



IJRTS TAKSHILA FOUNDATION

# RECENT DEVELOPMENTS IN MATHEMATICS & APPLIED SCIENCES

An Edited Book

ISBN: 978-81-964047-3-4

Chief Editor

**Dr. Neha Mittal**

Assistant Professor, Govt. College Julana, Jind, Haryana

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Invited Manuscripts

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MATHEMATICS & APPLIED SCIENCES**

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**26 MAY 2023**

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**Dr. Neha Mittal**

Assistant Professor, Govt. College Julana, Jind, Haryana, India

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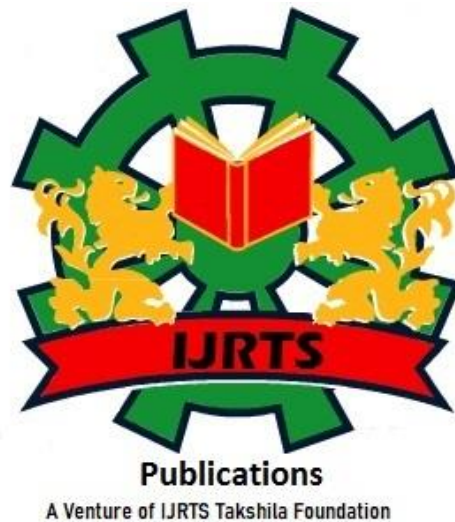
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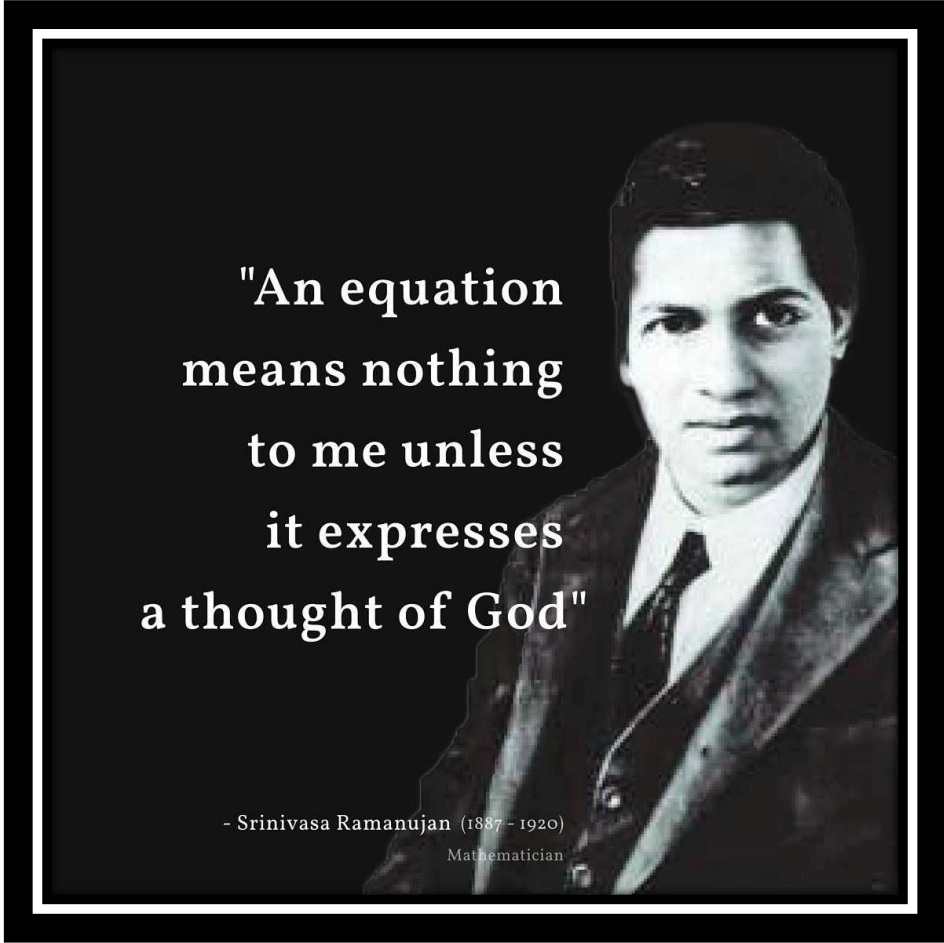
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**"An equation  
means nothing  
to me unless  
it expresses  
a thought of God"**

- Srinivasa Ramanujan (1887 - 1920)  
Mathematician

Srinivasa Ramanujan (born as Srinivasa Ramanujan Aiyangar, IPA: [sriːnivaːsa raːmaːnudz̪an ajːaŋgar], 22 December 1887 - 26 April 1920) was an Indian mathematician. Though he had almost no formal training in pure mathematics, he made substantial contributions to mathematical analysis, number theory, infinite series, and continued fractions, including solutions to mathematical problems then considered unsolvable. During his short life, Ramanujan independently compiled nearly 3,900 results (mostly identities and equations). Many were completely novel; his original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and mock theta functions, have opened entire new areas of work and inspired a vast amount of further research. Of his thousands of results, all but a dozen or two have now been proven correct.

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# Mathematics in the Modern World

Dr. Neha Mittal

Assistant Professor in Mathematics  
Govt. College Julana, Jind, Haryana, India  
nnehajain1986@gmail.com

## Abstract

Mathematics is often considered one of the most challenging subjects in school. But despite its reputation, it plays a crucial role in the modern world. From the technology we use every day to the complex systems that govern our society, mathematics is essential to understanding and solving the problems we face. In this chapter, we'll explore the role of mathematics in the modern world, including its relevance in science, technology, business, and society. We'll also discuss why math matters for individuals and how it can help us improve our lives.

## Introduction

Mathematics is a fundamental part of our everyday lives, and it is the language of science, technology, and engineering. From calculating the tip on a restaurant bill to designing the latest smartphone, mathematics is integral to many aspects of modern society. However, its influence extends beyond just practical applications. Mathematics can also help us understand the deeper complexities of the world, such as patterns in nature, social dynamics, and economic systems.

One area where math has made significant contributions is in the study of climate change. Through mathematical modelling, scientists can better understand the dynamics of the earth's climate system and predict the potential impacts of global warming. Similarly, math plays a vital role in understanding the spread of infectious diseases and developing effective strategies for disease control.

Mathematics is also a powerful tool for exploring complex social issues, such as income inequality and political polarization. By using mathematical models, researchers can identify underlying patterns and trends in data and develop more accurate predictions of future

outcomes. This can lead to more informed policy decisions and a better understanding of social dynamics.

At its core, math is about problem-solving and critical thinking. It teaches us to analyze data, identify patterns, and develop creative solutions to complex problems. These skills are essential in today's fast-paced, data-driven world and are highly valued in many industries, including finance, healthcare, and technology.

Our understanding of math and the constant desire to know more about everything in this world so that, today, the world as we know it would not be possible without maths.

Mathematics is an application of matter and contributes to all of our methodical and systematic behaviours.

It is Maths, for instance, that has brought order to the communities across this planet and prevented chaos and catastrophes. Many of our inherited human qualities are nurtured and developed by Maths theories, like our spatial awareness, our problem-solving skills, our power to reason (which involves calculated thinking) and even our creativity and communication.

### **Why Is Maths Important?**

Things that you wouldn't expect to bear any relation to Maths do in fact come down to an underlying need for mathematics and the structure it brings to our everyday lives.

Take shopping, cooking, buying a property, doing DIY, travelling, gambling, playing video games, driving and telling the time, for instance... none of these would be possible without the existence of Mathematics.

Furthermore, Maths is everywhere when you consider the educational and professional worlds.

Maths is relevant to a wide variety of academic subjects on the GCSE course and A Level curriculum, which not only means that a poor understanding of maths can cause students to struggle in many other subjects but also that it could limit their further studies options as well as their employability when they come to think about career options.

Whether you aspire to study sociology, psychology, physics, biology or even economics, maths is held in high regard, and you will be called on solve various maths problems, as part of your work.

### **Mathematics in Science:**

Mathematics and science go hand in hand, with math providing the language and tools for understanding the natural world. Without math, many of the scientific discoveries that have shaped our world would not be possible. Here are a few ways in which math is essential to science:

**Data analysis:** Scientists use mathematical tools to analyze data and identify patterns and trends. This is crucial for making accurate predictions and understanding complex systems.

**Modelling:** Mathematical models allow scientists to simulate real-world phenomena, such as climate change or the spread of disease. These models can help us understand the impact of different variables and predict how the system will behave in the future.

**Quantitative reasoning:** Math helps scientists think logically and systematically, making it easier to design experiments, interpret results, and draw conclusions.

### **Mathematics in Technology:**

Mathematics is also essential to modern technology, from smartphones and computers to advanced medical devices and space exploration. Here are a few ways in which math is crucial to technology:

**Coding:** Programming languages are based on mathematical logic, making math skills essential for software development and coding.

**Engineering:** Engineers use math to design and optimize everything from bridges to airplanes. Without math, these complex structures would not be possible.

**Data science:** Data scientists use math to analyze and interpret large datasets, helping businesses and organizations make better decisions.

### **Mathematics in Business:**

Mathematics plays a critical role in the world of business, from financial analysis to marketing research. Here are a few ways in which math is essential to business:

**Statistics:** Businesses use statistics to analyze data and make informed decisions. This can include everything from market research to forecasting sales.

**Accounting:** Accountants use math to prepare financial statements, balance budgets, and ensure compliance with tax laws.

**Investing:** Investors use math to analyze market trends and make investment decisions. This can include calculating risk and return, analyzing stock prices, and predicting market movements.

### **Mathematics in Society:**

Finally, math is crucial to understanding and solving the problems that society faces. Here are a few examples of how math is essential to society:

**Infrastructure:** Math is essential to designing and maintaining infrastructure, from roads and bridges to power grids and water systems.

**Policymaking:** Governments use math to inform policy decisions, such as determining the optimal tax rate or predicting the impact of a new law.

**Social justice:** Math can also be used to address social justice issues, such as determining fair voting districts or calculating reparations for historical injustices.

### **Conclusion**

Mathematics is a critical tool in the modern world. It is used in everything from engineering and architecture to medicine and finance. It helps us understand the world around us and make decisions based on data.

In the past, mathematics was used primarily for practical purposes. Today, we also use mathematics for its aesthetic value. We appreciate the beauty of a well-designed mathematical proof or the elegance of a mathematical formula.



Mathematics is a critical tool in the modern world. It is used in everything from engineering and architecture to finance and medicine. A strong understanding of mathematics is essential for success in any field.

There are many benefits to studying mathematics. It develops problem-solving skills, logical thinking, and spatial reasoning.

Mathematics also develops concentration and discipline. It can be difficult to master, but the rewards are well worth the effort.

Those who study mathematics have better job prospects and earn higher salaries than those who do not. mathematical skills are in high demand in many industries.

There is a great deal of satisfaction that comes from solving complex mathematical problems. It is a mentally stimulating activity that can be enjoyed by people of all ages.

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Publications

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# Semiconductor Diode and its Characteristics

Dr. Surender Kumar

Assistant Professor in Physics

Govt. College Julana, Jind, Haryana, India

iitatmotivation@gmail.com

## Abstract

This chapter deals with the physics behind the semiconductors. Here, the band structure of semiconductor is explained with its comparative study with the metals and insulators. The formation of energy band in case of semiconductor is explained from energy levels when inter-atomic distance between the semiconductors is decreased. Intrinsic semiconductors are also explained with the introduction of doping technique to enhance the level of conductivity in semiconductors. Formation and design of extrinsic semiconductors are also dealt with the concept of potential barrier and depletion region. Types of extrinsic semiconductors – P type and N type semiconductors are also given with technique of biasing. Forward and reverse biasing methods are explained to get the knowledge of voltage and current characteristics for each. Diode equation is given for getting the value of current of diode and lastly, uses of diodes in various field are explained.

## 1. Introduction

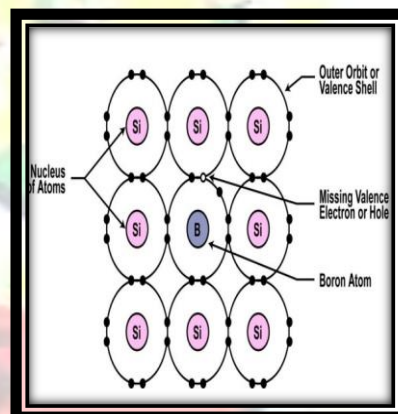
Electronics is the branch of science that deals with the phenomena of current flowing through a vacuum, gas, or semiconductor. The material itself is made up of very small particles known as atoms. A molecule comprises of a focal core of positive charge (nucleus) around which little adversely charged particles called electrons rotate in circular orbits. In 1913, Neil Bohr provided detailed explanation for the structure of an atom. The generation and control of an electrical current are the main objectives of electronic materials. Based on electronic behavior, materials can be divided into three broad categories – Conductors, Insulators and Semiconductors. Conductors have low obstruction which permits electrical flow stream without any problem. For example -- Copper, silver, gold, aluminum and so on. Insulators have high opposition which don't permit electrical flow stream through them. For example -- Wood, glass, ceramics, plastics, and so on. Semiconductors are those materials that has properties somewhere between the conductors and insulators i.e. at room temperature, they



behave as conductors while at absolute zero temperature, these behave as insulators. For example -- Silicon, carbon, germanium, and others. Silicon is awesome and most broadly utilized semiconductor. In good conductors, generally single valence electron is found in the outermost shell, which can be easily taken out resulting in the conduction of current across them. A semiconductor element's outer or valence orbit contains four electrons. The unique ability of semiconductor atoms is to link together to form a physical structure known as a crystal lattice. The outer electrons that link the atoms together are shared by all of them resulting in the formation of Covalent bonds between them. The crystal lattice structure of pure semiconductor materials, such as silicon, functions as an excellent insulator because all of the atoms are bound to one another and electrons cannot flow freely. The term "intrinsic" refers to pure semiconductor material that is found to be a good insulator. In order to enhance the conductivity of semiconductors, other atoms known as dopants or impurities must be added to the semiconductor by the process called as doping.

**N-Type Semiconductor:** By using the technique of Doping,

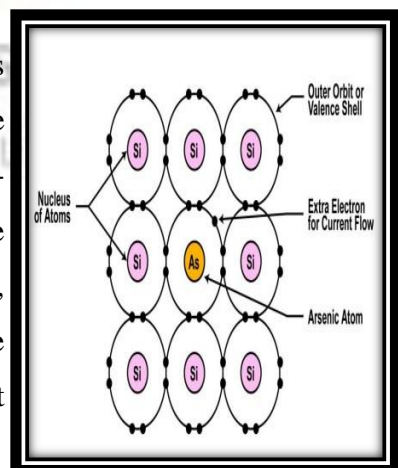
N-type semiconductor (normally called as Donors) is formed, in which impurity or dopant atoms are added to intrinsic semiconductor and these impurity atoms are from fifth group of periodic table i.e. pentavalent atoms (having five valence electrons). Example of these pentavalent impurities -- Arsenic, Phosphorus, Antimony etc. In these N-type semiconductors, four of the arsenic valence electrons



can bond with the four neighboring silicon atoms while the remaining fifth electron for each arsenic atom can now act as free electron and help in the conduction of Current thereby increasing the conductivity.

**P-Type Semiconductor:**

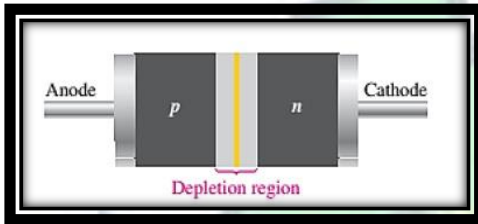
If intrinsic semiconductor is doped by third group elements as dopant or impurity, it will result in the formation of P-type semiconductor, popularly called as Acceptors. Example of P-type impurities i.e. trivalent atoms having three valence electrons in their outermost shell are listed as Aluminum, Boron, Gallium, Indium etc. In P-type semiconductors, the three electrons of dopant or impurity atoms form covalent





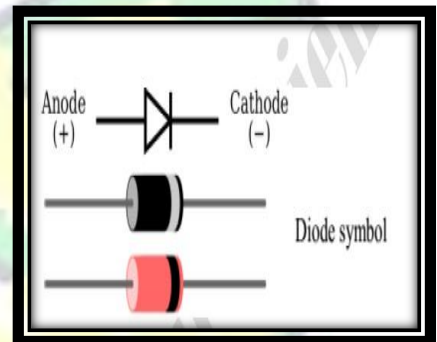
bonds with the three electrons of semiconductor atoms that are nearby. However, the bond lacks one electron resulting in the formation of Hole at the position where a fourth electron was ought to be found. To attract electrons from another source, the hole takes on a positive charge. For maintaining the flow of charge across semiconductor, these holes transform into a type of current carrier similar to an electron.

**2. Semiconductor Diode or P-N Junction Diode:**

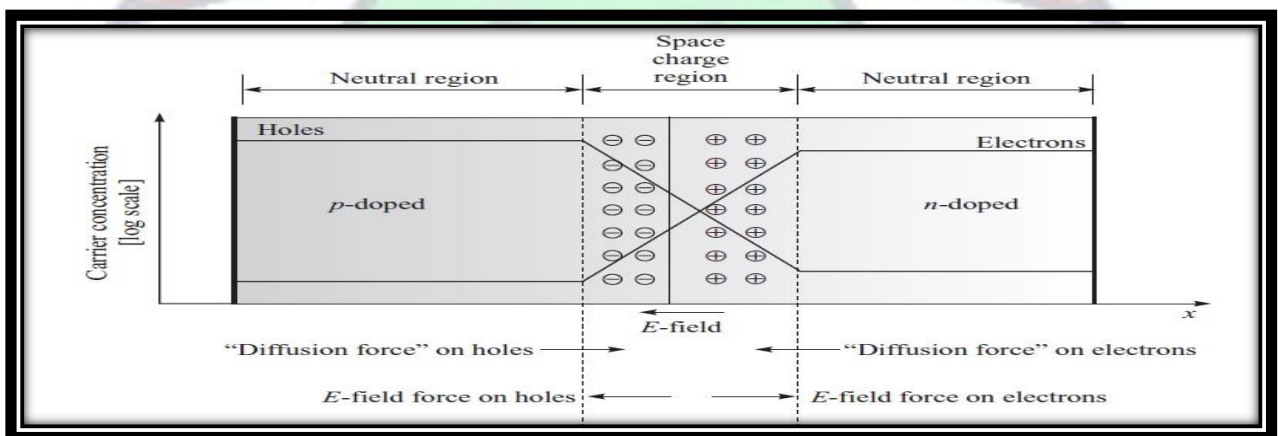


Whenever a p-type semiconductor material is joined to n-type semiconductor, then the contact surface is known as a P-N junction. The p-n junction is therefore called as semiconductor diode.

