THERMAL INSTABILITY IN A COUPLE STRESS NANOFLUID IN PRESENCE OF HORIZONTAL MAGNETIC FIELD

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Abstract

In this paper thermal instability of a horizontal layer of Couple Stress nanofluid in a porous medium is investigated under effect of horizontal magnetic field. It is assumed that nanoparticle flux is zero on the boundaries. Stationary convection is studied using normal mode technique. It is found that critical Rayleigh Number increases with an increase in the magnetic Chandrasekhar number as well as couple stress parameter. Stability of system has been investigated with effects of Lewis number, concentration Rayleigh number, modified diffusivity ratio and magnetic field. .The effects of various parameters on thermal Rayleigh Number have been presented graphically.

Paper Identification

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Introduction

In 1992, while working on microchannel liquid-nitrogen cooling of high heat load silicon mirrors, Choi [1] noticed that an excellent heat transfer could be achieved only at the increased pumping power. This finding proved to be a milestone in leading him to think of a new heat transfer enhancement approach. The aim was to achieve highest possible enhancement in thermal conductivity at the smallest possible concentration. In 1995 Choi [1] introduced a new class of fluids which were engineered colloidal suspensions containing nanometre – sized metallic particles suspended in the conventional heat transfer fluids and named them Nanofluids. In Non-Newtonian fluids , during last four decades , the Couple Stress fluids attracted the attention of research workers. Couple stresses appear in noticeable magnitudes in Polymer Solutions (liquids with larger molecules) .Theory for Couple Stress fluid was proposed by Stokes ([3], [4]).The field equations for the couple stress vector were discussed by Cosserat and Cosserat [2]. Stability of couple stresses binary fluid mixture having vertical temperature and concentration gradients was discussed by Rachana [5].

The onset of convection in a horizontal layer of a porous medium saturated by nanofluid was studied by Nield [6]. The model incorporated the effects of Brownian Motion and Thermophoresis. It was found that critical thermal Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top - heavy or bottom – heavy. Thermal instability of rotating nanofluid layer was studied by Yadav [10]. Galerkin Method was used to obtain the analytical expressions for both non- oscillatory and oscillatory cases. The influence of various nanofluid parameters and rotation on onset of convection was analysed and it was found that rotation has a stabilizing effect depending on the various nanofluid parameters. The effect of the magnetic field on flow of electrically conducting fluid through a vertical plate is of great importance such as metal casting, the cooling systems of electronic devices . In order to get enhanced heat performance of such devices, the use of nanofluid can be considered as a working medium. When the space between the plates is filled with the electric conductive nanofluid the flow and temperature fields can be controlled using a magnetic field. The effect of magnetic field on the onset of nanofluid convection induced by internal heating was studied by Yadav[11]. Effect of magnetic field on thermal convection of a porous nanofluid layer using Darcy Law was considered by Gupta [12]. It was found that magnetic field stabilizes the nanofluid layer appreciably while porosity hastens the onset of convection. Magnetic Field effect on unsteady nanofluid flow and heat transfer using Buongiorno Model was studied by Sheikholeslami[13].The graphical and analytical investigation was carried out for different governing parameters. Further, analytical and numerical study of the stability of a mono diffusive convection in a Darcy porous layer saturated by a Maxwellian nanofluid was done by Jaimala [7]. It was found that the mode of convection is changed in presence of salt and heat transfer is most active in Soret induced convection.

Keeping in view the importance of Couple Stress nanofluid in a porous medium attempt has been made to study the thermal instability in a horizontal layer of Couple Stress nanofluid in presence of horizontal magnetic field.

Mathematical Formulation of the Problem

We consider an infinite isotropic porous layer of incompressible Maxwellian couple stress viscoelastic fluid confined between two horizontal planes z* = 0 and z* = d where the temperatures at the lower and upper boundaries are T_h^* and T_c^* respectively, T_h^* being greater than T_c^*. A uniform vertical magnetic field acts on the system $H^* = (H_0^*, 0, 0)$. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks).

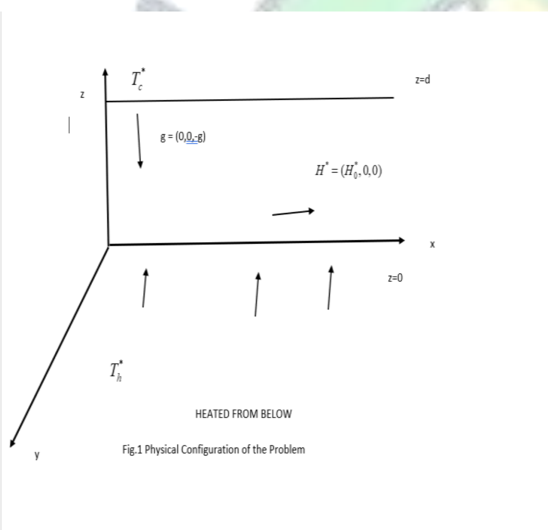

Fig.1 Physical Configuration of the Problem

Governing equations are $\nabla^* . \boldsymbol{V_D^*} = 0$, (1)

