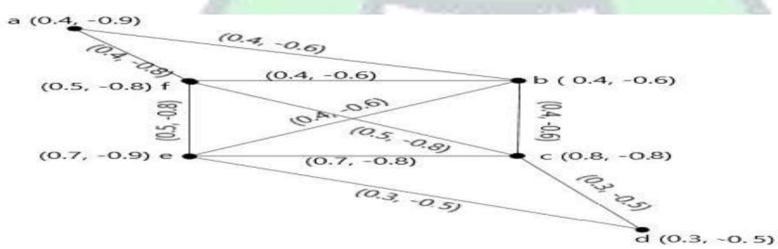
Consider the above example 2.2.2, The non – split clique dominating set DNSC (G) = {e, f, g, h} The non – split clique supremacy number γNSC(G) = 1.8

Theorem

A non – split clique dominating set exists for every complete BFG.

Proof

Let G = (A, B) be a complete BFG. Clearly every vertex $vi \in$ V, where



i= 1, 2……n is dominated by every other vertex and every vertex $vi \in V$ is complete. Let a vertex {v1} □ DC be minimal clique dominating set with minimum fuzzy cardinality γNSC(G). Since, every vertex is complete. Therefore, the induced bipolar fuzzy subgraph ⟨V-$v1$⟩is connected. Hence the Non – split clique dominating set exists for every complete BFG.

Theorem

Let G = (A, B) be any BFG, then $\gamma_{CD}(G) = \gamma_{NSC}(G)$.

Proof

The proof can be illustrated by the above example 2.2.3

Properties of Regular Clique supremacy in bipolar fuzzy graph

In this Section, the parametric condition of regular clique dominating set of a bipolar fuzzy graph is defined and some theorem are derived.

Definition

For any bipolar fuzzy graph G = (A, B). The clique dominating set D_C of G said to be a regular clique dominating set of a BFG, G if all the vertices of D_C has the same degree and it is denoted by D_{RC} (G)

A regular clique dominating set D_{RC} (G) of M is said to be minimal regular clique dominating set if no proper subset of D_{RC} (G) is a dominating set of G.

The minimum fuzzy cardinality of a minimal regular clique dominating set in G is called the regular clique supremacy number of G and is denoted by $\gamma_{RC}(G)$.

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