The term junction alludes to the limit interface where the two locales of the semiconductor meet. A diode is a basic electrical device that permits the progression of charge flow just in one direction just like a valve or may be said to be acting like a switch. These are made from a single semiconductor base by applying the technique of doping to it which may include process for ion implantation, dissemination of dopants, or technique of epitaxy for growing one kind of semiconductor layer over other resulting in the PN junction diode formation. These are also known as the basic building structures of all semiconductor based electronic devices.



**Origin of Depletion region:**



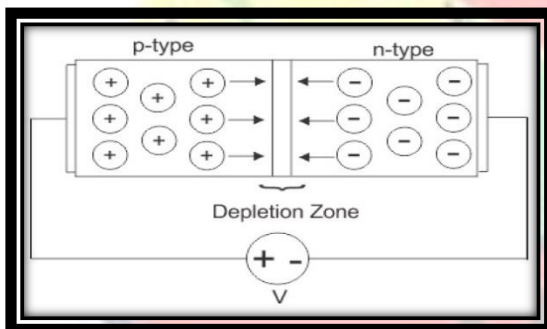
**Fig. 2: Origin of Depletion region between P-N Junction Diode**

In the above fig.2 showing origin of depletion region between P-N Junction, it is shown that the electric field developed due to the movement of charge carriers from one region to other is opposite to the direction of diffusion of charge carriers i.e. electrons from N to P side and holes from P to N side of P-N Diode. Thus, it can be summarized that two opposite phenomena are occurring due to production of PN Junction Diode in which the appearing depletion region act as the potential barrier which stops the further process of diffusion of charge carrier from one side to another.

### 3 Principle of P-N Junction Diode:

The principle behind the working of PN Junction diode is the ability of diode to transform the current in only one direction i.e. when P type is connected to positive terminal and N type is connected to negative terminal of battery. it results in the conduction of charge carrier across it i.e. it is said to be in ON position while in the reverse position, it do not conducts the charge carries and said to be in OFF position.

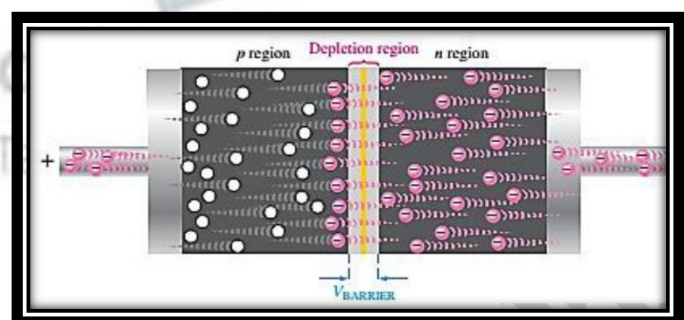
#### 3.1 Forward Biasing of P-N Junction Diode



In general, Biasing is the way of applying dc voltage to the given PN Junction diode such that this applied voltage compels the diode to either permit the majority charge carrier to pass through it or stops the flow of majority charge carrier across it. Forward bias is the state of PN

Junction Diode in which it permits the majority charge carrier to pass through it and the minimum external voltage at which this happens, is known as the cut-in or knee voltage or bias voltage. Here, the negative side of external battery is connected to the N type region and the other side of battery is connected to the P type region of junction diode.

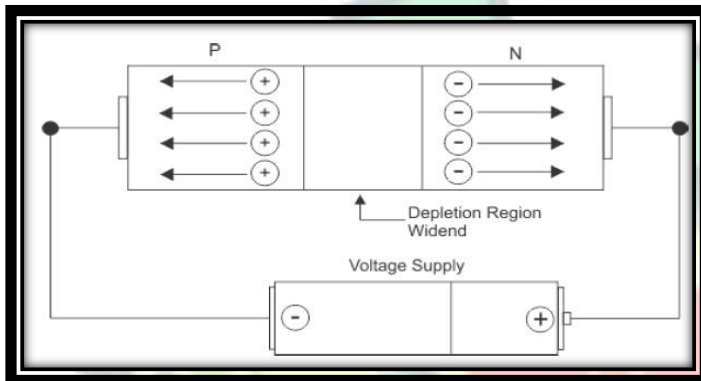
The applied value of external bias voltage must be greater than the potential barrier appearing at the junction of PN Diode i.e. it must be more than 0.3Volt in case of germanium semiconductor and 0.7 Volt in case of silicon semiconductor PN diodes.



From the negative terminal of applied external battery, free electrons are repelled and made to cross the depletion region and reach the other side of diode i.e. it is responsible for the flow of majority charge carrier as electrons from N to P side of the junction diode. The applied

external voltage or Bias voltage gives enough energy to these free electrons of N side so that they cross the depletion region and move to the other side i.e. P side of diode. Also, the positive terminal of battery at P side attracts these coming charge carriers i.e. electrons towards it. Similarly, the holes flow as majority charge carriers from P side to N side of the junction diode resulting in the working of PN Junction diode in forward bias condition.

### 3.2 Reverse Biasing of P-N Junction Diode



In this condition of biasing, majority charge carriers are prevented from going one side to other side of junction diode. The positive terminal of external battery is connected to the N type region of the PN diode and the negative terminal

is connected to the P type region of diode. Also, it is worth to point here that due to this type of biasing arrangement, depletion region keeps on widening resulting in the stopping if flow of majority charge carriers from one side to another. The positive terminal of the external battery connected at N type region attracts the majority charge carriers of that region i.e. free electrons away from the junction region resulting in widening of depletion region. Similarly, in P-type side majority charge carrier i.e. holes are attracted towards the connected negative terminal side of the battery, which further helps in more widening of the depletion region. This arrangement is helpful in stopping the flow of majority charge carriers but allows the flow of minority charge carriers i.e. holes from N type region and electrons from P type region towards opposite ends resulting in very small amount of current arising due to flow of these minority charge carriers, which can be ignored at this stage as it is very small.

### 4 Characteristics of PN Junction Diode (V-I Graph)

For getting the characteristics of P-N junction diode, we will apply the voltage across the ends of diode and note down the current variation with the help of ammeter and by plotting the graph by taking voltage along X-axis and current along Y-axis, we will get the desired graph depicting the characteristics of given diode. Here, the external battery applied is having voltage  $V$ . P-N Junction diode is taken along whose ends this external battery  $V$  is applied. One external resistance  $R$  is also applied in series which is working as a current limiting factor in this arrangement. For P-N junction diode made up of silicon semiconductor, the

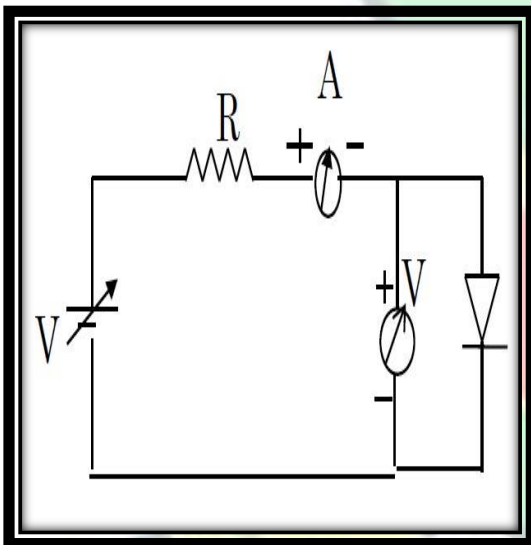


potential barrier or knee voltage is 0.7 Volt and for Ge semiconductor diode, this barrier voltage has value equal to 0.3 Volt.

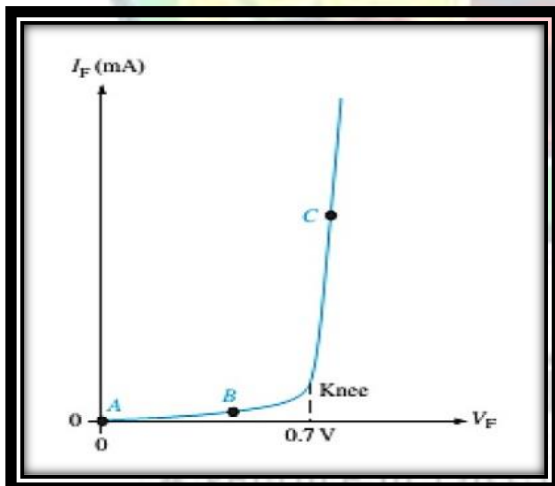
There are two modes of biasing in which we can study the V-I characteristics of given P-N junction diode:

1. Forward biased P-N Diode
2. Reverse Biased P-N Diode

#### 4.1 Forward Biasing (V-I characteristics)



For studying the V-I characteristics in forward biased mode of PN junction diode, P side of diode is connected to positive terminal of battery and N side is connected to negative terminal of battery. Voltmeter is connected in parallel to note down the voltage applied and the ammeter connected in series for getting the values of current. Resistance R is also connected in series. The value of current obtained in this condition is called as **Forward current ( $I_F$ )**.

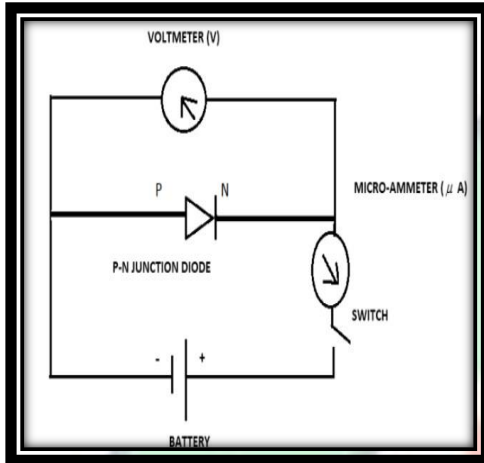


In the graph obtained for forward biased junction diode, it is seen that at zero voltage across ends of diode,  $V_F = 0$ , no current is obtained i.e.  $I_F = 0$ . Gradually, the applied voltage i.e. forward voltage is increased and the corresponding forward current is noted down with the help of ammeter. The resistance R is serving here the task of limiting the forward current ( $I_F$ ) up to such a safe value that will not allow the diode beyond limit and will prevent it

from damage. By careful study of the graph obtained for forward biased PN diode, it was seen that the current starts appearing only when applied voltage crosses the potential barrier voltage i.e. 0.7 Volt. After crossing the potential barrier, there is rapid increase in the forward current with the increase in forward voltage  $V_F$ . It is also to be noted here that the resistance

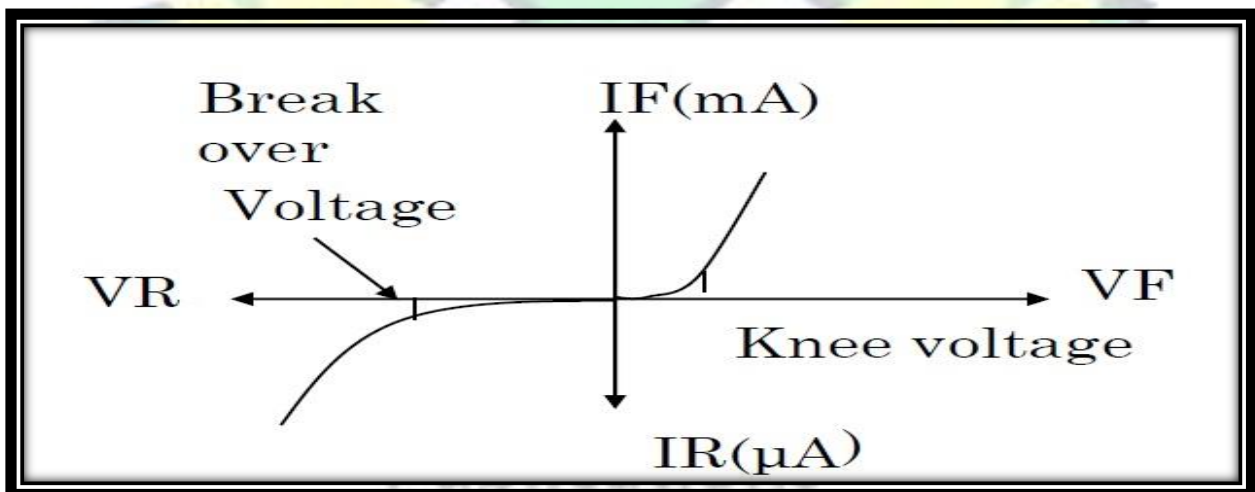
of forward biased PN junction diode is not static and is rather dynamic in nature, whose value keeps on varying during the entire process.

#### 4.2 Reverse Biasing (V-I characteristics)



For obtaining the desired graph for voltage-current characteristics of reverse biased PN Junction diode, P type of diode is connected to negative terminal of battery while positive terminal of external battery is applied to N side of given PN junction diode. With applied zero reverse voltage, no reverse current is found in ammeter i.e.  $I_R = 0$  for applied  $V_R = 0$ . As the reverse biased voltage is gradually increased, very small amount of reverse current is obtained due to the

flow of minority charge carriers across the ends of diode. At a particular reverse voltage called as breakdown voltage, there is tremendous increase in the value of reverse bias current. This tremendous increase is due to phenomena of Zener breakdown and Avalanche breakdown of PN junction diode. Beyond the value of this breakdown voltage, the value of reverse current  $I_R$  shows tremendous growth in value.



**Figure 4: Voltage-Current (V-I) characteristics for both forward and reverse biased PN junction diode**

#### 5. PN Junction Diode Equation

The amount of current obtained in the given PN Junction diode can be obtained with the help of Diode current equation expressed as:

$$I = I_0 \{ \exp. \left( \frac{V}{\eta V_T} \right) - 1 \} \text{ ----- (1)}$$

Here,

I = Amount of Current obtained in PN Diode

I<sub>0</sub> = Amount of saturation current obtained in reverse mode.

V = Voltage applied across PN junction diode

η = Constant for given Semiconductor diode i.e.

η = 1 in case of Germanium semiconductor

η = 2 in case of Silicon semiconductor.

V<sub>T</sub> = Voltage taken as equivalent for temperature, calculated

as T/11,600 (temperature T is taken in Kelvin

## 6. Uses of PN Junction diode

- a) It can be used as half wave rectifier and full wave rectifier.
- b) It can be used as a switch in electric circuits.
- c) It has wide applications in clipping and clamping circuits.
- d) It can be used as signal detector in signal receivers.

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# Inertial Accelerated Algorithm for Variational Inequality and Generalized Variational Inequality Problems

Nishu Gupta

Assistant Professor in Mathematics  
Govt. College Julana, Jind, Haryana, India  
nishugupta2771@gmail.com

## Abstract

In this chapter, we find the common solution of variational inequality problem and generalized variational inequality problem. We prove strong convergence result using inertial type iterative algorithm under certain mild conditions in the frame of real Hilbert spaces. The suggested algorithm does not require prior knowledge of Lipschitz constant of the associated pseudomonotone operator. This result is improvement and generalization of many recent results in this direction.

## AMS Classification

49J40; 47J20; 47H10; 47J25; 90C33

## 1. Introduction

Stampacchia [1] in 1964, proposed the variational inequality problem (VIP) in finite dimensional spaces for researching a large variety of interesting problems emerging in various fields like partial differential equations, optimization theory, economic equilibrium mechanics and game theory; see [2-5]. VIPs have been used in various areas of theory and practice because of its importance in optimization and the nonlinear analysis; see, forexample, [6-9].

VIP identifies a point  $\tilde{u}$  in a nonempty closed convex subset  $C$  of Hilbert space  $H$  such that

$$\langle g(\tilde{u}), u - \tilde{u} \rangle \geq 0 \text{ for all } u \in C, \quad (1)$$

where  $g : C \rightarrow H$  is a mapping. Solution set of VIP is represented by  $VI(C, g)$ . We know that VIP can be solved as fixed point problem (FPP) i.e.



$$\tilde{u} \in VI(C, g) \text{ if and only if } \tilde{u} = P_C(I - \gamma g)\tilde{u}, \quad (2)$$

where  $\gamma > 0$  and  $P_C$  is a metric projection of  $H$  onto  $C$ ; see [10]. For studying variational inequality problem, one of the most commonly used method is projection method and this method can be devoted to establish the equivalence between fixed point problem and VIP. A great deal of algorithms and a lot of work have been considered for solving generalized variational inequality problems (GVIPs) or variational inequality problems (VIPs) by many authors, see [11-19]. GVIP is the problem of identifying  $\tilde{u} \in C$  such that

$$\langle f(\tilde{u}), h(u) - h(\tilde{u}) \rangle \geq 0 \text{ for all } u \in C, \quad (3)$$

where  $f : C \rightarrow H$  and  $h : C \rightarrow C$  are nonlinear operators. If  $h = I$ , then GVIP reduces to VIP. For further new results in this direction, see [20-26].

In this chapter, we are interested in obtaining a common solution of VIP and GVIP which is described as identifying  $\tilde{u} \in H_1$  such that

$$\begin{cases} \tilde{u} \in GVI(C_1, f, h), \\ h(\tilde{u}) \in VI(C, g), \end{cases} \quad (4)$$

where  $f : C_1 \rightarrow H_1$ ,  $g : h_1 \rightarrow H_1$  and  $h : C_1 \rightarrow C_1$  are nonlinear operators. Let solution set of problem (4) be represented by  $\Omega$ .

Inertial extrapolation technique was introduced by Polyak [27] in 1964 and it is employed to fasten the rate of convergence of iterative algorithms [28-32]. In this chapter, we describe a novel inertial type algorithm for generating a common solution of VIP and GVIP.

## 2. Preliminaries

In this section, we state some useful and well-known results that are essential to prove our main theorem. Assume that  $H$  is a real Hilbert space with inner product  $\langle \cdot, \cdot \rangle$  and norm  $\|\cdot\|$  and  $C$  is a nonempty closed convex subset of Hilbert space  $H$ . We use the notations of weak and strong convergence as “ $\rightharpoonup$ ” and “ $\rightarrow$ ”, respectively.