$$\frac{1}{K}(\mu - \mu_c \nabla^{*2})\boldsymbol{V_D^*} = (1 + \lambda^* \frac{\partial}{\partial t^*})[\{-\nabla^* p^* + (\phi^* \rho_P + (1-\phi^*)\left\{\rho\left(1 - \beta_T(T^* - T_c^*) - \beta_c(C^* - C_c^*)\right)\right\})\boldsymbol{g}]$$
$$+ \frac{\mu_e}{4\pi}(\nabla^* \times \boldsymbol{H^*}) \times \boldsymbol{H^*}] \quad , \qquad (2)$$

$$\left(\rho c\right)_m \frac{\partial T^*}{\partial t^*} + \left(\rho c\right)_f \boldsymbol{V_D^*}.\nabla^* T^* = k_m \nabla^{*2} T^* + \in (\rho c)_p \left[D_B \nabla^* \phi^* . \nabla^* T^* + (\frac{D_T}{T_c^*}) \nabla^* T^* . \nabla^* T^* \right], \qquad (3)$$

$$\frac{\partial C^*}{\partial t^*} + \frac{1}{\in} \boldsymbol{V_D^*}.\nabla^* C^* = D_S \nabla^{*2} C^* + D_{CT} \nabla^{*2} T^* \ , \qquad (4)$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\in} \boldsymbol{V_D^*}.\nabla^* \phi^* = D_B \nabla^{*2} \phi^* + \frac{D_T}{T_c^*} \nabla^{*2} T^* . \tag{5}$$

The modified Maxwell equations are

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{\in} \left(\boldsymbol{V_D^*}.\nabla^* \right) \right) \boldsymbol{H^*} = \left(\boldsymbol{H^*}.\nabla^* \right) \frac{1}{\in} \boldsymbol{V_D^*} + \eta \nabla^{*2} \boldsymbol{H^*} \tag{6}$$

and $\nabla^* \cdot \boldsymbol{H^*} = 0$, $\eta = \dfrac{1}{4\pi \mu_e \sigma'}$, (7)

where $\boldsymbol{V_D^*} = (u^*, v^*, w^*)$ is Darcian velocity, K is permeability, μ_c is couple stress viscosity, λ^* is relaxation time, ϕ^* is concentration of nanoparticles, ρ_P is mass density of nanoparticles, ρ is density of base fluid, T^* is the temperature, C^* is the salt concentration, C_c^* is the reference concentration of the salt, T_C^* is reference temperature, $(\rho c)_m$ is effective heat capacity of the medium, $(\rho c)_f$ is effective heat capacity of fluid, $(\rho c)_p$ is effective heat capacity of material constituting nanoparticles, c is specific heat of nanofluid, k_m is effective thermal conductivity of porous medium, $\in$ is porosity, D_S is the solute diffusivity coefficient, D_{CT} is the Soret coefficient of salt, D_B is Brownian diffusion coefficient, D_T is thermophoretic diffusion coefficient, μ_e is magnetic permeability and σ' is electrical conductivity of nanofluid and η is electrical resistivity of the nanofluid.

Physical realistic boundary conditions on the nanoparticle volume fraction are considered which state that the flux of nanoparticle concentration is zero at the boundary. Therefore, the boundary conditions are

$$\boldsymbol{V_D^*} = 0, \; T^* = T_h^*, \; C^* = C_h^*, D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_T}{T_c^*} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = 0 \tag{8}$$

$$\boldsymbol{V_D^*} = 0, \; T^* = T_C^*, \; C^* = C_c^* \;, D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_T}{T_c^*} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = d \;, \tag{9}$$

We now make the physical quantities non-dimensional as follows :

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{d}, \; t = \frac{t^* \alpha_m}{\sigma d^2}, \; (u, v, w) = \frac{(u^*, v^*, w^*) d}{\alpha_m}, \; p = \frac{p^* K}{\mu \alpha_m}, \; \phi = \frac{\phi^* - \phi_0^*}{\phi_0^*}, T = \frac{T^* - T_c^*}{T_h^* - T_c^*},$$

$$\lambda = \frac{\lambda^* \alpha_m}{d^2}, \; (H_x, H_y, H_z) = \frac{(H_x^*, H_y^*, H_z^*)}{H_0^*},$$

where

ϕ_0^* is reference scale for volumetric fraction of nanoparticles,

$\alpha_m\left(=\dfrac{k_m}{(\rho c)_f}\right)$ is the thermal diffusivity of the porous medium

and $\sigma\left(=\dfrac{(\rho c)_m}{(\rho c)_f}\right)$ is the heat capacity ratio parameter.