**Lemma 2.1.** [33] Let  $H$  be a real Hilbert space, then



(i)  $2\langle p, q \rangle = \|p\|^2 + \|q\|^2 - \|p - q\|^2 = \|p + q\|^2 - \|p\|^2 - \|q\|^2$  for all  $p, q \in H$ .

(ii)  $\|p + q\|^2 \leq \|p\|^2 + 2\langle q, p + q \rangle$  for all  $p, q \in H$ .

(iii)  $\|\chi p + (1 - \chi)q\|^2 = \chi\|p\|^2 + (1 - \chi)\|q\|^2 - \chi(1 - \chi)\|p - q\|^2$  for all  $p, q \in H$  and  $\chi \in [0, 1]$ .

**Lemma 2.2.** [34] Let  $G: H \rightarrow H$  be pseudomonotone on  $C$  and  $L$ -Lipschitz continuous on  $H$  such that the solution set  $VIP(C, G)$  of variational inequality problem is nonempty. For  $w \in H, \zeta \in (0, 1)$  and  $\lambda > 0$ , define

$$\begin{cases} u = P_C(w - \lambda G(w)), \\ v = P_Q(w - \lambda G(u)), \\ Q = \{x \in H; \langle w - \lambda G(w) - u, x - u \rangle \leq 0\}, \\ \gamma = \begin{cases} \min \left\{ \frac{\zeta \|w - u\|}{\|G(w) - G(u)\|}, \lambda \right\} & \text{if } G(w) \neq G(u), \\ \lambda & \text{if } G(w) = G(u). \end{cases} \end{cases}$$

Then for all  $\tilde{t} \in VIP(C, G)$

$$\|v - \tilde{t}\|^2 \leq \|w - \tilde{t}\|^2 - \left(1 - \zeta \frac{\lambda}{\gamma}\right) \|w - u\|^2 - \left(1 - \zeta \frac{\lambda}{\gamma}\right) \|u - v\|^2.$$

**Lemma 2.3.** [35] Let  $\{r_n\} \subset [0, \infty), \{\delta_n\} \subset (0, 1)$  and  $\{\chi_n\}$  be three real sequences satisfying

$$r_{n+1} \leq (1 - \delta_n)r_n + \delta_n \chi_n \text{ for all } n \geq 0. \tag{5}$$

Further, if  $\sum_{n=0}^{\infty} \delta_n = \infty$  and  $\limsup_{n \rightarrow \infty} \chi_n \leq 0$ , then  $\lim_{n \rightarrow \infty} r_n = 0$ .

**Lemma 2.4.** [36] Assume that there exists a subsequence  $\{n_i\}$  of  $\{n\}$  satisfying  $\chi_{n_i} < \chi_{n_{i+1}} \forall i \in \mathbb{N}$  for a real number sequence  $\{\chi_n\}$ . Then a nondecreasing sequence  $\{m_l\} \subset \mathbb{N}$  exists such that  $m_l \rightarrow \infty$  and

$$\chi_{m_l} \leq \chi_{m_l+1} \text{ and } \chi_l \leq \chi_{m_l+1}, \tag{6}$$

for all (sufficiently large) numbers  $l \in \mathbb{N}$ , where  $m_l = \max\{q \leq l : \chi_q < \chi_{q+1}\}$ .

### 3. Main Result

In this section, we present a new inertial type iterative algorithm to find the common solution of VIP and GVIP. Assume that

- i.  $h : C_1 \rightarrow C_1$  is  $\lambda$ -strongly monotone and weakly continuous.
- ii.  $f : C_1 \rightarrow H_1$  is  $\theta$ -inverse strongly  $h$ -monotone operator.
- iii.  $\varphi : C_1 \rightarrow C_1$  is  $\rho$ -contractive operator.
- iv.  $g : H_1 \rightarrow H_1$  is  $L$ -Lipschitz continuous on  $H_1$  and pseudomonotone on  $C_1$  such that  $\limsup_{n \rightarrow \infty} \langle g(p_n), q - q_n \rangle \leq \langle g(\tilde{p}), q - \tilde{q} \rangle$  for every sequence  $\{p_n\}, \{q_n\}$  in  $H_1$  converging weakly to  $\tilde{p}$  and  $\tilde{q}$  respectively.

Now, we present our iterative algorithm for solving Problem (4).

**Algorithm 3.1.** Choose  $\zeta_0 > 0, \zeta \in (0, 1), \{\alpha_n\} \subset (0, 1)$  such that  $\lim_{n \rightarrow \infty} \alpha_n = 0$  and  $\sum_{n=1}^{\infty} \alpha_n = \infty$  and  $\{\tau_n\} \subset (0, 1)$  such that  $0 < \liminf_{n \rightarrow \infty} \tau_n \leq \limsup_{n \rightarrow \infty} \tau_n < 1$  and  $\sum_{n=0}^{\infty} \tau_n = \infty$ . Let  $\{\kappa_n\}$  be real number sequence in  $(0, \infty)$  such that  $0 < \liminf_{n \rightarrow \infty} \kappa_n \leq \limsup_{n \rightarrow \infty} \kappa_n < 2\theta$ . Let  $x_0 \in C_1$  and define sequence  $\{x_n\}$  as

$$\left\{ \begin{array}{l} h(v_n) = h(x_n) + \Theta_n(x_n - x_{n-1}), \\ y_n = \alpha_n \phi(v_n) + (1 - \alpha_n) P_{C_1}(h(v_n) - \kappa_n f(v_n)), \\ z_n = P_{C_1}(y_n - \zeta_n g(y_n)), \\ u_n = P_{C_n}(y_n - \zeta_n g(z_n)), \\ h(x_{n+1}) = (1 - \tau_n)h(x_n) + \tau_n u_n, \end{array} \right. \tag{7}$$

where

$$C_n = \{w \in H_1 : \langle y_n - \zeta_n g(y_n) - z_n, w - z_n \rangle \leq 0\}, \quad (8)$$

$$\zeta_{n+1} = \begin{cases} \min \left\{ \frac{\zeta \|y_n - z_n\|}{\|g(y_n) - g(z_n)\|}, \zeta_n \right\} & \text{if } g(y_n) \neq g(z_n), \\ \zeta_n & \text{if } g(y_n) = g(z_n), \end{cases} \quad (9)$$

and  $0 \leq \Theta_n \leq \tilde{\Theta}$  with

$$\Theta_n = \begin{cases} \min \left\{ \frac{\tilde{\Theta}}{2}, \frac{\alpha_n}{\|x_n - x_{n-1}\|} \right\} & \text{if } x_n \neq x_{n-1} \\ \frac{\tilde{\Theta}}{2} & \text{otherwise} \end{cases}$$

where  $\alpha_n = o(\frac{1}{n})$  is positive sequence i.e.  $\lim_{n \rightarrow \infty} \frac{\alpha_n}{n} = 0$ .

Now, we prove Strong convergence of the suggested algorithm.

**Theorem 3.2.** Assume that the solution set  $\Omega$  as defined in equation (4) is nonempty and  $0 < \rho < \lambda < 2\theta$ , then  $\{x_n\}$  generated by Algorithm 3.1 strongly converges to an element  $\tilde{t} \in \Omega$  satisfying  $\langle \phi(\tilde{t}) - h(\tilde{t}), h(x) - h(\tilde{t}) \rangle \leq 0$  for all  $x \in \Omega$ .

**Proof.** Firstly, we prove  $\{x_n\}$  is bounded. Let  $h(\tilde{t}) \in \Omega$ . Since  $\lim_{n \rightarrow \infty} \frac{\alpha_n}{n} = 0$ , there exists

$M_1 > 0$  such that  $\alpha_n \leq \frac{1}{n} M_1$ . Now,

$$\begin{aligned} \|h(x_n) - h(\tilde{t})\| &\leq \|h(x_n) - h(\tilde{t})\| + \Theta_n \|x_n - x_{n-1}\| \\ &\leq \|h(x_n) - h(\tilde{t})\| + \alpha_n \\ &\leq \|h(x_n) - h(\tilde{t})\| + \frac{1}{n} M_1. \end{aligned} \quad (10)$$

Since  $f$  is  $\theta$ -inverse strongly  $h$ -monotone and  $0 < \liminf_{n \rightarrow \infty} \kappa_n \leq \limsup_{n \rightarrow \infty} \kappa_n < 2\theta$ , therefore

$$\begin{aligned}
 & \| (h(v_n) - \kappa_n f(v_n)) - (h(\tilde{t}) - \kappa_n f(\tilde{t})) \|^2 \\
 & \quad = \| h(v_n) - h(\tilde{t}) \|^2 - 2\kappa_n \langle f(v_n) - f(\tilde{t}), h(v_n) - h(\tilde{t}) \rangle \\
 & \quad \quad + \kappa_n^2 \| f(v_n) - f(\tilde{t}) \|^2 \\
 & \quad \leq \| h(v_n) - h(\tilde{t}) \|^2 - 2\kappa_n \theta \| f(v_n) - f(\tilde{t}) \|^2 + \kappa_n^2 \| f(v_n) - f(\tilde{t}) \|^2 \\
 & \quad \leq \| h(v_n) - h(\tilde{t}) \|^2 + \kappa_n(\kappa_n - 2\theta) \| f(v_n) - f(\tilde{t}) \|^2 \tag{11} \\
 & \quad \leq \| h(v_n) - h(\tilde{t}) \|^2 . \tag{12}
 \end{aligned}$$

From the  $\lambda$  – strong monotonicity of  $h$  , we deduce

$$\| h(p) - h(q) \| \geq \lambda \| p - q \| \text{ for all } p, q \in C_1.$$

As  $\phi$  is  $\rho$  – contraction and  $h$  is  $\lambda$  – strongly monotone operator, therefore using equation (12), we get

$$\begin{aligned}
 \| y_n - h(\tilde{t}) \| &= \| {}_n\phi(v_n) + (1 - {}_n)P_{C_1} (h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \| \\
 &= \| {}_n\phi(v_n) + (1 - {}_n)P_{C_1} (h(v_n) - \kappa_n f(v_n)) - P_{C_1} (h(\tilde{t}) - \kappa_n f(\tilde{t})) \| \\
 &\leq \| {}_n(\phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t})) + (1 - {}_n)((h(v_n) - \kappa_n f(v_n)) \\
 &\quad - (h(\tilde{t}) - \kappa_n f(\tilde{t}))) \| \\
 &\leq {}_n \| \phi(v_n) - \phi(\tilde{t}) \| + {}_n \| \phi(\tilde{t}) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \| \\
 &\quad + (1 - {}_n) \| (h(v_n) - \kappa_n f(v_n)) - (h(\tilde{t}) - \kappa_n f(\tilde{t})) \| \\
 &\leq {}_n \rho \| v_n - \tilde{t} \| + {}_n \| \phi(\tilde{t}) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \| + (1 - {}_n) \| h(v_n) - h(\tilde{t}) \| \\
 &\leq {}_n \frac{\rho}{\lambda} \| h(v_n) - h(\tilde{t}) \| + {}_n \| \phi(\tilde{t}) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \| + (1 - {}_n) \| h(v_n) - h(\tilde{t}) \| \\
 &\leq [1 - (1 - \frac{\rho}{\lambda}) {}_n] \| h(v_n) - h(\tilde{t}) \| + {}_n (\| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \|). \tag{13}
 \end{aligned}$$

Also, from equation (11), we have

$$\begin{aligned}
 \| y_n - h(\tilde{t}) \|^2 &\leq \| {}_n(\phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t})) \\
 &\quad + (1 - {}_n)((h(v_n) - \kappa_n f(v_n)) - (h(\tilde{t}) - \kappa_n f(\tilde{t}))) \|^2 \\
 &\leq {}_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2 \\
 &\quad + (1 - {}_n) \| (h(v_n) - \kappa_n f(v_n)) - (h(\tilde{t}) - \kappa_n f(\tilde{t})) \|^2 \\
 &\leq {}_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2
 \end{aligned}$$

$$+(1-\zeta_n) [\|h(v_n) - h(\tilde{t})\|^2 + \kappa_n(\kappa_n - 2\theta) \|f(v_n) - f(\tilde{t})\|^2]. \quad (14)$$

Further, from Lemma 2.2 and definition of  $\{t_n\}$ , we have

$$\|u_n - h(\tilde{t})\|^2 \leq \|y_n - h(\tilde{t})\|^2 - \left(1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}\right) \|y_n - z_n\|^2 - \left(1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}\right) \|z_n - u_n\|^2. \quad (15)$$

Since  $g$  is  $L$ -Lipschitz continuous on  $H_1$ , we have

$$\|g(y_n) - g(z_n)\| \leq L \|y_n - z_n\|. \quad (16)$$

Then by induction for all  $n \geq 0$ , we have  $\zeta_n \geq \min\left(\frac{\zeta}{L}, \zeta_0\right) > 0$ . By the definition of  $\zeta_{n+1}$ , we have  $\zeta_{n+1} \leq \zeta_n$  for all  $n \geq 0$  which implies that the limit of  $\{\zeta_n\}$  exists. Let  $\lim_{n \rightarrow \infty} \zeta_n = \zeta^*$ , then

$\zeta^* \geq \min\left(\frac{\zeta}{L}, \zeta_0\right) > 0$ . From  $\lim_{n \rightarrow \infty} \zeta_n = \zeta^*$ , we get

$$\lim_{n \rightarrow \infty} \left(1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}\right) = 1 - \zeta > 0. \quad (17)$$

Therefore, there exists  $n_0 \in \mathbb{N}$  such that  $1 - \zeta \frac{\zeta_n}{\zeta_{n+1}} > 0$  for all  $n \geq n_0$ . From equations (15) and

(17), we have

$$\|t_n - h(\tilde{t})\| \leq \|y_n - h(\tilde{t})\| \text{ for all } n \geq n_0. \quad (18)$$

Now, using equations (10), (13) and (18), we get



$$\begin{aligned}
 \|h(x_{n+1}) - h(\tilde{t})\| &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\| + \tau_n \|u_n - h(\tilde{t})\| \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\| + \tau_n \left[ \left(1 - \left(1 - \frac{\rho}{\lambda}\right)_n\right) \|h(v_n) - h(\tilde{t})\| \right. \\
 &\quad \left. + \left(\| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \| \right) \right] \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\| + \tau_n \left[ \left(1 - \left(1 - \frac{\rho}{\lambda}\right)_n\right) \|h(x_n) - h(\tilde{t})\| \right] \\
 &\quad + \tau_n M_1 + \tau_n \| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \tau_n \| f(\tilde{t}) \| \\
 &= \left[ \left(1 - \left(1 - \frac{\rho}{\lambda}\right)_n \tau_n\right) \|h(x_n) - h(\tilde{t})\| \right] \\
 &\quad + \left(1 - \frac{\rho}{\lambda}\right) \tau_n \left[ \frac{M_1 + \| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \|}{1 - \frac{\rho}{\lambda}} \right] \\
 &\leq \max \left\{ \|h(x_n) - h(\tilde{t})\|, \frac{M_1 + \| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \|}{1 - \frac{\rho}{\lambda}} \right\} \\
 &\leq \max \left\{ \|h(x_0) - h(\tilde{t})\|, \frac{M_1 + \| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \|}{1 - \frac{\rho}{\lambda}} \right\}. \tag{19}
 \end{aligned}$$

It follows that

$$\begin{aligned}
 \|x_n - \tilde{t}\| &\leq \frac{1}{\lambda} \|h(x_n) - h(\tilde{t})\| \\
 &\leq \frac{1}{\lambda} \max \left\{ \|h(x_0) - h(\tilde{t})\|, \frac{M_1 + \| \phi(\tilde{t}) - h(\tilde{t}) \| + 2\theta \| f(\tilde{t}) \|}{1 - \frac{\rho}{\lambda}} \right\}.
 \end{aligned}$$

Hence,  $\{x_n\}$  and  $\{h(x_n)\}$  are bounded. Consequently,  $\{y_n\}, \{z_n\}, \{u_n\}$  and  $\{h(v_n)\}$  are also bounded.

**Case 1:** Suppose, there exists a  $n_1 \in \mathbb{N}$  such that  $\{\|h(x_n) - h(\tilde{t})\|\}$  is decreasing for  $n \geq n_1$ .