On replacing $\boldsymbol{V_D}$ by $\boldsymbol{V}$, non-dimensional form of equations (1) to (6) together with boundary conditions (8) and (9) can be written as

$$\nabla.\boldsymbol{V}=0, \tag{10}$$

$$(\boldsymbol{V}- \quad \nabla^2\boldsymbol{V})=\left(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)[\left(-\nabla p-R_m\hat{\boldsymbol{e}}_{\boldsymbol{z}}-R_n\phi\hat{\boldsymbol{e}}_{\boldsymbol{z}}+R_aT\hat{\boldsymbol{e}}_{\boldsymbol{z}}+\frac{R_s}{L_n}C\hat{\boldsymbol{e}}_{\boldsymbol{z}}\right)+\frac{P_r}{P_{rM}}QD_a(\nabla\times\boldsymbol{H})\times\boldsymbol{H}], \tag{11}$$

$$\frac{\partial T}{\partial t}+(\boldsymbol{V}.\nabla)T=\nabla^2T+\frac{N_B}{Le}\nabla\phi.\nabla T+\frac{N_AN_B}{Le}\nabla T.\nabla T, \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\in}\boldsymbol{V}.\nabla C=\frac{1}{L_n}\nabla^2C+N_{ct}\nabla^2T, \tag{13}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\in}(\boldsymbol{V}.\nabla)\phi=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2T, \tag{14}$$

$$\frac{1}{\sigma}\frac{\partial\boldsymbol{H}}{\partial t}+\frac{1}{\in}(\boldsymbol{V}.\nabla)\boldsymbol{H}=\frac{1}{\in}(\boldsymbol{H}.\nabla)\boldsymbol{V}+\frac{\mathrm{P}_r}{P_{rM}}\nabla^2\boldsymbol{H}, \tag{15}$$

$$\boldsymbol{V}=0,\ \mathrm{T=1,\ C=1,}\ \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \quad \text{at} \quad z=0 \tag{16}$$

$$\boldsymbol{V}=0,\ \mathrm{T=0,\ C=0,}\ \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \quad \text{at} \quad z=1, \tag{17}$$

Here,

$R_a=\dfrac{\rho g\beta Kd\left(T_h^*-T_c^*\right)}{\mu\alpha_m}$ (Thermal Rayleigh-Darcy number)

$R_n=\dfrac{(\rho_P-\rho)\phi_0^*gKd}{\mu\alpha_m}$ (Concentration Rayleigh-Darcy number)

$R_m=\dfrac{\rho_P\phi_0^*+\rho(1-\phi_0^*)gKd}{\mu\alpha_m}$ (Basic density Rayleigh-Darcy number)

$$R_s = \frac{\rho\beta_c gdK(C_h^* - C_c^*)}{\mu D_s}$$ (Solutal Rayleigh-Darcy number)

$$P_r = \frac{\mu}{\rho\alpha_m}$$ (Prandtl number) $$P_{rM} = \frac{\mu}{\rho\eta}$$ (Magnetic Prandtl number)

$$Q = \frac{\mu_e H_0^{*2} d^2}{4\pi\mu\eta}$$ (Magnetic Chandrasekhar number)

$$D_a = \frac{K}{d^2}$$ (Darcy number)

$$= \frac{\mu_C}{\mu d^2}$$ (Couple-Stress parameter)

$$N_A = \frac{D_T(T_h^* - T_c^*)}{D_B T_c^* Q_0^*}$$ (Modified diffusivity ratio)

$$N_B = \frac{(\rho c)_\rho \in Q_0^*}{(\rho c)_f}$$ (Modified particle density increment)

$$N_{ct} = \frac{D_{CT}(T_h^* - T_c^*)}{\alpha_m(C_h^* - C_c^*)}$$ (Soret parameter)

$$Ln = \frac{\alpha_m}{D_s}$$ (Thermo-solutal Lewis number)

and $$Le = \frac{\alpha_m}{D_B}$$ (Thermo- nanofluid Lewis number)

Basic State

Time independent basic state of nanofluid is described as

$$\boldsymbol{V} = 0,\ p = p_b(z),\ T = T_b(z),\ \phi = \phi_b(z),\ \boldsymbol{H} = \hat{\boldsymbol{e}}_{\boldsymbol{x}} \tag{18}$$

The basic volume fraction and temperature of nanoparticles satisfy the following equations:

$$\frac{d^2\phi_b}{dz^2} + N_A \frac{d^2T_b}{dz^2} = 0, \tag{19}$$

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{Le}\frac{d\phi_b}{dz}\frac{dT_b}{dz} + \frac{N_A N_B}{Le}\left(\frac{dT_b}{dz}\right)^2 = 0, \tag{20}$$

and $$\frac{1}{L_n}\frac{d^2C_b}{dz^2} + N_{CT}\frac{d^2T_b}{dz^2} = 0 \tag{21}$$

The boundary conditions are

$$\boldsymbol{V} = 0\ ,\quad T_b\left(z\right) = 1,\ \ C_b(z) = 1,\quad \frac{d\phi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z=0 \tag{22}$$

$$\boldsymbol{V} = 0,\quad T_b\left(z\right) = 0,\ C_b(z) = 0\ ,\quad \frac{d\phi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z=1\ . \tag{23}$$

On solving the basic temperature and concentration equations (19)-(21) and using boundary conditions (22) and (23), we get the following basic temperature and volume fraction :

$T_b = 1 - z$, $\phi_b = \phi_0 + N_A z$, and $C_b = 1 - z$

where the primes indicate the perturbations and are functions of x , y , z and t.

Perturbed State

On superimposing small perturbations on the basic state given by (18), we write

$V = V', p = p_b + p', T = T_b + T', \phi = \phi_b + \phi', H = \hat{e}_x + H'$,

where the primes indicate the perturbations and are functions of x , y , z and t.

Neglecting the products of primed quantities, the linearised perturbation equation of couple stress nanofluid are obtained as

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t} - \frac{P_r}{P_{rM}}\nabla^2\right)\left[(\nabla^2 - \ \nabla^4)w' - \left(1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left(R_a\nabla_H^2 T' - R_n\nabla_H^2\phi'\right)\right]$$

$$= (1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t})Q\frac{P_r}{P_{rM}}\frac{D_a}{\in}\nabla^2\frac{\partial^2 w'}{\partial x^2}, \tag{24}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' - \frac{N_A N_B}{Le}\frac{\partial T'}{\partial z} - \frac{N_B}{Le}\frac{\partial \phi'}{\partial z},$$

(25)

$$\frac{1}{\sigma}\frac{\partial C'}{\partial t} - \frac{w'}{\in} = \frac{1}{Ln}\nabla^2 C' + N_{CT}\nabla^2 T'\ \ , \tag{26}$$

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t} + \frac{1}{\in}N_A w' = \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2 T'\ . \tag{27}$$

The boundary conditions are

$$w' = 0\ ,\ T' = 0\ ,\ C' = 0\ ,\ \frac{\partial \phi'}{\partial z} + N_A\frac{\partial T'}{\partial z} = 0 \quad \text{at} \quad z = 0 \text{ and } z = 1\ . \tag{28}$$

Linear Stability Analysis

Analysing the perturbations into the normal modes and assuming that the perturbation quantities are of the form

$$\left(w',T',\phi',C'\right)=\left[W(z),\Theta(z),\Phi(z),\Psi(z)\right]e^{st+ilx+imy},$$

where l and m are dimensionless wave numbers in x and y directions respectively and

s$\left(=\omega_r+i\omega_i\right)$ is complex time constant. Substituting this form of perturbation, the linearised equations in dimensionless form are as follows:

$$\left[\frac{s}{\sigma}\left(D^2-\alpha^2\right)-\frac{P_r}{P_{rM}}\left(D^2-\alpha^2\right)^2-Q\left(1+\frac{\lambda s}{\sigma}\right)\frac{P_r}{P_{rM}}\frac{D_a}{\in}l^2\left(D^2-\alpha^2\right)-\frac{s}{\sigma}(D^2-\alpha^2)^2+\frac{P_r}{P_{rM}}(D^2-\alpha^2)^3\right]W$$
$$-R_a\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_r}{P_{rM}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Theta$$
$$+R_n\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_r}{P_{rM}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Phi+\frac{R_s}{Ln}\alpha^2(1+\frac{\lambda s}{\sigma})[\frac{s}{\sigma}-\frac{P_r}{P_{rM}}(D^2-\alpha^2)]\Psi=0 \quad (29)$$

$$W+\left(D^2-\alpha^2-s-\frac{N_AN_B}{Le}D\right)\Theta-\frac{N_B}{Le}D\Phi=0, \quad (30)$$

$$\frac{W}{\in}+N_{CT}(D^2-\alpha^2)\Theta+[\frac{1}{Ln}(D^2-\alpha^2)-\frac{s}{\sigma}]\Psi=0 \quad , \quad (31)$$

$$\frac{N_A}{\in}W-\frac{N_A}{Le}(D^2-\alpha^2)\Theta-\left[\frac{1}{Le}(D^2-\alpha^2)-\frac{s}{\sigma}\right]\Phi=0 \quad (32)$$

The boundary conditions are

$$W=0=\Theta\ ,\ \Psi=0\ ,\ D\Phi+N_AD\Theta=0 \quad \text{at} \quad z=0 \text{ and } z=1\ , \quad (33)$$

where $D=\frac{d}{dz},\alpha=(l^2+m^2)^{1/2}$

Solution

Equations (29) to (32) together with the boundary conditions in (33) constitute a linear eigen-value problem of the system which is solved by the Galerkin – type weighted residual technique. The variables are written in a series of base functions as :

$$W=\sum_{k=1}^{N}A_kW_k\ ,\ \Theta=\sum_{k=1}^{N}B_k\Theta_k\ ,\ \Phi=\sum_{k=1}^{N}C_k\Phi_k\ ,\ \Psi=\sum_{k=1}^{N}D_k\Psi_k \quad ,$$

where A_k,B_k,C_k,D_k are unknown constants with k= 1 ,2,3….N. Using boundary conditions given by equation (33) base functions are assumed as