Hence  $\{\|h(x_n) - h(\tilde{t})\|\}$  is convergent because it is monotonic and bounded. Now,

$$h(x_{n+1}) - h(x_n) = \tau_n (u_n - h(x_n)),$$

i.e.

$$\langle h(x_{n+1}) - h(x_n), h(x_n) - h(\tilde{t}) \rangle = \tau_n \langle u_n - h(x_n), h(x_n) - h(\tilde{t}) \rangle.$$

Therefore, using Lemma (2.3, (i)), we obtain

$$\begin{aligned} \|h(x_{n+1}) - h(\tilde{t})\|^2 - \|h(x_{n+1}) - h(x_n)\|^2 - \|h(x_n) - h(\tilde{t})\|^2 &= \tau_n (\|u_n - h(\tilde{t})\|^2 \\ &\quad - \|u_n - h(x_n)\|^2 - \|h(x_n) - h(\tilde{t})\|^2). \end{aligned} \quad (20)$$

Hence, using equation (18), we have

$$\begin{aligned} \|h(x_{n+1}) - h(\tilde{t})\|^2 - \|h(x_n) - h(\tilde{t})\|^2 &= \tau_n (\|u_n - h(\tilde{t})\|^2 - \|u_n - h(x_n)\|^2 \\ &\quad - \|h(x_n) - h(\tilde{t})\|^2) + \tau_n^2 \|u_n - h(x_n)\|^2 \\ &= \tau_n (\|u_n - h(\tilde{t})\|^2 - \|h(x_n) - h(\tilde{t})\|^2) \\ &\quad - \tau_n (1 - \tau_n) \|u_n - h(x_n)\|^2 \\ &\leq \tau_n (\|y_n - h(\tilde{t})\|^2 - \|h(x_n) - h(\tilde{t})\|^2) \\ &\quad - \tau_n (1 - \tau_n) \|u_n - h(x_n)\|^2. \end{aligned} \quad (21)$$

Also, from Lemma (2.3,(ii)) and definition of  $\Theta_n$ , we have

$$\begin{aligned} \|h(v_n) - h(\tilde{t})\|^2 &\leq \|h(x_n) - h(\tilde{t})\|^2 + 2\Theta_n \langle x_n - x_{n-1}, h(v_n) - h(\tilde{t}) \rangle \\ &\leq \|h(x_n) - h(\tilde{t})\|^2 + 2\Theta_n \|x_n - x_{n-1}\| \|h(v_n) - h(\tilde{t})\| \\ &\leq \|h(x_n) - h(\tilde{t})\|^2 + 2\alpha_n \|h(v_n) - h(\tilde{t})\| \\ &\leq \|h(x_n) - h(\tilde{t})\|^2 + 2_n M_1 \|h(v_n) - h(\tilde{t})\|. \end{aligned} \quad (22)$$

Now, using equations (14) and (18), we have

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$$\begin{aligned}
 \|h(x_{n+1}) - h(\tilde{t})\|^2 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \|u_n - h(\tilde{t})\|^2 \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2 \\
 &\quad + \tau_n (1 - \tau_n) \|h(v_n) - h(\tilde{t})\|^2 \\
 &\quad + \tau_n (1 - \tau_n) \kappa_n (\kappa_n - 2\theta) \|f(v_n) - f(\tilde{t})\|^2 \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2 \\
 &\quad + \tau_n [\|h(x_n) - h(\tilde{t})\|^2 + 2M_1 \|h(v_n) - h(\tilde{t})\|] \\
 &\quad + \tau_n (1 - \tau_n) \kappa_n (\kappa_n - 2\theta) \|f(v_n) - f(\tilde{t})\|^2 \\
 &= \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2 \\
 &\quad + 2\tau_n M_1 \|h(v_n) - h(\tilde{t})\| \\
 &\quad + \tau_n (1 - \tau_n) \kappa_n (\kappa_n - 2\theta) \|f(v_n) - f(\tilde{t})\|^2. \tag{23}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 0 &\leq \tau_n (1 - \tau_n) \kappa_n (2\theta - \kappa_n) \|f(v_n) - f(\tilde{t})\|^2 \\
 &\leq r_n - r_{n+1} + \tau_n \| \phi(v_n) - h(\tilde{t}) + \kappa_n f(\tilde{t}) \|^2 + 2\tau_n M_1 \|h(v_n) - h(\tilde{t})\|, \tag{24}
 \end{aligned}$$

where  $r_n = \|h(x_n) - h(\tilde{t})\|^2$ . Taking limit as  $n \rightarrow \infty$ , we obtain

$$\lim_{n \rightarrow \infty} \|f(v_n) - f(\tilde{t})\| = 0. \tag{25}$$

Let  $w_n = h(v_n) - \kappa_n f(v_n) - (h(\tilde{t}) - \kappa_n f(\tilde{t}))$ . Then from equations (5) and (12), we get

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$$\begin{aligned}
 \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \|^2 &= \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - P_{C_1}(h(\tilde{t}) - \kappa_n f(\tilde{t})) \|^2 \\
 &\leq \langle P_{C_1}(h(v_n) - \kappa_n f(v_n)) - P_{C_1}(h(\tilde{t}) - \kappa_n f(\tilde{t})), w_n \rangle \\
 &\leq \langle P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}), w_n \rangle \\
 &= \frac{1}{2} [\| w_n \|^2 + \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \|^2 \\
 &\quad - \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \\
 &\quad - \kappa_n (f(v_n) - f(\tilde{t})) \|^2] \\
 &\leq \frac{1}{2} [\| h(v_n) - h(\tilde{t}) \|^2 + \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \|^2 \\
 &\quad - \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \|^2 \\
 &\quad - \kappa_n^2 \| (f(v_n) - f(\tilde{t})) \|^2 \\
 &\quad + 2\kappa_n \langle h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)), f(v_n) - f(\tilde{t}) \rangle],
 \end{aligned}$$

(26)

which implies

$$\begin{aligned}
 \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \|^2 &\leq \| h(v_n) - h(\tilde{t}) \|^2 - \kappa_n^2 \| (f(v_n) - f(\tilde{t})) \|^2 \\
 &\quad - \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \|^2 \\
 &\quad + 2\kappa_n \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \| \cdot \| f(v_n) - f(\tilde{t}) \|.
 \end{aligned}$$

(27)

Moreover, using equation (27), we get

$$\begin{aligned}
 \| y_n - h(\tilde{t}) \|^2 &\leq \| \phi(v_n) - h(\tilde{t}) \|^2 + (1 - \kappa_n) \| P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}) \|^2 \\
 &\leq \| \phi(v_n) - h(\tilde{t}) \|^2 + (1 - \kappa_n) [\| h(v_n) - h(\tilde{t}) \|^2 \\
 &\quad - \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \|^2 \\
 &\quad + 2\kappa_n \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \| \cdot \| f(v_n) - f(\tilde{t}) \rangle].
 \end{aligned}$$

(28)

Using equations (15) and (28), we obtain

$$\begin{aligned}
 \|h(x_{n+1}) - h(\tilde{t})\|^2 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \|u_n - h(\tilde{t})\|^2 \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n [\|y_n - h(\tilde{t})\|^2 \\
 &\quad - (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|y_n - z_n\|^2 - (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|z_n - t_n\|^2] \\
 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n (1 - \tau_n) \|h(v_n) - h(\tilde{t})\|^2 \\
 &\quad + \tau_n \| \phi(v_n) - h(\tilde{t}) \|^2 \\
 &\quad - \tau_n (1 - \tau_n) \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\|^2 \\
 &\quad + 2\kappa_n \tau_n \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\| \cdot \|f(v_n) - f(\tilde{t})\| \\
 &\quad - \tau_n [(1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|y_n - z_n\|^2 + (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|z_n - t_n\|^2] \\
 &\leq \|h(x_n) - h(\tilde{t})\|^2 + 2\tau_n M_1 (1 - \tau_n) \|h(v_n) - h(\tilde{t})\| \\
 &\quad + \tau_n \| \phi(v_n) - h(\tilde{t}) \|^2 \\
 &\quad - \tau_n (1 - \tau_n) \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\|^2 \\
 &\quad + 2\kappa_n \tau_n \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\| \cdot \|f(v_n) - f(\tilde{t})\| \\
 &\quad - \tau_n [(1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|y_n - z_n\|^2 + (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|z_n - t_n\|^2], \tag{29}
 \end{aligned}$$

which implies

$$\begin{aligned}
 0 &\leq \tau_n (1 - \tau_n) \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\|^2 \\
 &\leq r_n - r_{n+1} + 2\tau_n M_1 (1 - \tau_n) \|h(v_n) - h(\tilde{t})\| + \tau_n \| \phi(v_n) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_n \tau_n \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\| \cdot \|f(v_n) - f(\tilde{t})\|
 \end{aligned}$$

and

$$\begin{aligned}
 0 &\leq \tau_n (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|y_n - z_n\|^2 + \tau_n (1 - \zeta \frac{\zeta_n}{\zeta_{n+1}}) \|z_n - u_n\|^2 \\
 &\leq r_n - r_{n+1} + 2\tau_n M_1 (1 - \tau_n) \|h(v_n) - h(\tilde{t})\| \\
 &\quad + \tau_n \| \phi(v_n) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_n \tau_n \|h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n))\| \cdot \|f(v_n) - f(\tilde{t})\|.
 \end{aligned}$$

Taking limit as  $n \rightarrow \infty$ , we obtain

$$\lim_{n \rightarrow \infty} \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \| = 0 \tag{30}$$

and

$$\lim_{n \rightarrow \infty} \| y_n - z_n \| = \lim_{n \rightarrow \infty} \| z_n - u_n \| = 0. \tag{31}$$

Further,

$$\begin{aligned} \| y_n - h(v_n) \| &= \| \alpha_n \phi(v_n) + (1 - \alpha_n) P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(v_n) \| \\ &\leq \alpha_n \| \phi(v_n) - h(v_n) \| + (1 - \alpha_n) \| h(v_n) - P_{C_1}(h(v_n) - \kappa_n f(v_n)) \|. \end{aligned}$$

Since  $\lim_{n \rightarrow \infty} \alpha_n = 0$ , using equation (30), we obtain

$$\lim_{n \rightarrow \infty} \| y_n - h(v_n) \| = 0. \tag{32}$$

Also,

$$\begin{aligned} \| h(v_n) - h(x_n) \| &= \Theta_n \| x_n - x_{n-1} \| \\ &\leq \alpha_n \leq M_1. \end{aligned}$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\lim_{n \rightarrow \infty} \| h(v_n) - h(x_n) \| = 0. \tag{33}$$

From boundedness of  $\{x_n\}$ , we can take a subsequence  $\{x_{n_k}\}$  of  $\{x_n\}$  converging weakly to some  $p \in H_1$ . Due to weak continuity of  $h$ , we can say  $h(x_{n_k}) \rightharpoonup h(p) \in C_1$ . From equation

(33), we can say that  $h(v_{n_k}) = h(p)$ . Let  $x \in C_1$ , then from definition of  $z_n$  and equation (6), we have

$$\langle y_{n_k} - \zeta_{n_k} g(y_{n_k}) - z_{n_k}, x - z_{n_k} \rangle \leq 0 \text{ for all } x \in C_1 \text{ and } k \in \mathbb{N}.$$

Since  $\zeta_{n_k} > 0$  for every  $k \in \mathbb{N}$ , it follows

$$\langle g(y_{n_k}), x - z_{n_k} \rangle \geq \frac{\langle y_{n_k} - z_{n_k}, x - z_{n_k} \rangle}{\zeta_{n_k}}. \quad (34)$$

From  $\lim_{k \rightarrow \infty} \|y_{n_k} - z_{n_k}\| = 0$  and  $\lim_{k \rightarrow \infty} \zeta_{n_k} = \zeta^* > 0$  and by boundedness of  $\{z_{n_k}\}$ , we have

$$\lim_{k \rightarrow \infty} \frac{\langle y_{n_k} - z_{n_k}, x - z_{n_k} \rangle}{\zeta_{n_k}} = 0.$$

Using equation (34), assumption (iv) and weak convergence of  $\{y_{n_k}\}$  and  $\{z_{n_k}\}$  to  $h(p)$ , we have

$$0 \leq \lim_{k \rightarrow \infty} \langle g(y_{n_k}), x - z_{n_k} \rangle \leq \langle g(h(p)), x - h(p) \rangle,$$

i.e.  $h(p) \in VI(C_1, g)$ . Let  $b_n = P_{C_1}(h(v_n) - \kappa_n f(v_n))$ , then by definition of  $\{w_n\}$  and equation (11), we have

$$\langle h(v_{n_k}) - \kappa_{n_k} f(v_{n_k}) - b_{n_k}, h(x) - b_{n_k} \rangle \leq 0 \text{ for all } x \in C_1 \text{ and } k \in \mathbb{N}.$$

Since  $\kappa_{n_k} > 0$  for every  $k \in \mathbb{N}$ , it follows

$$\langle f(v_{n_k}), h(x) - b_{n_k} \rangle \geq \frac{\langle h(v_{n_k}) - b_{n_k}, h(x) - b_{n_k} \rangle}{\kappa_{n_k}}. \quad (35)$$

From equation (30),  $0 < \liminf_{k \rightarrow \infty} \kappa_{n_k} \leq \limsup_{k \rightarrow \infty} \kappa_{n_k} < 1$  and by boundedness of  $\{b_{n_k}\}$ , we have

$$\lim_{k \rightarrow \infty} \frac{\langle h(v_{n_k}) - b_{n_k}, x - b_{n_k} \rangle}{\kappa_{n_k}} = 0.$$

Using equation (35), we have

$$0 \leq \lim_{k \rightarrow \infty} \langle f(v_{n_k}), h(x) - b_{n_k} \rangle \leq \langle f(p), h(x) - h(p) \rangle,$$

i.e.  $p \in GVI(C_1, f, h)$ . Hence  $p \in \Omega$ .

Using equations (6) and (32), we have

$$\begin{aligned} \limsup_{n \rightarrow \infty} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle &= \limsup_{n \rightarrow \infty} \langle \phi(\tilde{t}) - h(\tilde{t}), h(v_n) - h(\tilde{t}) \rangle \\ &= \langle \phi(\tilde{t}) - h(\tilde{t}), h(p) - h(\tilde{t}) \rangle \\ &\leq 0. \end{aligned} \tag{36}$$

Now, using Lemma (2.3, (ii)), we get

$$\begin{aligned} \|y_n - h(\tilde{t})\|^2 &= \|{}_n(\phi(v_n) - h(\tilde{t})) + (1 - {}_n)(P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t}))\|^2 \\ &\leq (1 - {}_n)^2 \|P_{C_1}(h(v_n) - \kappa_n f(v_n)) - h(\tilde{t})\|^2 \\ &\quad + 2 {}_n \langle \phi(v_n) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\leq (1 - {}_n)^2 \|h(v_n) - h(\tilde{t})\|^2 + 2 {}_n \langle \phi(v_n) - \phi(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\quad + 2 {}_n \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\leq (1 - {}_n)^2 \|h(v_n) - h(\tilde{t})\|^2 + 2 {}_n \frac{\rho}{\lambda} \|h(v_n) - h(\tilde{t})\| \cdot \|y_n - h(\tilde{t})\| \\ &\quad + 2 {}_n \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\leq (1 - {}_n)^2 \|h(v_n) - h(\tilde{t})\|^2 + {}_n \frac{\rho}{\lambda} \|h(v_n) - h(\tilde{t})\|^2 \\ &\quad + {}_n \frac{\rho}{\lambda} \|y_n - h(\tilde{t})\|^2 + 2 {}_n \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle. \end{aligned} \tag{37}$$

It follows that

$$\begin{aligned} \|y_n - h(\tilde{t})\|^2 &\leq \left[1 - \frac{2(1 - \rho/\lambda)}{1 - {}_n \rho/\lambda}\right] \|h(v_n) - h(\tilde{t})\|^2 + \frac{{}_n^2}{1 - {}_n \rho/\lambda} \|h(v_n) - h(\tilde{t})\|^2 \\ &\quad + \frac{2 {}_n}{1 - {}_n \rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\leq \left[1 - \frac{2(1 - \rho/\lambda)}{1 - {}_n \rho/\lambda}\right] \|h(x_n) - h(\tilde{t})\|^2 + \frac{{}_n^2}{1 - {}_n \rho/\lambda} \|h(v_n) - h(\tilde{t})\|^2 \end{aligned}$$



$$+ \frac{2}{1 - \rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle + 2\alpha_n \|h(v_n) - h(\tilde{t})\|. \quad (38)$$

Hence using equation (18), we obtain

$$\begin{aligned} \|h(x_{n+1}) - h(\tilde{t})\|^2 &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \|u_n - h(\tilde{t})\|^2 \\ &\leq (1 - \tau_n) \|h(x_n) - h(\tilde{t})\|^2 + \tau_n \|y_n - h(\tilde{t})\|^2 \\ &\leq \left[1 - \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda}\right] \|h(x_n) - h(\tilde{t})\|^2 \\ &\quad + \frac{2\tau_n}{1 - \rho/\lambda} \|h(v_n) - h(\tilde{t})\|^2 \\ &\quad + \frac{2\tau_n}{1 - \rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle \\ &\quad + 2\tau_n \alpha_n \|h(v_n) - h(\tilde{t})\| \\ &\leq \left[1 - \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda}\right] \|h(x_n) - h(\tilde{t})\|^2 \\ &\quad + \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda} \left[\frac{\tau_n}{2(1 - \rho/\lambda)} \|h(v_n) - h(\tilde{t})\|^2\right. \\ &\quad \left. + \frac{1}{1 - \rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle\right. \\ &\quad \left. + \frac{\alpha_n}{1 - \rho/\lambda} \|h(v_n) - h(\tilde{t})\|\right]. \end{aligned} \quad (39)$$

Set  $M_2 = \sup_{n \in \mathbb{N}} \|h(v_n) - h(\tilde{t})\|$ . Therefore,

$$\begin{aligned} \|h(x_{n+1}) - h(\tilde{t})\|^2 &\leq \left[1 - \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda}\right] \|h(x_n) - h(\tilde{t})\|^2 \\ &\quad + \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda} \left[\frac{\tau_n}{2(1 - \rho/\lambda)} M_2^2\right. \\ &\quad \left. + \frac{1}{1 - \rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle + \frac{\alpha_n}{1 - \rho/\lambda} M_2\right], \end{aligned} \quad (40)$$

which implies

$$r_{n+1} \leq (1 - \delta_n) r_n + \delta_n \chi_n, \quad (41)$$

where

$$\delta_n = \frac{2(1 - \rho/\lambda)\tau_n}{1 - \rho/\lambda}$$

and

$$\chi_n = \frac{n}{2(1-\rho/\lambda)} M_2^2 + \frac{1}{1-\rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_n - h(\tilde{t}) \rangle + \frac{\alpha_n}{n} \cdot \frac{1-n\rho/\lambda}{1-\rho/\lambda} M_2.$$

From equation (36), we have  $\lim_{n \rightarrow \infty} \chi_n \leq 0$ . Also,  $\sum_{n=0}^{\infty} \delta_n = \infty$ . Hence using Lemma 2.3, we conclude that

$$\lim_{n \rightarrow \infty} r_n = \lim_{n \rightarrow \infty} \|h(x_n) - h(\tilde{t})\|^2 = 0.$$

Also,

$$\|x_n - \tilde{t}\|^2 \leq \frac{1}{\lambda} \|h(x_n) - h(\tilde{t})\|.$$

Therefore,

$$\lim_{n \rightarrow \infty} \|x_n - \tilde{t}\| = 0.$$

Hence, we conclude that  $x_n \rightarrow \tilde{t}$ .