$W_k=\Theta_k=\Psi_k=\sin k\pi z$ and $\Phi_k=-N_A\sin k\pi z$

Taking first approximation (N=1) , we have

$W = A_1 \sin \pi z$, $\Theta = B_1 \sin \pi z$, $\Phi = -N_A C_1 \sin \pi z$, $\Psi = D_1 \sin \pi z$

Substituting these expressions , equations (29)-(32) can be written in matrix equation as

$$\begin{pmatrix} \frac{s}{\sigma}\delta^2 + \frac{P_r}{P_{rM}}\delta^4 + \frac{sC}{\sigma}\delta^4 + \frac{P_r}{P_{rM}}C\delta^6 + (1+\frac{\lambda s}{\sigma})\frac{QP_r}{P_{rM}}\frac{D_a}{\in}\delta^2\pi^2 & -R_a\alpha^2(1+\frac{\lambda s}{\sigma})(\frac{s}{\sigma}+\frac{P_r}{P_{rM}}\delta^2) & -R_n N_A \alpha^2 (1+\frac{\lambda s}{\sigma})(\frac{s}{\sigma}+\frac{P_r}{P_{rM}}\delta^2) \\ 1 & -(\delta^2+s) & 0 \\ \frac{1}{\in} & \frac{\delta^2}{Le} & -(\frac{\delta^2}{Le}+\frac{s}{\sigma}) \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = 0 \tag{34}$$

For a non-trivial solution of the equation the determinant of coefficient matrix must necessarily be zero, which leads to the following Rayleigh number

$$Ra = \frac{\sigma}{\in \alpha^2}\left[\frac{\begin{array}{l} R_s\alpha^2(\lambda s+\sigma)(\sigma A\delta^2+s)(\sigma\delta^2+sLe)(\delta^2+s)\{\delta^2(\in N_{CT}-1)-s\} \\ -R_n N_A \alpha^2(\lambda s+\sigma)(\delta^2\sigma+sLn)(A\delta^2\sigma+s)\{\delta^2(\in+Le)+sLe\} \\ +\in(\delta^2\sigma+sLn)(\delta^2\sigma+sLe)(\delta^2+s)\{A\sigma\delta^4+Bl^2\delta^2(\sigma+\lambda\delta)+s\delta^2+s\quad\delta^4+A\quad\sigma\delta^6\} \end{array}}{(\sigma\delta^2+sLn)(\sigma\delta^2+sLe)(\lambda s+\sigma)(A\delta^2\sigma+s)}\right], \tag{35}$$

where $\delta^2 = \pi^2 + \alpha^2$.

Stationary Convection

For stationary convection, we write s=0 in equation (35). This leads to the following Rayleigh number

$$Ra^{st} = \frac{\delta^4}{\alpha^2} - \left(1+\frac{Le}{\in}\right)R_n N_A + \frac{QD_a l^2\delta^2}{\in\alpha^2} - \frac{R_s}{\in}(1-\in N_{CT}) + \frac{\delta^6}{\alpha^2}. \tag{36}$$

The above relation expresses the stationary Rayleigh number as a function of the parameters ,
Q, R_n , $\in$, D_a and dimensionless wave number α .

To obtain the critical Rayleigh number, we write $\frac{dR_a^{st}}{d\alpha} = 0$.

The critical wave number is then given by the equation

$$2 \quad (\alpha^2)^3 + (3\pi^2 \quad +1)(\alpha^2)^2 - (\pi^6 + \quad \pi^4 + \frac{QD_a}{\in}\pi^4) = 0 \qquad (37)$$

which shows that the critical wave number depends on couple stress parameter, porosity parameter and the magnetic field.

Results and Discussion

To study the effects of the couple stress parameter , Lewis number Le, modified diffusivity ratio N_A , nanoparticle concentration Rayleigh number R_n and porosity parameter $\in$ on stationary convection , we observe the behaviour of $\frac{\partial R_a^{st}}{\partial}, \frac{\partial R_a^{st}}{\partial Q}, \frac{\partial R_a^{st}}{\partial Le}, \frac{\partial R_a^{st}}{\partial \in}, \frac{\partial R_a^{st}}{\partial N_A}$ *and* $\frac{\partial R_a^{st}}{\partial R_n}$ analytically.

From equation (36), we have:

$\frac{\partial R_a^{st}}{\partial} = \frac{(\pi^2 + \alpha^2)^3}{\alpha^2}$ which is same as obtained by Jaimala [14].

Clearly $\frac{\partial R_a^{st}}{\partial} > 0, \frac{\partial R_a^{st}}{\partial Q} > 0, \frac{\partial R_a^{st}}{\partial Le} < 0, \frac{\partial R_a^{st}}{\partial N_A} < 0, \frac{\partial R_a^{st}}{\partial R_n} < 0$

which shows that the couple stress and the magnetic field have a stabilizing effect while the Lewis number, modified diffusivity and nanoparticle concentration Rayleigh number have destabilizing effect on stationary convection.

If $R_n = 0, Q = 0$, R_s =0, then

$$Ra^{st} = \frac{\delta^4}{\alpha^2} + \frac{\delta^6}{\alpha^2} = \frac{(\pi^2 + \alpha^2)^2}{\alpha^2}(1 + \quad \delta^2)$$

which is the same as obtained by Shivakumara [15].

To obtain the critical Rayleigh number, we put $\frac{dR_a^{st}}{d\alpha} = 0$. This yields the following equation for determining the critical wave number :

$$2 \quad (\alpha^2)^3 + (3\pi^2 \quad +1)(\alpha^2)^2 - (\pi^6 \quad + \pi^4 + \frac{QD_a}{\in}\pi^4) = 0$$

which shows that the critical wave number depends on couple stress parameter, porosity and magnetic field. The stationary convection curves for Rayleigh number R_a versus the wave number α are shown in Figs 2(a)-(g) by assigning fixed values,

$$= 5 , N_A = 4,\ D_a = 0.2,\ Le = 10,\ R_n = 4,\ \in = 0.4,\ Q = 800,\ R_s = 5,\ N_{ct} = 0.1$$

with variations in one of these parameters.

Fig. 2(a) shows the neutral stability curves for different values of the couple stress parameter keeping other parameters fixed. It is clear from Fig. 2(a), that the minimum value of the Rayleigh number increases with an increase in the value of , showing thereby, that the effect of couple stress is to stabilise the system.

Fig. 2(b) shows the effect of Darcy number. The increase in Darcy number increases the Rayleigh number resulting in delay in convection.

Fig. 2(c) displays the effect of porosity parameter $\in$. Porosity has stabilizing as well as the destabilizing effect. Initially, there is increase in Rayleigh number with increases in porosity and after a certain wave number behaviour gets reversed.

Fig. 2(d) illustrates the behaviour of Rayleigh number for different values of Lewis number. There is decrease in stationary Rayleigh number with increase in Le.

The effect of R_n on Rayleigh number is shown in Fig. 2(e). Different Curves show that Rayleigh number is decreased with increase in R_n. It means that the instability is promoted by the thermal Rayleigh number R_n. The graphs for Rayleigh number R_a^{st} against the wave number α for various values of N_A and fixed values of other parameters are in Fig. 2(f). It is evident that N_A advances the onset of stationary convection.

The graphs for Rayleigh number R_a^{st} against the wave number α for various values of Q and fixed values of other parameters are in Fig 2(g). It is clear from the figure that there is a significant increase in the value of critical Rayleigh number with increase in Q. Thus, the magnetic field stabilises the nanofluid layer .and the increase in magnetic field increases the stabilising effect.

It is clear from Fig. 2(h) that the critical Rayleigh number is decreased on increasing the solutal Rayleigh number , thus resulting in an early convection.

Fig. 2(i) shows the effect of Soret parameter . Critical Rayleigh number increases with increase in Soret parameter and hence responsible for promoting the stability of the flow.

Conclusion

In this analysis, the linear double diffusive convection in a porous medium saturated by a couple stress nanofluid layer under the effect of horizontal magnetic field, using Horton-Roger-Lapwood problem based on Darcy Model has been studied providing the following results:

- Relaxation parameter has no effect on stationary convection.
- Critical wave number depends on couple stress parameter as well as magnetic field.
- Couple stress has stabilizing effect on stationary convection.
- As compared to ordinary fluids, convection sets up earlier in nanofluid.
- Magnetic field stabilises the nanofluid layer.
- Porosity has stabilizing as well as destabilizing effect on convection.
- Soret parameter promotes the stability of the flow.
- Increase in solutal Rayleigh number decreases the oscillatory Rayleigh number,