**Case 2:** Suppose that there is no such  $n_1 \in \mathbb{N}$  such that  $\{\|h(x_n) - h(\tilde{t})\|\}$  is decreasing for all  $n \geq n_1$ . Hence there is a subsequence  $\{r_{n_q}\}$  of  $\{r_n\}$  such that

$$r_{n_q} < r_{n_{q+1}} \quad \forall l \in \mathbb{N}. \tag{42}$$

Then by Lemma 2.4, there is a non-decreasing sequence  $\{n_l\} \in \mathbb{N}$  such that  $n_l \rightarrow \infty$  as  $l \rightarrow \infty$  and

$$r_{n_l} \leq r_{n_{l+1}} \quad \text{and} \quad r_l \leq r_{n_{l+1}}. \tag{43}$$

From equation (23), we obtain

$$\begin{aligned} r_{n_{l+1}} &\leq r_{n_l} + \tau_{n_l, n_l} \| \phi(v_{n_l}) - h(\tilde{t}) + \kappa_{n_l} f(\tilde{t}) \|^2 \\ &\quad + 2\tau_{n_l, n_l} M_1 \| h(v_{n_l}) - h(\tilde{t}) \| + \tau_{n_l} (1 - \kappa_{n_l}) (\kappa_{n_l} - 2\theta) \| f(v_{n_l}) - f(\tilde{t}) \|^2, \end{aligned}$$

which implies

$$\begin{aligned} 0 &\leq -\tau_{n_l} (1 - \kappa_{n_l}) (\kappa_{n_l} - 2\theta) \| f(v_{n_l}) - f(\tilde{t}) \|^2 \\ &\leq r_{n_l} - r_{n_{l+1}} + \tau_{n_l, n_l} \| \phi(v_{n_l}) - h(\tilde{t}) + \kappa_{n_l} f(\tilde{t}) \|^2 + 2\tau_{n_l, n_l} M_1 \| h(v_{n_l}) - h(\tilde{t}) \| \\ &\leq \tau_{n_l, n_l} \| \phi(v_{n_l}) - h(\tilde{t}) + \kappa_{n_l} f(\tilde{t}) \|^2 + 2\tau_{n_l, n_l} M_1 \| h(v_{n_l}) - h(\tilde{t}) \|. \end{aligned}$$

Taking limit as  $l \rightarrow \infty$ , we obtain

$$\lim_{l \rightarrow \infty} \|f(v_{n_l}) - f(\tilde{t})\| = 0.$$

Again from equation (29), we obtain

$$\begin{aligned}
 0 &\leq \tau_{n_l} (1 - \delta_{n_l}) \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \|^2 \\
 &\leq r_{n_l} - r_{n_l+1} + 2\tau_{n_l} M_1 (1 - \delta_{n_l}) \| h(v_{n_l}) - h(\tilde{t}) \| + \tau_{n_l} \| \phi(v_{n_l}) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_{n_l} \tau_{n_l} \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \| \cdot \| f(v_{n_l}) - f(\tilde{t}) \| \\
 &\leq 2\tau_{n_l} M_1 (1 - \delta_{n_l}) \| h(v_{n_l}) - h(\tilde{t}) \| + \tau_{n_l} \| \phi(v_{n_l}) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_{n_l} \tau_{n_l} \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \| \cdot \| f(v_{n_l}) - f(\tilde{t}) \|
 \end{aligned}$$

and

$$\begin{aligned}
 0 &\leq \tau_{n_l} \left(1 - \zeta \frac{\zeta_{n_l}}{\zeta_{n_l+1}}\right) \| y_{n_l} - z_{n_l} \|^2 + \tau_{n_l} \left(1 - \zeta \frac{\zeta_{n_l}}{\zeta_{n_l+1}}\right) \| z_{n_l} - u_{n_l} \|^2 \\
 &\leq r_{n_l} - r_{n_l+1} + 2\tau_{n_l} M_1 (1 - \delta_{n_l}) \| h(v_{n_l}) - h(\tilde{t}) \| + \tau_{n_l} \| \phi(v_{n_l}) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_{n_l} \tau_{n_l} \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \| \cdot \| f(v_{n_l}) - f(\tilde{t}) \| \\
 &\leq 2\tau_{n_l} M_1 (1 - \delta_{n_l}) \| h(v_{n_l}) - h(\tilde{t}) \| + \tau_{n_l} \| \phi(v_{n_l}) - h(\tilde{t}) \|^2 \\
 &\quad + 2\kappa_{n_l} \tau_{n_l} \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \| \cdot \| f(v_{n_l}) - f(\tilde{t}) \|.
 \end{aligned}$$

Taking limit as  $l \rightarrow \infty$ , we obtain

$$\lim_{l \rightarrow \infty} \| h(v_{n_l}) - P_{C_1}(h(v_{n_l}) - \kappa_{n_l} f(v_{n_l})) \| = 0$$

and

$$\lim_{l \rightarrow \infty} \| y_{n_l} - z_{n_l} \| = 0 = \lim_{l \rightarrow \infty} \| z_{n_l} - u_{n_l} \|.$$

In a similar way, we can show

$$\lim_{l \rightarrow \infty} \| y_{n_l} - h(v_{n_l}) \| = \lim_{l \rightarrow \infty} \| h(v_{n_l}) - h(x_{n_l}) \| = 0$$

and  $\{x_{n_l}\}$  converges weakly to  $p \in \Omega$ . Also, we can show that

$$\limsup_{l \rightarrow \infty} \langle \phi(\tilde{t}) - h(\tilde{t}), y_{n_l} - h(\tilde{t}) \rangle \leq 0. \tag{44}$$

Further from equation (41), we obtain

$$r_{n_l+1} \leq (1 - \delta_{n_l}) r_{n_l} + \delta_{n_l} \chi_{n_l}, \tag{45}$$

where



$$\delta_{n_l} = \frac{2(1-\rho/\lambda)\tau_{n_l}}{1-\rho/\lambda}$$

and

$$\chi_{n_l} = \frac{n_l}{2(1-\rho/\lambda)} M_2^2 + \frac{1}{1-\rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_{n_l} - h(\tilde{t}) \rangle + \frac{\alpha_{n_l}}{n_l} \cdot \frac{1-\rho/\lambda}{1-\rho/\lambda} M_2.$$

From equation (45), we have

$$\begin{aligned} \delta_{n_l} r_{n_l} &\leq r_{n_l} - r_{n_l+1} + \delta_{n_l} \chi_{n_l} \\ &\leq \delta_{n_l} \chi_{n_l}. \end{aligned} \tag{46}$$

Using  $\delta_{n_l} > 0$ , we obtain  $r_{n_l} < \chi_{n_l}$  i.e.

$$\|h(x_{n_l}) - h(\tilde{t})\|^2 \leq \frac{n_l}{2(1-\rho/\lambda)} M_2^2 + \frac{1}{1-\rho/\lambda} \langle \phi(\tilde{t}) - h(\tilde{t}), y_{n_l} - h(\tilde{t}) \rangle + \frac{\alpha_{n_l}}{n_l} \cdot \frac{1-\rho/\lambda}{1-\rho/\lambda} M_2. \tag{47}$$

Using  $\lim_{l \rightarrow \infty} \alpha_{n_l} = 0$  and equation (44), we conclude  $r_{n_l} \rightarrow 0$  as  $l \rightarrow \infty$ . This together with (45)

gives  $r_{n_l+1} \rightarrow 0$  as  $l \rightarrow \infty$ . Further from equation (43), we conclude  $\lim_{l \rightarrow \infty} r_l = 0$  i.e.

$\lim_{l \rightarrow \infty} \|h(x_l) - h(\tilde{t})\| = 0$ , which implies  $\lim_{l \rightarrow \infty} \|x_l - \tilde{t}\| = 0$  and thus  $x_q \rightarrow \tilde{t}$ . This completes the

proof.

**Conclusion:** In this research, we conclude our contribution as:

- We know that every monotone operator is pseudomonotone, but the converse is not always true. Therefore, for solving VIP, we have considered the underlying operator  $g$  as pseudomonotone which extends many results in the literature where the operator is assumed to be monotone.
- Our proposed algorithm does not require prior estimate of Lipschitz constant of pseudomonotone operator. This is very important because in practice, it is very difficult to approximate this Lipschitz constant.
- The strong convergence proved by our algorithm makes it a good convergent method for approximating a common solution of VIP and GVIP.
- For proving convergence result, we have embedded inertial term which accelerate the convergence speed of the algorithms.

## References

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## **Nanomaterial : Synthesis and Applications**

Sahil

Assistant Professor in Chemistry

Govt. College Julana, Jind, Haryana, India

mehta.sahil560@gmail.com

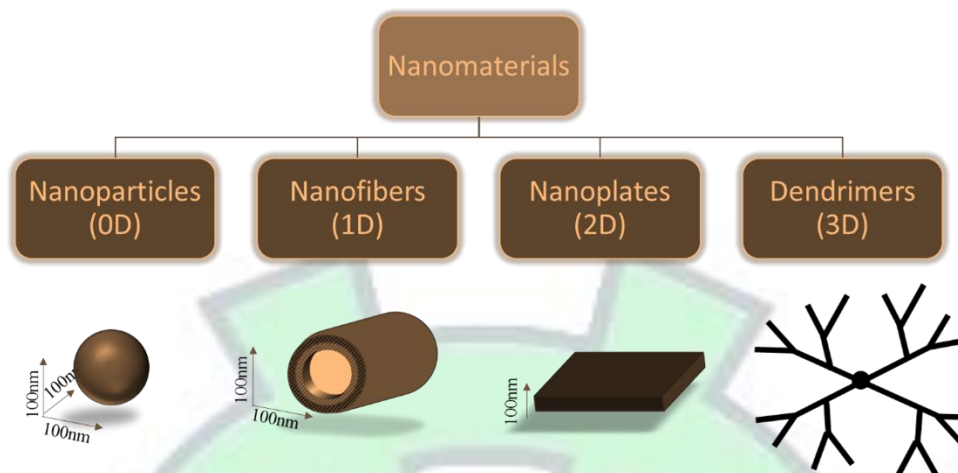
### **Abstract**

Nanomaterials have emerged as one of the most promising topics in a variety of disciplines, including physics, chemistry, engineering, and biology, due to the numerous breakthroughs that will alter the direction of nanotechnology advancements in a vast array of applications. The term "nanomaterials" is used to characterize the creation and utilization of materials with at least one dimension between 1 and 100 nanometers. The term "nano" derives from the Greek word "nanos" (or Latin "nanus"), which means "Dwarf;" however, the scientific definition of "nano" is one in a billion. This chapter provides a concise introduction to the general characteristics and applications of nanomaterials. In addition, this chapter will discuss the various nanomaterial synthesis techniques.

### **Introduction**

Nanomaterials[1] are a fascinating class of materials that have captured the attention of scientists, engineers, and researchers across various disciplines. At the heart of nanomaterials lies their unique and intriguing feature: they possess at least one dimension at the nanoscale, typically ranging from 1 to 100 nanometers (one billionth of a meter). This nanoscale dimension imparts extraordinary properties and behaviors to these materials, distinct from their bulk counterparts. Despite having the same composition as their bulk counterparts, nanomaterials exhibit distinct physicochemical properties. When reduced to the nanoscale, these materials display altered characteristics compared to their macroscale forms. For instance, insulators can transform into conductors, such as silicon, and inert substances can become catalysts, like platinum. There is various classification of nanomaterial, one is based on number of dimensions out of nanoscale range. On basis of this classification nanomaterials are:





- (i) **Zero dimensional nanomaterials:** Materials in which all the dimension is in nanoscale range e.g. Nanoparticles, quantum dots.
- (ii) **One dimensional nanomaterials:** Materials in which two of the dimensions are in nanoscale range and one in macroscale e.g. Nanotubes
- (iii) **Two dimensional nanomaterials:** Materials in which one dimension is in nanoscale range and other two in macroscale e.g. Nanoplates
- (iv) **Three dimensional nanomaterials:** Materials in which none of the dimensions are in nanoscale range e.g. Dendrimers, polycrystalline

Various nanomaterials can be formed through distinct methods, broadly classified as 'top-down' and 'bottom-up' approaches[2]. Top-down techniques involve creating small structures from larger materials. For instance, the synthesis of Graphene oxide from graphite using the Hummers method exemplifies a top-down approach. On the other hand, bottom-up techniques involve the self-assembly of atoms or molecules to form the desired nanostructures. An example of this is the chemical vapor deposition of semiconductors. Nanomaterials play a pivotal role in advancing innovative energy generation and storage devices. The unique quantum effects at the nanoscale significantly enhance electron transport and band design in these materials, resulting in exceptional device performance. Nanomaterials find applications in various fields, including photovoltaics, batteries, supercapacitors, electrocatalysts, and photoelectrochemical catalysts.

In the realm of photovoltaics, wide-bandgap nanomaterials derived from II-VI and III-V elements have garnered considerable attention for their potential in energy generation.

Nanomaterial-based solar cells, such as organic solar cells with nano-crystalline films, polymer thin films incorporating inorganic nanostructures, nanostructured dye-sensitive solar cells, and perovskite solar cells, exhibit improved electron-hole pair creation probabilities due to the increased photon path within the nanostructures. An example of this advancement can be seen in the use of  $\text{BiFeO}_3$  nanostructures as additives for organic halide perovskite  $\text{CH}_3\text{NH}_3\text{PbI}_3$ . This addition leads to enhanced charge carrier mobility and electron diffusion length, resulting in an increased short circuit current for the perovskite solar cell. These developments highlight the potential of nanomaterials in revolutionizing energy-related technologies.

The utilization of nanomaterials in batteries offers significant advantages, such as increased surface area for both electrode and electrolyte materials. Nanoparticles can enhance conductivity, enabling smoother charge flow, resulting in higher capacity and shorter charge/discharge cycles. Several nanomaterials have been explored for their potential to achieve higher lithium storage densities compared to standard graphite or metal electrodes. Examples include  $\text{LiMn}_2\text{O}_4$  and  $\text{LiCo}_2\text{O}_4$  nanoparticles, carbon-coated silicon nanowires, and Phosphorene-graphene hybrid materials. Additionally, the addition of nanoparticles like  $\text{Al}_2\text{O}_3$ ,  $\text{SiO}_2$ , or  $\text{ZrO}_2$  to solid polymer gel can significantly boost the electrolyte's conductivity and storage capacity. For high-performance Li-S batteries, 2D  $\text{MoS}_2$  acts as an efficient protective layer for Li metal anodes. Supercapacitors also benefit from nanostructuring, with carbon nanomaterials like graphene and carbon nanotubes, as well as nanostructured metal oxides like  $\text{MnO}_2$ , showing promise as electrode materials. Their large surface area, high mesoporosity, good electrical properties, and electrolyte accessibility make them excellent candidates for high-performance supercapacitors. Nanomaterials are also extensively applied in chemical energy storage, such as hydrogen generation, utilizing nano catalysts for electrocatalytic and photoelectrocatalytic processes. Materials like  $\text{MoS}_2$ , NiS, FeP (electrocatalysts), and  $\text{BiVO}_4$ ,  $\text{Fe}_2\text{O}_3$ ,  $\text{WO}_3$  (photoelectrocatalysts) reduce the activation energy and overpotential, leading to improved catalytic activity. In sensing applications, nanomaterials offer enhanced sensitivity and resolution due to their higher active surface area, allowing ultrasensitive detection of low concentrations of analytes. The improved conductivity and electron transfer enable the simultaneous detection of multiple analytes.

The field of nanomaterials has witnessed rapid advancements in recent years, enabled by groundbreaking research and cutting-edge technologies. Nanomaterials exhibit a wide range

of applications due to their exceptional electrical, optical, mechanical, and chemical properties. These materials hold immense promise in revolutionizing numerous industries, from electronics and energy to medicine and environmental sustainability.

### **Synthesis of Nanomaterials**

The synthesis of nanomaterials involves the production and manipulation of materials at the nanoscale level, typically ranging from 1 to 100 nanometers. Various techniques have been developed to create nanomaterials with precise control over their size, shape, composition, and properties. The choice of synthesis method depends on the desired material and its intended applications. Here are some common methods used for synthesizing nanomaterials:

#### **Chemical Methods:**

1. **Sol-Gel Method:** The sol-gel method is a versatile and widely used chemical technique for the synthesis of nanomaterials, particularly oxide-based nanoparticles. It involves the conversion of a colloidal solution (sol) into a solid (gel) network through hydrolysis and condensation reactions. This process allows for precise control over the composition, size, shape, and structure of the resulting nanomaterials. The sol-gel method offers several advantages for nanomaterial synthesis:
  - (i) **Precise control:** The method allows for precise control over the composition, size, and structure of the nanoparticles by adjusting the precursor materials, reaction conditions, and drying methods.
  - (ii) **Homogeneity:** The resulting nanoparticles are typically highly homogeneous in size and distribution.
  - (iii) **Versatility:** The sol-gel method can be used to synthesize a wide range of nanomaterials, including metal oxides, metal silicates, and mixed oxides.
  - (iv) **Scalability:** The synthesis can be easily scaled up for large-scale production of nanomaterials.

The sol-gel method has found numerous applications in various fields, such as catalysis, sensors, coatings, and biomedical applications, owing to its ability to produce nanomaterials with tailored properties suitable for specific purposes. However, it is essential to carefully control the reaction parameters to avoid aggregation or unwanted phase transformations during the synthesis process.



2. **Co-precipitation:** Nanoparticles are formed by simultaneously precipitating two or more reactants from a solution, resulting in the formation of a nanocrystalline solid. The synthesis typically begins with the preparation of two or more metal salt solutions, which are then mixed under controlled conditions, such as temperature, pH, and stirring rate. The mixing of the solutions leads to the rapid formation of insoluble metal hydroxides or metal oxide precursors. These precursors subsequently undergo nucleation and growth processes, resulting in the formation of nanoparticles. The co-precipitation method enables the incorporation of multiple metal ions in a single nanoparticle, which can be advantageous for creating nanomaterials with tailored properties. By adjusting the composition and molar ratios of the metal salts in the initial solution, the resulting nanoparticles can exhibit unique characteristics and functionalities.
3. **Microemulsion:** Nanoparticles are formed in a stable solution called a microemulsion, which contains water, oil, and a surfactant. Controlled reaction conditions lead to the formation of nanosized particles. The microemulsion method is an effective technique for the synthesis of nanomaterials with precise control over size, shape, and composition. It involves the formation of a stable microemulsion, which is a thermodynamically stable system consisting of nanometer-sized droplets of one liquid dispersed in another immiscible liquid, stabilized by surfactants or co-surfactants. The size and shape of the resulting nanoparticles can be precisely controlled by adjusting the composition and concentration of the reactants and surfactants. The microemulsion method offers several advantages for nanomaterial synthesis. Firstly, it allows for the synthesis of nanoparticles with a narrow size distribution, reducing the need for post-synthesis size separation. Secondly, the confined reaction environment protects the nanoparticles from aggregation and growth, resulting in well-defined and stable nanomaterials. Additionally, the method can be easily scaled up for large-scale production of nanomaterials. The microemulsion method is widely used for the synthesis of various nanomaterials, including metal and metal oxide nanoparticles, quantum dots, and nanocomposites. It finds applications in catalysis, drug delivery, electronic devices, and many other fields where precise control over nanomaterial properties is essential.
4. **Chemical Vapor Deposition (CVD):** In the CVD process, precursor gases are introduced into a reaction chamber, where they undergo chemical reactions to produce

volatile compounds or radicals. These reactive species then adsorb onto the substrate surface, where they react and deposit to form the desired nanomaterial. The substrate is typically heated to a specific temperature to promote the chemical reactions and control the growth of the nanomaterial. There are different variations of CVD, each tailored to produce specific nanomaterials. Some common types of CVD include:

- a) **Thermal CVD:** In thermal CVD, high temperatures are used to thermally decompose the precursor gases and deposit the nanomaterial onto the substrate. This method is suitable for the growth of high-quality films and nanowires.
- b) **Plasma-Enhanced CVD (PECVD):** PECVD involves the use of plasma to enhance the decomposition of precursor gases and promote chemical reactions at lower temperatures. This method is more suitable for depositing materials on temperature-sensitive substrates.
- c) **Metal-Organic CVD (MOCVD):** MOCVD uses metal-organic precursors, which contain metal-carbon bonds, to deposit metal-containing thin films and nanomaterials. It is commonly used in the semiconductor industry for growing epitaxial layers.