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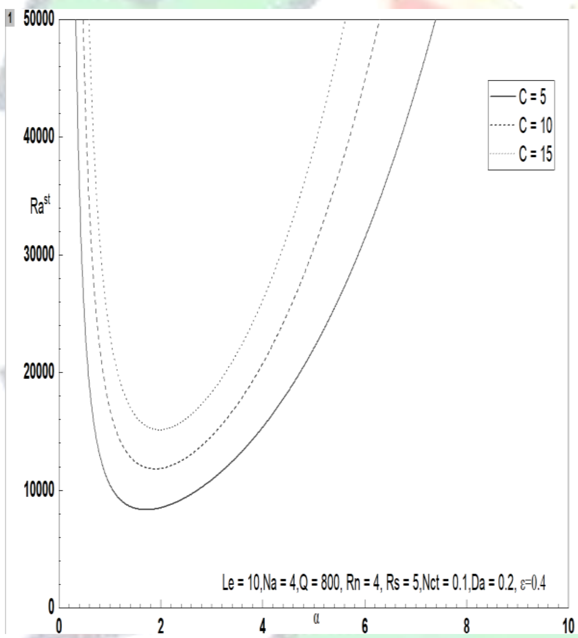
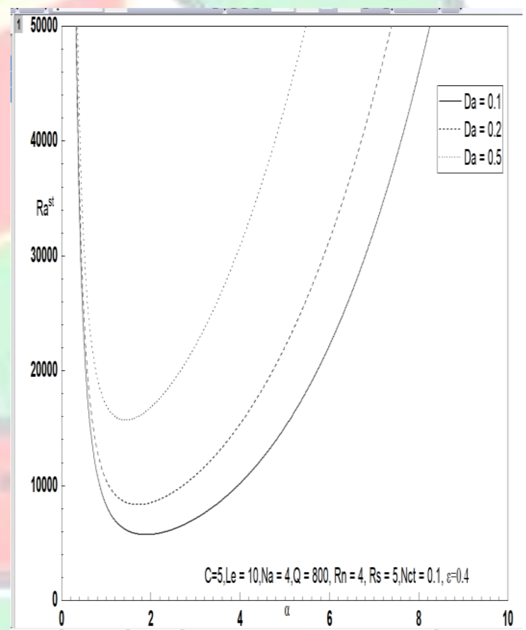
</td><td>(b)
</td></tr>
<tr><td>(c)</td><td>(d)</td></tr>
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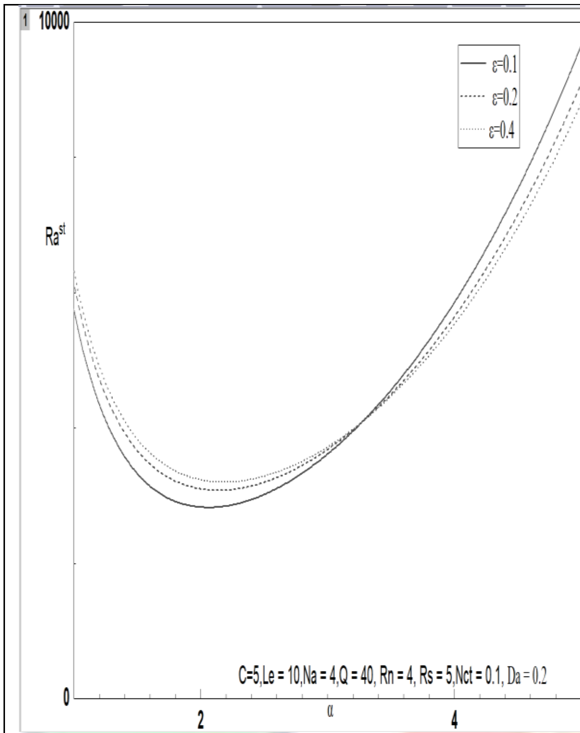


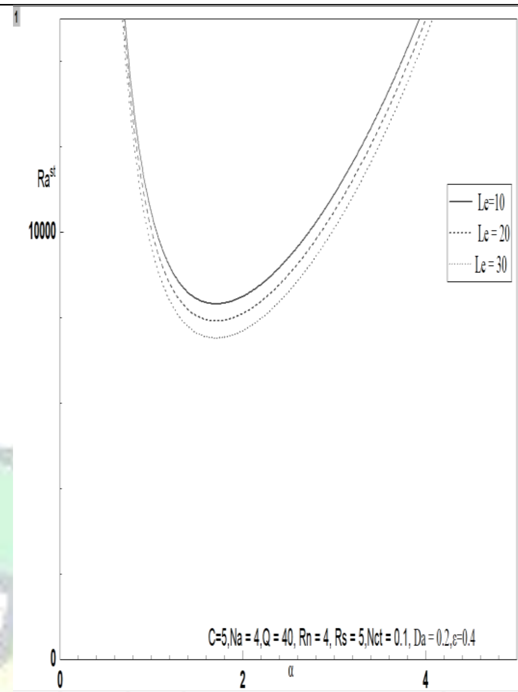
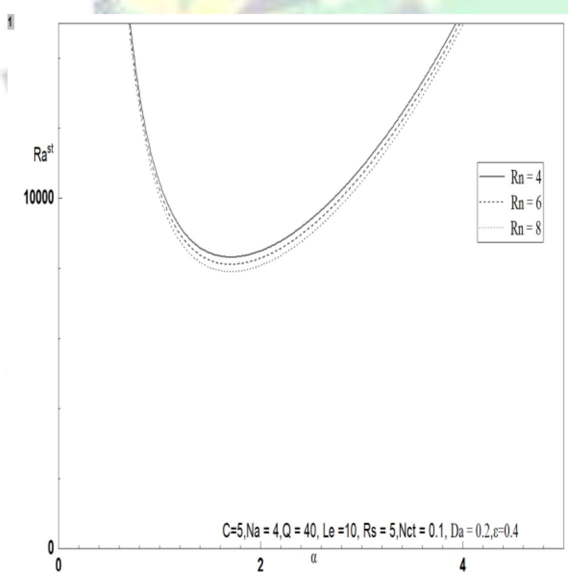
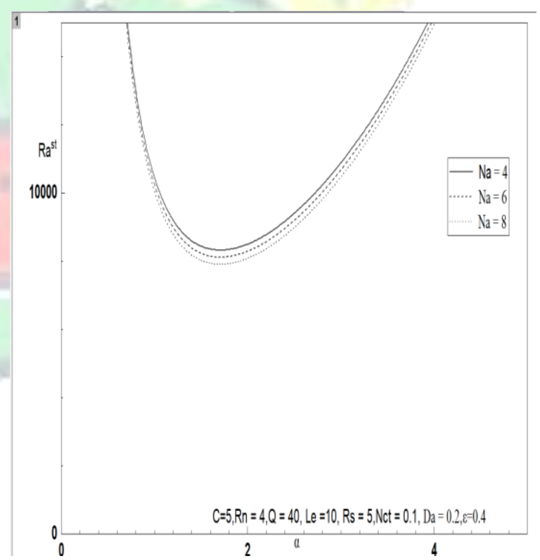


Fig. 2: Linear stationary convection with wave number α (taking m =0) for different values of

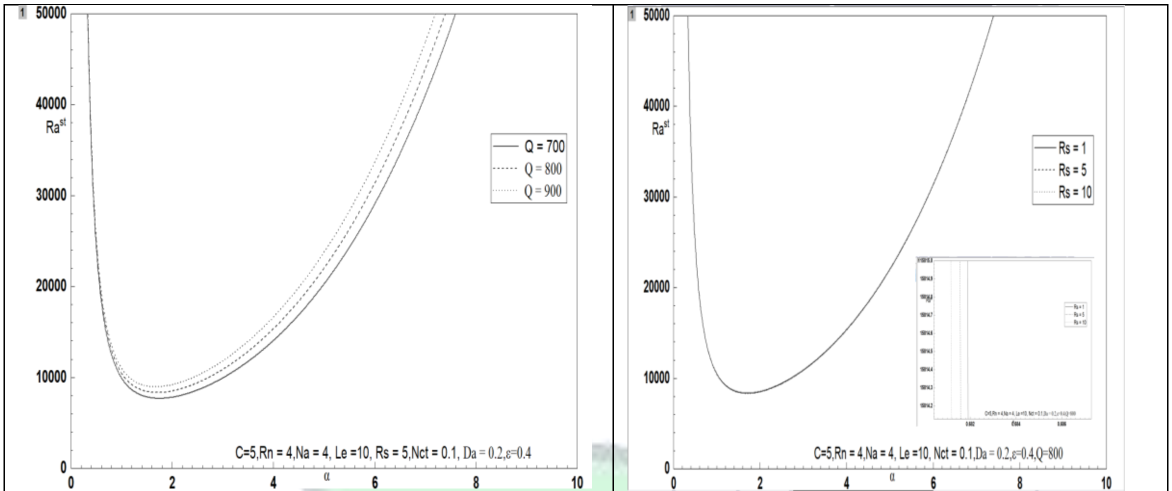
(a) (b) D_a (c) ε (d) Le

(e) (f)





(g) (h)



(i)

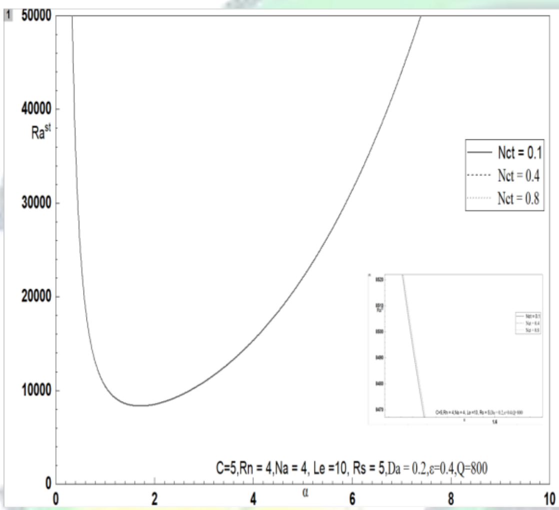


Fig. 2: Linear stationary convection with wave number α (taking m =0) for different values of

(e) *Rn* (f) N_A (g) *Q* (h) R_s (i) N_{ct}

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