CVD offers several advantages for nanomaterial synthesis. It allows for excellent control over the growth process, resulting in uniform and well-defined nanoscale structures. The method is highly scalable, making it suitable for large-scale production of nanomaterials for industrial applications. Furthermore, CVD can be used to deposit nanomaterials on various substrates, including silicon, glass, and flexible materials, enabling their integration into diverse devices and systems.

#### **Physical Methods:**

1. **Mechanical Milling:** High-energy ball milling or attrition milling is used to break down bulk materials into nanoparticles through mechanical forces.
2. **Laser Ablation:** A laser is used to vaporize a target material, and the vapor condenses to form nanoparticles.
3. **Sputtering:** In this method, ions are accelerated towards a target material, causing the ejection of atoms from the target, which then condense into nanoparticles.
4. **Physical Vapor Deposition (PVD):** PVD techniques, such as evaporation or magnetron sputtering, deposit atoms or molecules onto a substrate to form nanomaterials.



### **Biological Methods:**

1. **Biomolecule-Assisted Synthesis:** Biomolecules like proteins, enzymes, or DNA are used as templates or reducing agents to synthesize nanomaterials.
2. **Green Synthesis:** Plant extracts or microorganisms are employed to synthesize nanoparticles in an eco-friendly manner.

### **Template-Based Methods:**

1. **Nanocasting:** Nanomaterials are synthesized using a sacrificial template, which is removed after the formation of the desired nanostructure.
2. **Hard Template Synthesis:** Nanoporous materials, such as alumina or silica templates, are used to create nanomaterials with specific sizes and shapes.

Each of these methods offers distinct advantages and limitations, and researchers often combine multiple approaches to achieve precise control over nanomaterial properties. The field of nanomaterial synthesis continues to evolve rapidly, paving the way for exciting new materials with a wide range of applications in various industries. As research progresses, nanomaterials are expected to play an increasingly significant role in shaping the technologies of the future.

### **Application of Nanomaterials**

Nanomaterials have a wide range of applications in various fields due to their unique properties and functionalities at the nanoscale. Important applications of nanomaterials are[3]:

#### **❖ Electronics and Optoelectronics**

Nanomaterials have revolutionized the fields of electronics and optoelectronics, offering unprecedented opportunities for advanced device design and performance[4]. In electronics, nanomaterials like graphene and carbon nanotubes have emerged as key building blocks for high-speed and low-power transistors. Their exceptional charge carrier mobility and electrical conductivity enable the development of smaller and more efficient electronic devices. Additionally, nanomaterials with tunable bandgaps, such as quantum dots and nanowires, are paving the way for next-generation optoelectronic devices. These materials find applications in light-emitting diodes (LEDs), lasers, and photodetectors, where their size-dependent

optical properties and quantum confinement effects enable enhanced light emission and detection. Plasmonic nanomaterials, such as metallic nanoparticles, exhibit unique interactions with light, enabling the design of enhanced optical sensors and imaging systems. Furthermore, the development of flexible and transparent nanomaterial-based electronics has opened up new possibilities for wearable devices and flexible displays. In the realm of energy, nanomaterials play a crucial role in photovoltaics, where they improve light absorption and charge transport in solar cells, leading to more efficient energy conversion. The multifaceted properties of nanomaterials continue to drive advancements in electronics and optoelectronics, promising a future of faster, smaller, and more energy-efficient electronic devices, as well as innovative light-based technologies with widespread applications in various industries.

#### ❖ **Energy Generation and Storage**

Nanomaterials have emerged as key players in the field of energy generation and storage, offering transformative solutions to address global energy challenges[5, 6]. In energy storage, nanomaterials play a critical role in improving the performance of batteries and supercapacitors. Nanoscale electrode materials, such as metal oxides and carbon-based nanomaterials, enable higher charge storage capacity and faster charge-discharge rates. Additionally, nanostructuring of battery components enhances ion diffusion pathways, reducing internal resistance and improving overall energy efficiency. Furthermore, nanomaterials are used in the development of advanced supercapacitors, where their high surface area and fast charge-discharge kinetics result in devices with high power density and rapid energy storage capabilities. In the realm of energy generation, nanomaterials have revolutionized photovoltaic technologies. Semiconductor nanocrystals, such as quantum dots, provide a means to tune the bandgap to capture a broader spectrum of light and boost the efficiency of solar cells. Moreover, nanostructured thin films, like perovskite nanocrystals, have shown great promise in achieving high power conversion efficiencies in solar cell devices. Nanomaterials also play a crucial role in enhancing the performance of energy conversion devices, such as fuel cells. Catalyst nanoparticles with tailored compositions and morphologies improve the kinetics of electrochemical reactions, leading to higher fuel cell efficiency and lower operating temperatures. Additionally, nanomaterials have enabled advancements in hydrogen storage technologies, as metal hydrides and nanoporous materials offer high hydrogen storage capacity and rapid uptake and release kinetics. The integration of

nanomaterials in energy storage and generation applications not only enhances performance but also leads to compact and lightweight devices, making them more practical for various applications, including portable electronics, electric vehicles, and renewable energy systems. While the potential of nanomaterials in energy generation and storage is immense, challenges related to cost, scalability, and safety need to be addressed for large-scale commercialization. Nevertheless, ongoing research and development in nanomaterials continue to drive progress in the quest for cleaner, more efficient, and sustainable energy solutions, promising a greener and brighter future.

#### ❖ Catalysis

Nanomaterials have revolutionized the field of catalysis, offering remarkable advancements in diverse industrial processes and environmental applications[7, 8]. The unique properties of nanomaterials, such as high surface area, size-dependent reactivity, and tunable surface chemistry, make them highly efficient catalysts. Nanocatalysts play a pivotal role in accelerating chemical reactions and converting raw materials into valuable products with enhanced selectivity and efficiency. In heterogeneous catalysis, metal nanoparticles supported on various nanostructured materials serve as active sites, providing an ideal platform for chemical transformations. Nanocatalysts have found applications in petroleum refining, chemical synthesis, and pollutant removal from exhaust gases. The tailored size and shape of nanocatalysts enable precise control over reaction rates and product distributions, making them highly desirable for fine chemical synthesis. In environmental catalysis, nanomaterials effectively remove pollutants from air and water by catalyzing oxidation or reduction reactions. Moreover, photocatalysts based on nanomaterials, such as titanium dioxide and zinc oxide nanoparticles, can efficiently harness solar energy to degrade organic pollutants and disinfect water. Additionally, nanomaterials are employed in enzymatic catalysis, where they enhance enzyme immobilization and stability, leading to more sustainable and efficient biocatalytic processes. The growing integration of nanomaterials in catalytic applications holds the promise of significantly reducing energy consumption and waste generation, paving the way for cleaner and greener chemical processes with positive implications for both industry and the environment.



❖ **Biomedical and Healthcare**

Nanomaterials have sparked a revolutionary wave in biomedical and healthcare applications, ushering in a new era of personalized medicine and advanced diagnostic and therapeutic tools[9]. The unique properties of nanomaterials, such as their small size, high surface area, and ability to interact with biological molecules, have opened up numerous possibilities in the field of medicine. In drug delivery, nanomaterials serve as versatile carriers that can encapsulate and deliver therapeutic agents to specific target sites in the body, enabling precise and controlled drug release. This targeted drug delivery approach reduces systemic side effects and enhances the efficacy of treatments for various diseases, including cancer, infections, and inflammatory disorders. Moreover, nanomaterials play a crucial role in medical imaging. Nanoparticles, quantum dots, and nanoscale contrast agents enhance the resolution and sensitivity of imaging techniques like magnetic resonance imaging (MRI), computed tomography (CT), and fluorescence imaging. These nanomaterial-based imaging agents enable early disease detection and non-invasive monitoring of treatment responses. In regenerative medicine, nanomaterials contribute to tissue engineering and wound healing. Scaffold materials at the nanoscale provide a suitable environment for cell growth, promoting tissue regeneration and repair. Nanofibers and nanoparticles loaded with growth factors or stem cells can accelerate healing processes and improve the outcomes of tissue regeneration therapies. Additionally, nanomaterials have shown potential in the development of point-of-care diagnostic devices, providing rapid and sensitive detection of biomarkers for diseases like infectious agents and cancer. Nanosensors integrated into wearable devices and implantable sensors allow continuous monitoring of health parameters and enable early detection of health issues. Furthermore, nanomaterials have proven to be invaluable tools in targeted therapies and precision medicine. Functionalized nanoparticles can deliver therapeutic agents to specific diseased cells, making treatments more effective while sparing healthy tissues. In combination with personalized genomic and proteomic data, nanomaterials facilitate tailored therapies that consider individual patient characteristics. While the application of nanomaterials in biomedical and healthcare fields shows great promise, it is essential to address potential safety concerns and ensure the biocompatibility of these materials. Rigorous research and stringent regulatory oversight are necessary to harness the full potential of nanomaterials in revolutionizing medical treatments, enhancing diagnostics, and ultimately improving patient outcomes.

### ❖ Environmental Remediation

Nanomaterials have emerged as powerful tools for environmental remediation, providing innovative and efficient solutions to address pollution and contamination challenges [10, 11]. Their unique properties, such as large surface area, tunable reactivity, and high adsorption capacity, make them ideal candidates for tackling various environmental pollutants. Nanomaterials, particularly metal and metal oxide nanoparticles, are widely used to remove heavy metals, organic pollutants, and other harmful substances from air, water, and soil. In water treatment, nanomaterials act as adsorbents and catalysts to remove contaminants. Metal nanoparticles, such as iron and zero-valent iron (ZVI) nanoparticles, are effective in reducing or degrading toxic organic pollutants and heavy metals. Nanoscale titanium dioxide ( $\text{TiO}_2$ ) and other photocatalytic materials are utilized for the degradation of organic compounds under sunlight or UV irradiation. Additionally, nanomaterial-based membranes and filters are employed to purify water from microorganisms, pollutants, and other impurities. In air purification, nanomaterials play a crucial role in removing volatile organic compounds (VOCs), nitrogen oxides ( $\text{NO}_x$ ), and particulate matter from the atmosphere. Nanostructured metal oxides and nanocarbon materials, such as activated carbon nanotubes, are effective adsorbents for capturing and neutralizing harmful air pollutants. Furthermore, nanomaterials are applied to soil remediation, where they help in the removal of heavy metals and organic contaminants from contaminated soils. Nano-sized particles can enhance the mobility and bioavailability of soil-bound pollutants, making it easier to extract or degrade them using suitable techniques. Nanomaterials also aid in the remediation of oil spills and industrial waste. Nanoscale materials can absorb oil and hydrophobic pollutants, facilitating the cleanup of oil-contaminated water surfaces. Moreover, nanomaterial-based catalysts are used in advanced oxidation processes to break down persistent organic pollutants and convert them into less harmful substances. One of the main advantages of using nanomaterials in environmental remediation is their ability to target specific contaminants, thus minimizing the impact on non-target species and reducing waste generation. However, the environmental and health implications of releasing nanomaterials into the environment require careful consideration and risk assessment. Ongoing research and responsible deployment of nanomaterials in environmental remediation will pave the way for cleaner and more sustainable approaches to address environmental pollution and safeguard our ecosystems.



## Other Applications

1. **Coatings and Surface Treatments:** Nanomaterials are used in coatings and surface treatments to enhance properties like scratch resistance, UV protection, and antimicrobial activity. They find applications in automotive, aerospace, and consumer products.
2. **Textiles and Fabrics:** Nanomaterials are integrated into textiles to enhance properties such as water repellency, stain resistance, and UV protection. They are also used in wearable technology for monitoring health and fitness.
3. **Food Packaging:** Nanomaterials are used in food packaging to improve barrier properties and prolong the shelf life of food products. They can prevent the migration of gases and moisture, thus preserving the freshness of the contents.
4. **Water Purification:** Nanomaterials are used in water treatment processes to remove contaminants, including heavy metals and organic pollutants, from drinking water and wastewater[10, 11].
5. **Aerospace and Defense:** Nanomaterials find applications in the aerospace and defense industries, including lightweight and strong nanocomposites for aircraft and protective coatings for military equipment.

These are just a few examples of the diverse applications of nanomaterials. As nanotechnology continues to advance, the potential for new and innovative applications across various industries is vast, promising to revolutionize technology, healthcare, energy, and environmental sustainability. However, it is essential to address any potential environmental and health impacts associated with the use of nanomaterials responsibly.

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Publications

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# Techniques for Electrochemical Characterisation of Nanomaterials

Abhishek Sharma

Assistant Professor in Chemistry  
Govt. College Julana, Jind, Haryana, India  
absharma@gmail.com

## Abstract

Nanomaterials are materials with at least one external dimension measuring between 1 and 100 nm. According to the European Commission's definition, at least half of the particles in the number size distribution must have a particle size of 100 nm or less. Using cyclic voltammetry (CV), the electrochemical properties associated with the electroactive surface area are investigated. CV is a highly adaptable electrochemical technique that identifies the mechanism of a redox system and the transport properties of an electroactive species in solution.

## Introduction

Electrochemical characterization plays a crucial role in understanding the unique properties and behavior of nanomaterials, enabling their successful integration into various electrochemical devices and applications. Nanomaterials exhibit distinct electrochemical properties due to their high surface area, size confinement, and unique electronic structure, which often differ significantly from their bulk counterparts. As a result, traditional electrochemical techniques designed for bulk materials may not be directly applicable to nanomaterials. To fully exploit the potential of nanomaterials in electrochemical applications, specialized techniques have been developed for their characterization. These techniques provide valuable insights into the electrochemical behavior, charge transfer kinetics, redox processes, and surface interactions of nanomaterials, aiding in the design and optimization of nanomaterial-based electrochemical devices. In this article, we will explore several essential electrochemical techniques employed for the characterization of nanomaterials. We will discuss cyclic voltammetry, differential pulse voltammetry, electrochemical impedance spectroscopy, and spectroelectrochemistry, highlighting their principles, applications, and



advantages in probing the unique electrochemical properties of nanomaterials. Through a comprehensive understanding of these electrochemical techniques and their applications, researchers and engineers can harness the full potential of nanomaterials to develop advanced electrochemical devices for energy storage, sensors, catalysis, and other cutting-edge applications. Furthermore, we will address the challenges and considerations specific to the electrochemical characterization of nanomaterials, contributing to the ongoing efforts to unlock new possibilities in the field of nanotechnology.

## Cyclic Voltammetry

Cyclic voltammetry is a powerful electrochemical technique utilized to study the current response of redox-active analytes when subjected to a triangular voltage waveform[1-3], as illustrated in Fig. 1(i). The voltage at which the direction of the potential is reversed is termed the switching voltage. To ensure diffusion-controlled oxidation or reduction of one or more analytes, the range of switching voltage is carefully selected. The direction of the voltage scan can be either positive or negative, depending on the composition of the sample. The measured current response in cyclic voltammetry is influenced by the concentration of redox species present in the solution and can be effectively described by combining Faraday's law and Fick's first law of diffusion.

$$i_d = nFAD_0\left(\frac{\partial C_0}{\partial x}\right)_0$$

Where  $i_d$  is the diffusion-limited current,  $A$  is the electrode area,  $D_0$  is the diffusion coefficient of the analyte and  $(\partial C / \partial x)$  is concentration gradient at the electrode surface. This combination allows researchers to gain insights into the electrochemical behavior of the analytes and the kinetics of their redox processes. Cyclic voltammetry finds wide application in various fields, such as analytical chemistry, materials science, and biochemistry. It is particularly valuable for investigating the electrochemical properties of materials, the determination of redox potentials, and the study of electrode kinetics. The technique's ability to provide detailed information about electron transfer processes and redox reactions makes it an indispensable tool in the characterization and understanding of electroactive species.

The duck-shaped plot generated by cyclic voltammetry is indeed called a cyclic voltammogram. In Fig. 1(i), a cyclic voltammogram of 6mM  $K_3Fe(CN)_6$  in a 1M  $KNO_3$

solution is shown, revealing valuable information about the electrochemical behavior of the analyte. At the starting potential of +0.8V, a small anodic current is observed, which is due to the oxidation of water to oxygen. As the scan continues, this current soon becomes zero. Between 0.8V and +0.4V, no significant current is observed as no reducible or oxidizable species are present within this potential range. Upon making the potential more cathodic (i.e., negative) than +0.4V, the current begins to exponentially increase. This increase corresponds to the oxidation of the analyte species occurring at the working electrode surface. The process is electrochemically controlled, and the current exhibits a linear relationship with the potential due to a constant concentration gradient of the analyte species near the electrode surface within the double layer. At point D, the current reaches its peak maximum, known as the cathodic peak current ( $i_{pc}$ ), which occurs at the cathodic peak potential ( $E_{pc}$ ). Beyond this point, the process shifts to mixed control. More negative potentials would normally increase the current; however, this is counteracted by a decreased analyte flux at the electrode surface. As a result, the current decreases until point F is reached. When the scan is reversed, the current continues to be cathodic because the potentials are still negative enough to cause the reduction of  $Fe(CN)_6^{3-}$ . As the potential sweeps in the positive direction, it eventually reaches a value where the reduced analyte accumulated at the electrode surface undergoes re-oxidation. The process for oxidation mirrors the reduction process but with an opposite scan direction and an anodic peak ( $i_{pa}$ ) at the anodic peak potential ( $E_{pa}$ ). For a reversible electrode reaction, the anodic and cathodic peak currents are approximately equal in magnitude but have opposite signs. The difference in peak potentials, denoted as  $\Delta E_p$ , for a reversible electrode reaction at 25°C is expected to be

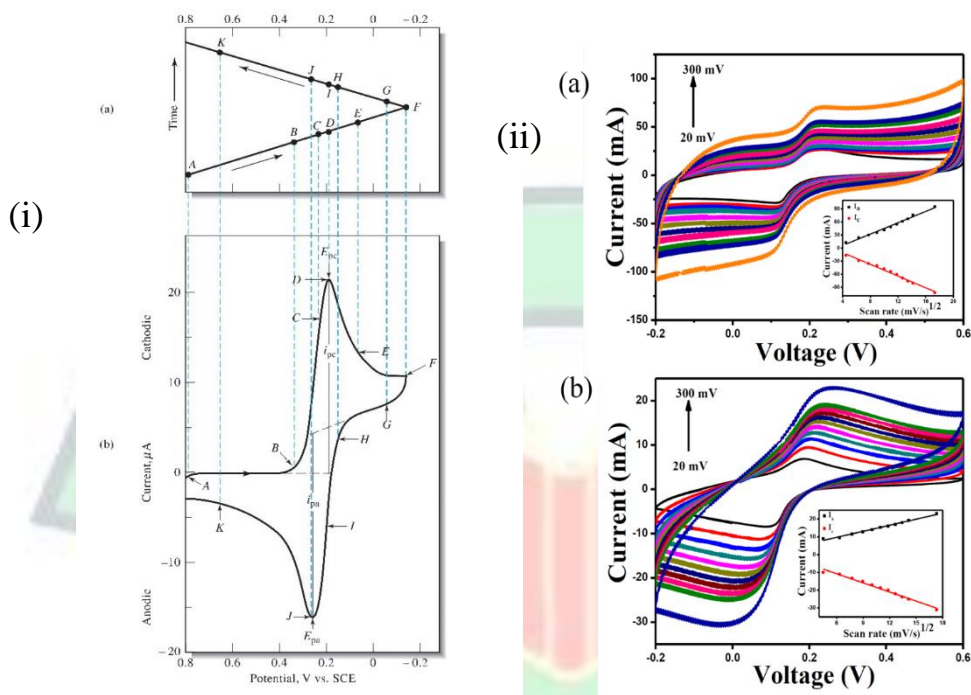
$$\Delta E_p = |E_{pa} - E_{pc}| = \frac{0.0592}{n}$$

where  $n$  is the number of electrons involved in the half-reaction. For irreversible reaction, electron transfer kinetics are slow, so  $\Delta E_p$  exceeds to expected value. At slower scan rate, an irreversible reaction may appear to be reversible, but increasing the scan rate, increase the value of  $\Delta E_p$  shows sign of irreversibility. Quantitative information can be determined from Randles-Sevcik equation, which at 25°C is

$$i_p = 2.686 \times 10^5 n^{3/2} A c D^{1/2} \nu^{1/2}$$



where  $i_p$  is the peak current in amperes,  $A$  is the electrode area in  $\text{cm}^2$ ,  $D$  is the diffusion coefficient in  $\text{cm}^2/\text{s}$ ,  $c$  is the concentration in  $\text{mol}/\text{cm}^3$ , and  $v$  is the scan rate in  $\text{V}/\text{s}$ . Hence, with known value of scan rate, area of electrode and concentration, it is possible to determine



value of diffusion coefficient using CV. As an example, consider the cyclic voltammogram for PBNCs/SnO<sub>2</sub>

Fig. 1: (A) (i) Potential versus time waveform. (ii) Cyclic voltammogram for a solution that is 6.0 mM in  $\text{K}_3\text{Fe}(\text{CN})_6$  and 1.0 M in  $\text{KNO}_3$  [2](2) (a) & (b) Cyclic voltammetry curves of PBNCs/SnO<sub>2</sub> QDs/RGO and PBNCs/SnO<sub>2</sub> QDs nanocomposites at different scan rates (20-300  $\text{mV}/\text{s}$ ) in 0.1 M PBS (pH 7.4, 0.1 M KCl). Inset (a) & (b): peak currents ( $I_a$  &  $I_c$ ) vs. square root of the scan rate ( $v^{1/2}$ )[4].

QDs/RGO and PBNCs/SnO<sub>2</sub> QDs nanocomposite in Fig. 1(ii). Sahoo et al.[4] determined the diffusion coefficients for the PBNCs/SnO<sub>2</sub> QDs/RGO and PBNCs/SnO<sub>2</sub> QDs nanocomposite modified electrodes and found that PBNCs/SnO<sub>2</sub> QDs/RGO nanocomposite has larger diffusion coefficient for ion transport than PBNCs/SnO<sub>2</sub> QDs nanocomposite.

## Applications of CV

Cyclic voltammetry (CV) is a versatile electrochemical technique that finds various applications in scientific research, industry, and technology development. Some of the key applications of cyclic voltammetry include:

1. **Electrochemical Kinetics:** CV is used to study the rate and mechanism of electrochemical reactions, providing valuable insights into reaction kinetics and electron transfer processes.
2. **Redox Behavior of Compounds:** CV is employed to investigate the redox behavior of compounds, such as metal complexes, organic molecules, and biological species. It helps in determining redox potentials and understanding their electron transfer properties.
3. **Corrosion Studies:** CV is utilized to assess the corrosion behavior of materials and coatings. It helps in understanding the corrosion mechanism and evaluating the effectiveness of corrosion protection strategies.
4. **Sensor Development:** CV is used to design and characterize electrochemical sensors for detecting analytes in various applications, including environmental monitoring, medical diagnostics, and food safety.
5. **Battery and Fuel Cell Research:** CV is applied to study the electrochemical properties of batteries and fuel cells, assisting in the development of more efficient and sustainable energy storage and conversion systems.
6. **Electrode Material Analysis:** CV is employed to investigate the electrochemical properties of electrode materials, such as catalysts, to optimize their performance in applications like fuel cells, water electrolysis, and chemical synthesis.
7. **Analyte Quantification:** CV can be used for quantitative analysis, determining the concentration of analytes based on their redox currents or peak potentials.
8. **Environmental Analysis:** CV is utilized in environmental monitoring to detect and quantify pollutants and other analytes in water, soil, and air samples.
9. **Pharmaceuticals and Drug Development:** CV is employed in drug research to study redox processes involved in drug metabolism and to assess the electrochemical behavior of pharmaceutical compounds.
10. **Nanomaterial Characterization:** CV is applied to characterize the electrochemical properties of nanomaterials, providing crucial information for their use in nanoelectronics, nanosensors, and other nanotechnology applications.

Overall, cyclic voltammetry is a powerful and widely used electrochemical technique with a broad range of applications across various scientific and industrial fields. Its ability to provide valuable information about electrochemical processes and materials makes it an indispensable tool in research and technology development.

### **Differential Pulse Voltammetry**

Differential Pulse Voltammetry (DPV) is an electrochemical technique used to measure the current response of redox-active analytes as a function of the applied potential[5]. It is a variation of cyclic voltammetry that offers enhanced sensitivity and resolution, making it particularly useful for the detection and quantification of trace analytes in complex samples. The DPV technique involves applying a series of potential pulses to the working electrode. Each potential pulse consists of two segments: a staircase potential scan (pulse-on) and a relaxation time (pulse-off). During the pulse-on segment, the potential is quickly increased or decreased to a specific value and then held constant for a short period. This rapid potential change results in a transient current response from the electrochemical reactions occurring at the electrode surface. The pulse-off segment follows the pulse-on and allows the system to relax to a stable baseline before the next pulse. During this relaxation period, the current response is measured, which corresponds to the faradaic current resulting from the redox reactions of the analyte species. Typically, a small pulse of 50 mV is applied during the last 50-100 ms of the lifetime of the period of the excitation signal as shown in Fig. 2. The base potential value in DPV is chosen such that there is no faradaic reaction and it is increased between pulses with equal increments. Current measurements are made alternately at two points: one at  $S_1$ , and other at  $S_2$  at the end of the pulse. The difference between them is recorded as a function of the linearly increasing excitation potential. A DPV curve results as a peak and the height of peak is directly proportional to concentration. In DPV, the peak potential,  $E_p$ , can be approximately identified with  $E_{1/2}$ . With increasing irreversibility of reaction,  $E_p$  deviates from  $E_{1/2}$  as the base of the peak widens and its height decreases. DPV is often used with a Dropping Mercury Electrode (DME) or a Static Mercury Drop Electrode (SMDE). When these electrodes are used, it is called Differential Pulse Polarography (DPP).



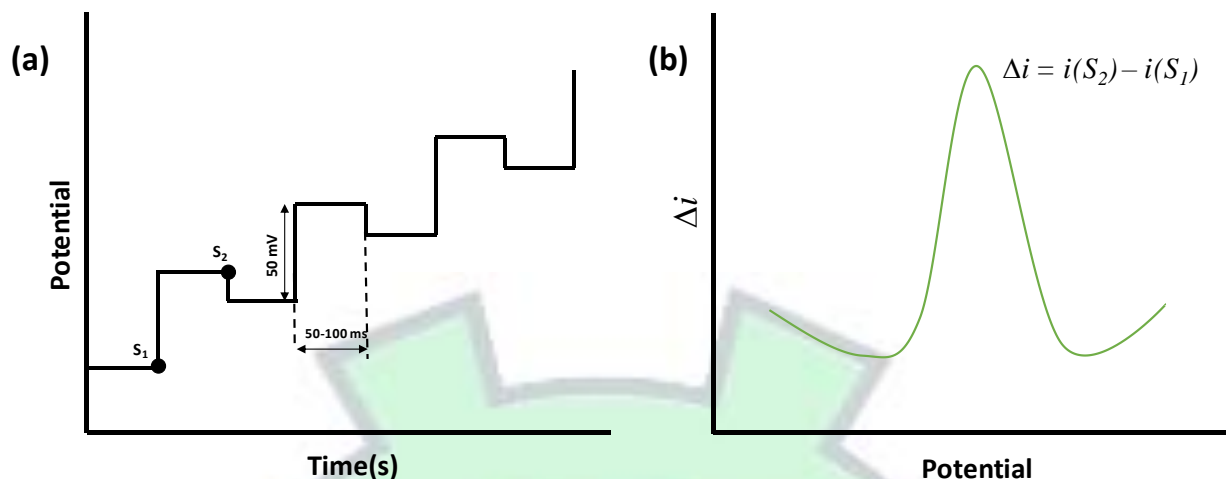


Fig. 2: (a) Diagram of the application of pulses in the differential pulse voltammetry (DPV) technique (b) Typical response of a differential pulse voltammogram

DPV offers several advantages over other electrochemical techniques:

- Enhanced Sensitivity: DPV provides superior sensitivity, making it ideal for detecting low concentrations of analytes.
- Reduced Background Noise: The pulsed nature of the technique reduces background noise, leading to better signal-to-noise ratios.
- Improved Resolution: DPV allows for better separation of peak currents, enabling the discrimination of closely spaced redox peaks.
- Minimal Interference: DPV is less susceptible to interference from capacitive currents or faradaic currents from other species in the sample.

### Applications of DPV

- Environmental Monitoring: DPV is used for the detection of pollutants and contaminants in environmental samples, such as water and soil.[6]
- Pharmaceutical Analysis: DPV is applied in drug development and pharmaceutical analysis to study redox reactions and assess the stability and purity of pharmaceutical compounds.[7]
- Food and Beverage Analysis: DPV is used to determine the presence and concentration of various compounds in food and beverage samples.
- Biosensors: DPV is incorporated into biosensor platforms for the detection of biomolecules, such as glucose, cholesterol, and DNA.



- e) Trace Metal Analysis: DPV is employed for the quantification of trace metals in complex matrices, including biological and environmental samples.

Overall, Differential Pulse Voltammetry is a powerful electrochemical technique that provides valuable information for a wide range of applications, including analytical chemistry, environmental science, pharmaceutical research, and biosensing. Its ability to offer enhanced sensitivity and resolution makes it a valuable tool in modern electrochemical analysis.

### **Electrochemical Impedance Spectroscopy**

Electrochemical Impedance Spectroscopy (EIS) is a powerful and widely used electrochemical technique for studying the electrochemical processes occurring at the electrode/electrolyte interface [8, 9]. It provides valuable information about the electrical properties and kinetics of the system under investigation. In EIS, a small sinusoidal potential excitation is applied to the electrochemical cell over a range of frequencies. The resulting current response is then measured and analyzed. The impedance of the system, which represents the frequency-dependent resistance, is determined from the ratio of the amplitude of the sinusoidal current response to the amplitude of the applied potential excitation. EIS is typically performed using small excitation signals to ensure that the system exhibits a linear or pseudo-linear response. This linear behavior allows for straightforward interpretation of the obtained impedance data. The current response to the sinusoidal potential excitation will also be sinusoidal at the same frequency but may be shifted in phase for a linear or pseudo-linear system. The phase shift in the current response provides information about the electrochemical processes occurring at the electrode/electrolyte interface. For example, the phase angle can indicate the presence of charge transfer processes, diffusion, or capacitive effects. EIS is particularly valuable for characterizing various electrochemical systems, including batteries, fuel cells, corrosion processes, sensors, and electrocatalysts. It can be used to determine parameters such as charge transfer resistance, double-layer capacitance, diffusion coefficients, and reaction kinetics. The versatility and non-destructive nature of EIS make it an essential tool for researchers and engineers working in the fields of electrochemistry, materials science, and energy storage, among others. The information obtained from EIS can help optimize the performance of electrochemical devices and

contribute to the development of more efficient and sustainable technologies. The impedance is represented as a complex number:

$$Z_{\omega} = \frac{E}{I} = \frac{E_0 e^{j\omega t}}{I_0 e^{j(\omega t + \phi)}} = Z_0 e^{j\phi} = Z_0 (\cos\phi + j\sin\phi)$$

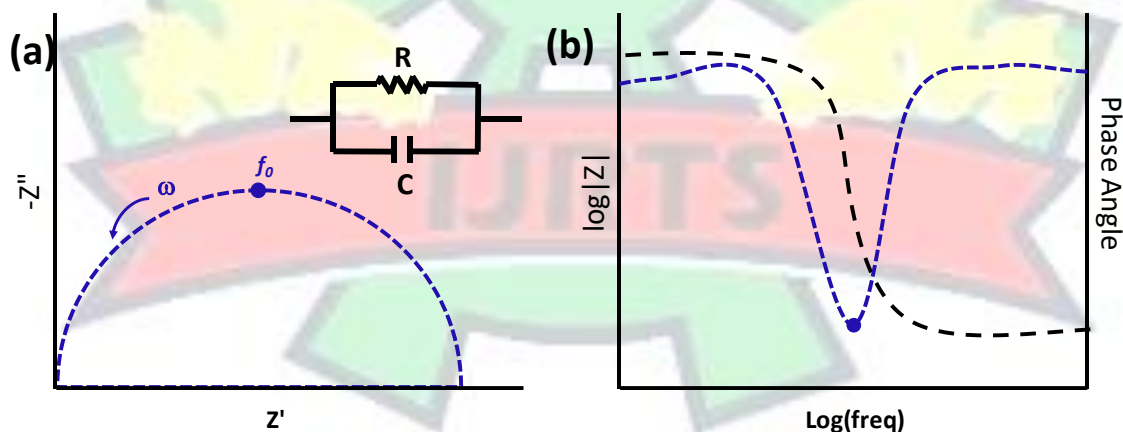
Where  $Z_0 \cos\phi$  is real impedance and  $Z_0 j\sin\phi$  is imaginary impedance. Impedance measurement produces numerical results, usually as real  $Z'$  and imaginary  $Z''$  or modulus  $|Z|$  & phase angle  $\phi$  as a function of frequency. Graphical inspection of obtained results makes it possible to identify electrical equivalent circuits. There are two fundamental types of graph:

**Complex Plane plots** (also called Argand Diagram or Nyquist Plot) (Fig 3(a))

If the negative of the imaginary part of  $Z_{\omega}$  is plotted versus the real part, we get a "Nyquist Plot".

**Bode Plots** (Fig 3(b))

The impedance is plotted with log frequency on the X-axis and both the absolute values of



the impedance ( $|Z|$ ) and the phase-shift on the Y-axis.

Fig. 3: (a) Nyquist Plot for parallel RC circuit (b) Bode Plot for Parallel RC circuit

### Application of EIS:

Electrochemical Impedance Spectroscopy (EIS) finds a wide range of applications in various fields due to its ability to provide valuable information about the electrical properties and kinetics of electrochemical systems. Some of the key applications of EIS include:

- a) Batteries and Fuel Cells: EIS is extensively used to study the performance and degradation mechanisms of batteries and fuel cells. It helps in evaluating electrode kinetics, ion transport, and impedance of the entire cell.
- b) Corrosion Studies: EIS is employed to investigate the corrosion behavior of metals and coatings, providing insights into the corrosion rate, protective properties of coatings, and the effectiveness of corrosion inhibitors.
- c) Electrode and Catalyst Characterization: EIS is applied to characterize the electrochemical behavior of various electrodes and catalysts, which is crucial for optimizing their performance in fuel cells, electrolyzers, and other electrochemical devices.
- d) Sensor Development: EIS is used to develop and analyze electrochemical sensors for detecting specific analytes in complex samples, such as in environmental monitoring, medical diagnostics, and food safety.
- e) Electrochemical Capacitors (Supercapacitors): EIS is used to study the electrical properties and performance of supercapacitors, aiding in the design of high-power energy storage devices.
- f) Coatings and Coated Materials: EIS is applied to evaluate the quality and integrity of coatings on various materials, providing information about the barrier properties and adhesion of the coatings.
- g) Bioelectrochemistry: EIS is used in bioelectrochemical studies to investigate electron transfer processes in biological systems and to understand the behavior of biofuel cells and biosensors.
- h) Analytical Chemistry: EIS is employed in analytical chemistry to determine parameters like diffusion coefficients, charge transfer resistances, and surface area of electrodes, which are important for quantitative analysis.
- i) Environmental Monitoring: EIS is utilized for monitoring and analyzing water quality, identifying pollutants, and assessing the health of aquatic ecosystems.
- j) Nanomaterials Characterization: EIS is applied to characterize the electrical properties of nanomaterials, providing insights into their potential applications in electronics, sensors, and energy storage.

These are just a few examples of the diverse applications of EIS. Its ability to provide detailed information about electrochemical processes, impedance, and charge transfer

mechanisms makes it an indispensable tool in various scientific and technological disciplines. EIS continues to play a vital role in advancing research, development, and optimization of electrochemical systems for a wide range of applications.

## Spectroelectrochemistry

Spectroelectrochemistry[10, 11] (SEC) refers to the branch of chemistry that combines Electrochemistry which allows us to correlate the electrical current produced during an electrochemical reaction with the concentration of a substance, and Spectroscopy that studies the interactions of electromagnetic radiation with matter. When these two techniques are combined, spectral measurements are made on molecules resulting from oxidation or reduction of the analyte at the electrode. Thus, it records both signals arising from electrochemical and spectrochemical changes simultaneously. Undergoing electrochemical reaction characterized spectroscopically allowed us to establish relationship between reaction mechanisms and their structures. The structural information unavailable from the electrochemical response is complemented by optical monitoring.



Fig.4: Schematics for Spectroelectrochemistry

### Applications of Spectroelectrochemistry:

- a) Understanding Redox Processes: Spectroelectrochemistry helps in studying the redox behavior of various species, such as metal ions, transition metal complexes, and

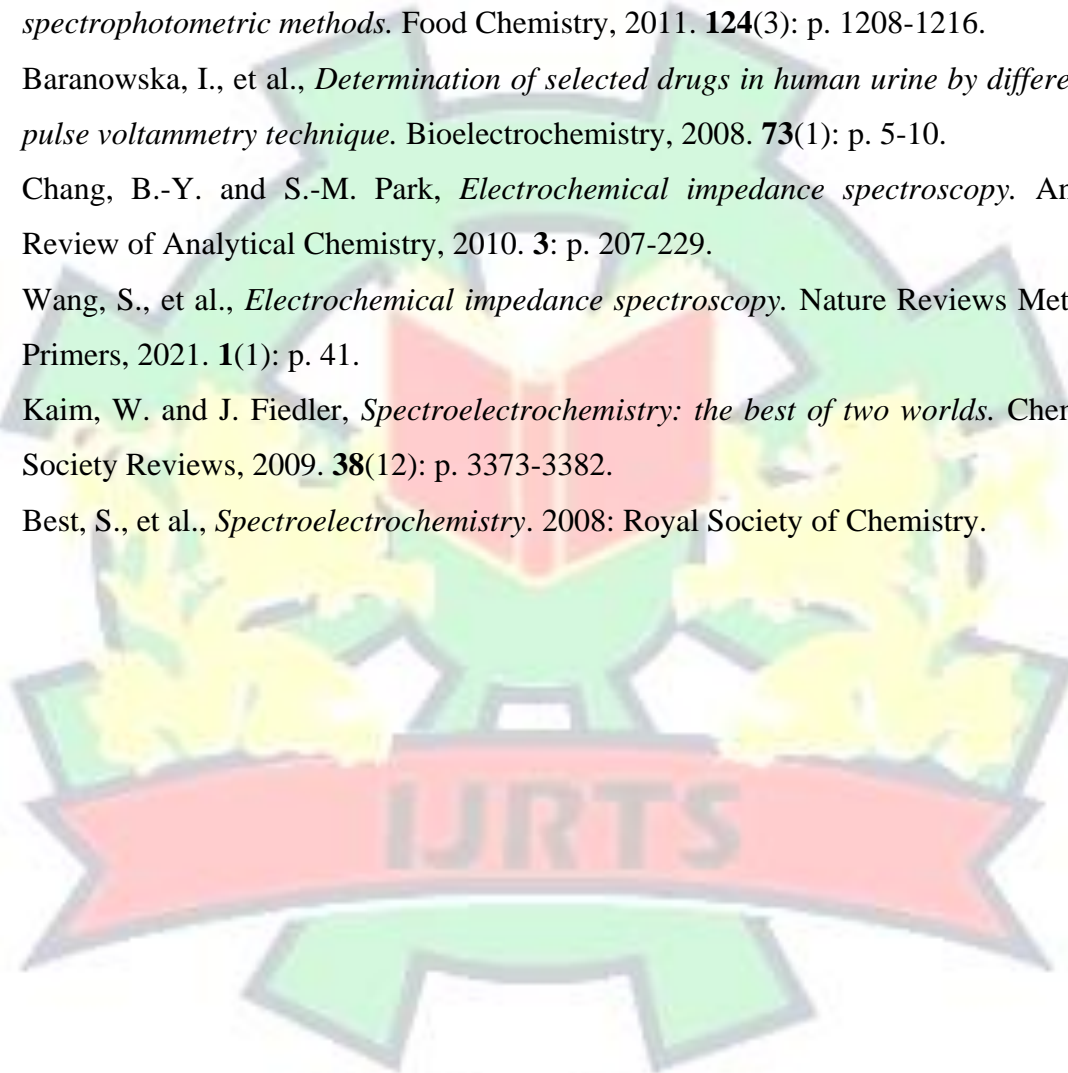


- organic molecules. It provides information about the oxidation and reduction processes and the changes in electronic configurations during these reactions.
- b) Mechanistic Studies: Spectroelectrochemistry aids in understanding the reaction mechanisms of electrochemical processes, including electron transfer, ligand substitution, and complex formation.
  - c) In-situ Characterization: Spectroelectrochemistry allows for the in-situ characterization of electrogenerated species, providing information that is not obtainable with traditional ex-situ techniques.
  - d) Battery and Fuel Cell Research: Spectroelectrochemistry is used in battery and fuel cell research to investigate the behavior of electrode materials and electrolytes during charge-discharge cycles.
  - e) Sensing and Biosensing: Spectroelectrochemistry is applied in sensing and biosensing applications to detect and study analytes or biomolecules in real-time, enhancing sensitivity and selectivity.
  - f) Photoelectrochemistry: Spectroelectrochemistry is used to study photoelectrochemical processes, such as photoinduced electron transfer and photocatalysis, combining the advantages of both spectroscopy and electrochemistry.
  - g) Nanomaterials Characterization: Spectroelectrochemistry aids in characterizing the electronic properties and charge transport mechanisms of nanomaterials, such as quantum dots and nanoparticles.

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**Publications**

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# Quantum Codes over the ring $Z_{11}$ obtained through $(1 + 9\zeta)$ - Constacyclic Codes over $Z_{11} + \zeta Z_{11}$

Meena

Assistant Professor in Mathematics  
Govt. College Julana, Jind, Haryana, India  
ranameena908@gmail.com

## Abstract

This paper contains quantum codes over the ring  $Z_{11}$  which were obtained through  $(1 + 9\zeta)$  - Constacyclic codes over  $Z_{11} + \zeta Z_{11}$ , their dual codes, Gray images and to illustrate results examples are given. An element  $(1 + 9\zeta)$  which is self inverse unit element of ring is taken. There are two maximal ideals of ring and due to Chinese Remainder theorem, ring can be expressed as direct sum of these two ideals. Next, defined two linear codes  $\mathfrak{T}_1$  and  $\mathfrak{T}_2$  over ring and ring  $Z_{11} + \zeta Z_{11}$  can be written as direct sum of these two linear codes. The decomposition of  $(1 + 9\zeta)$  - Constacyclic codes into negacyclic and cyclic codes over ring is given and then by help of CSS construction quantum codes are derived.

## Introduction

There has been a sea growth in the research of quantum codes. Quantum codes are used in quantum communication and quantum computing to protect quantum information from transmission noise on the channel. Quantum information can propagate faster than light under certain conditions, while classical information can't. Quantum information can't be duplicated but classical information can be. Quantum codes adequately defeat decoherence and provide an efficient way to overcome decoherence. Firstly, the existence of quantum error correction code was proven in [39] and individually in [41]. Later, a method for obtaining quantum codes through classical codes were used in [20]. Since then there has been an tremendous growth in the theory of quantum codes and several authors constructed many better quantum codes through cyclic codes over finite

field  $F_q$  ( $q$  is the power of some prime) with self orthogonal properties [42, 43, 44, 45, 46, 11, 21, 35].

The construction for finding quantum codes through cyclic codes of odd length over finite chain ring  $F_2 + uF_2$ , where  $u^2 = 0$  was first proposed in [23]. Inspired by this study, development for quantum codes through cyclic codes of odd length over finite chain ring  $F_4 + uF_4$ , where  $u^2 = 0$  was made in [51]. Also, in [53] a condition for the existence of quantum codes from cyclic codes over  $F_2 + uF_2 + u^2F_2$  with Lee metric was given. The CSS construction to finite commutative Frobenius rings was extended in [15]. In [11] received quantum codes from cyclic codes over  $F_2 + uF_2 + vF_2 + uvF_2$ . A new method to obtain quantum codes through cyclic codes over finite non-chain ring  $F_2 + vF_2$ , where  $v^2 = v$  of arbitrary length was discussed in [23]. Also, cyclic codes over  $F_3 + vF_3$ , where  $v^2 = 1$  was investigated in [27]. Recently, in [43] a method to obtain quantum codes through cyclic codes over the finite non-chain ring  $F_3 + vF_3$ , where  $v^2 = 1$  was discussed. In [29] the structure of cyclic codes over ring  $F_4 + vF_4$ , where  $v^2 = 1$  was given. These results conclude that some good or optimal quantum codes can be constructed by using cyclic codes over finite rings. Constacyclic codes naturally generalize the structural properties of cyclic codes and offer more flexibility in the construction of new quantum codes. Very recently, several researchers established a number of new quantum codes via  $F_p$  from the classical cyclic and constacyclic codes [43, 44, 45, 35, 40]. This motivates us to consider  $(1 + 9\zeta)$ -Constacyclic codes over the finite non-chain ring  $Z_{11} + \zeta Z_{11}$ , where  $\zeta^2 = \zeta$  to obtain good quantum codes. The rest of the paper is arranged in the following way, Section 2 is Preliminaries in which some fundamental properties and some essential definitions have been given. In Section 3 Gray Map over  $Z_{11} + \zeta Z_{11}$  is defined. In Section 4 we presented the development of quantum codes through  $(1 + 9\zeta)$ -constacyclic codes over the considered ring which are exemplified in section 4. In Section 5 the paper is concluded.



# Comparison Between Chemically Synthesized and Green Synthesized Corrosion Inhibitors for Metal Alloys

Anita Tanwar

Research Scholar, Department of Chemistry  
Banasthali Vidyapith, Rajasthan 304022, India  
anitanwar6789@gmail.com

## Introduction

Corrosion is a significant challenge that affects the durability and performance of metal alloys in various industrial applications. To mitigate corrosion, inhibitors are commonly employed to protect metal surfaces from oxidative degradation. Traditionally, chemically synthesized corrosion inhibitors have been the standard choice, offering high efficiency and precise formulation. However, increasing environmental concerns and stringent regulations have spurred interest in green synthesized inhibitors, which are derived from natural sources and are more eco-friendly. This chapter provides a comprehensive comparison of these two types of inhibitors, analyzing their mechanisms, advantages, limitations, and potential applications.

## Mechanisms of Corrosion Inhibition

### Chemically Synthesized Inhibitors

Chemically synthesized corrosion inhibitors primarily function through adsorption on the metal surface, forming a protective barrier that prevents interaction with corrosive agents. These inhibitors can be classified into three categories:

1. **Anodic Inhibitors:** Reduce the oxidation reaction by forming a passivation layer.
2. **Cathodic Inhibitors:** Decrease the reduction reaction, typically by precipitating insoluble compounds on the surface.
3. **Mixed Inhibitors:** Act on both anodic and cathodic sites simultaneously.

Common examples include organic compounds such as amines and imidazolines, as well as inorganic compounds like chromates and nitrates. These inhibitors are often tailored for specific applications, offering precise control over corrosion processes.

### **Green Synthesized Inhibitors**

Green corrosion inhibitors are derived from natural products such as plant extracts, algae, and bio-wastes. They operate primarily by adsorption mechanisms similar to their chemical counterparts, utilizing phytochemicals such as alkaloids, flavonoids, tannins, and saponins. These compounds interact with the metal surface, forming a protective layer that mitigates corrosion. Additionally, some green inhibitors release antioxidants, further enhancing their protective properties.

### **Advantages and Limitations**

#### **Chemically Synthesized Inhibitors**

##### **Advantages:**

- High efficiency and reliability under extreme conditions.
- Customizable formulations for specific industrial needs.
- Rapid action and long-term protection.

##### **Limitations:**

- Environmental toxicity due to harmful residues.
- High production and disposal costs.
- Potential health risks to workers handling these chemicals.

#### **Green Synthesized Inhibitors**

##### **Advantages:**

- Eco-friendly and biodegradable, reducing environmental impact.
- Low toxicity, making them safer for workers and ecosystems.

- Cost-effective, particularly when derived from agricultural or industrial by-products.

#### **Limitations:**

- Variability in composition and effectiveness due to natural source dependency.
- Limited thermal stability and shorter lifespan compared to synthetic counterparts.
- Requires further research to enhance efficiency and scalability.

#### **Performance Evaluation**

##### **Laboratory Studies**

Numerous studies have evaluated the performance of chemically and green synthesized inhibitors through electrochemical techniques such as potentiodynamic polarization and electrochemical impedance spectroscopy (EIS). Results indicate that chemically synthesized inhibitors often achieve higher inhibition efficiencies under controlled conditions. However, green inhibitors have demonstrated competitive performance in neutral or mildly corrosive environments.

##### **Field Applications**

In industrial settings, chemically synthesized inhibitors are preferred for high-stress environments, such as oil and gas pipelines or marine applications, due to their robust performance. Conversely, green inhibitors are gaining traction in low-to-moderate corrosion scenarios, particularly in industries prioritizing sustainability, such as wastewater treatment and agriculture.

##### **Case Studies**

###### **Chemically Synthesized Inhibitor Application**

A study on the use of zinc chromate in protecting steel structures revealed inhibition efficiencies exceeding 95% under acidic conditions. Despite the high performance, environmental regulations necessitated a shift toward less toxic alternatives.

###### **Green Synthesized Inhibitor Application**

Extracts from neem leaves were tested as corrosion inhibitors for mild steel in saline environments. The study reported an inhibition efficiency of 85%, highlighting the potential of plant-based inhibitors for sustainable applications.

### **Future Trends and Research Directions**

The ongoing transition toward sustainable industrial practices emphasizes the need for green corrosion inhibitors. Future research should focus on:

- Enhancing the efficiency and thermal stability of green inhibitors.
- Developing hybrid inhibitors that combine the strengths of chemical and green methods.
- Exploring novel natural sources and their phytochemical compositions.

Simultaneously, advancements in nanotechnology and material science offer opportunities to improve the formulation and application of both types of inhibitors. For instance, nanocomposites and encapsulated inhibitors may enhance performance while reducing environmental impact.

### **Conclusion**

The comparison of chemically synthesized and green synthesized corrosion inhibitors highlights a trade-off between efficiency and sustainability. While chemically synthesized inhibitors continue to dominate in high-performance applications, the rise of green inhibitors marks a significant step toward eco-friendly corrosion mitigation. Balancing performance with environmental responsibility will be key to addressing the evolving demands of modern industries.

This chapter underscores the importance of ongoing research and innovation in corrosion inhibitor technologies, paving the way for a more sustainable future in metal protection.



# Intersections Between AI and VR in the Modern World

Dr. Vipin Mittal

Director, Trinity World Educational Province  
71-75, Shelton Street, Covent Garden, London WC2H9JQ  
vipin.net@gmail.com

## Introduction

Artificial Intelligence (AI) and Virtual Reality (VR) are two of the most transformative technologies of the modern era. Each technology on its own has revolutionized industries, from entertainment and healthcare to education and engineering. When combined, AI and VR create a powerful synergy that unlocks unprecedented possibilities. This chapter explores the intersections between AI and VR, highlighting their integration, applications, and the profound impact they have on various aspects of the modern world.

## The Integration of AI and VR

The integration of AI and VR involves embedding intelligent algorithms into immersive virtual environments to enhance realism, interactivity, and personalization. Key elements of this integration include:

### 1. AI-Driven Content Generation

AI enables the dynamic generation of VR content, such as:

- **Procedural Environments:** Creating realistic landscapes, cities, or objects in virtual spaces.
- **Real-Time Adaptation:** Modifying VR scenarios based on user behavior and preferences.

### 2. Natural Interaction

AI-powered systems facilitate seamless interaction within VR through:

- **Voice Recognition:** Understanding and responding to user commands.
- **Gesture Recognition:** Tracking and interpreting hand movements and body language.

### 3. Personalization

AI algorithms analyze user data to tailor VR experiences, ensuring that they are:

- Relevant to individual preferences.
- Adaptable to skill levels or emotional states.

### Applications of AI and VR

The intersection of AI and VR has led to transformative applications across various domains:

#### 1. Healthcare

- **Surgical Training and Planning:** AI-enhanced VR simulations allow surgeons to practice complex procedures in realistic, risk-free environments.
- **Therapeutic Interventions:** Virtual environments powered by AI are used in treatments for PTSD, phobias, and anxiety disorders.
- **Diagnostics:** AI analyzes patient behavior within VR settings to detect neurological or psychological conditions.

#### 2. Education and Training

- **Immersive Learning:** AI adapts VR learning modules to match the pace and style of individual learners.
- **Skill Development:** Interactive VR simulations help trainees acquire technical skills, with AI providing real-time feedback and guidance.

#### 3. Entertainment and Gaming

- **Realistic NPCs:** AI-controlled characters in VR games exhibit lifelike behavior, enhancing engagement.
- **Dynamic Storylines:** AI adapts narratives within VR games based on player choices.

#### 4. Retail and E-Commerce

- **Virtual Shopping Experiences:** AI personalizes product recommendations in VR retail stores.
- **Customer Interaction:** Virtual assistants powered by AI enhance customer support within VR environments.

## 5. Urban Planning and Architecture

- **Design Simulation:** AI helps architects visualize and optimize designs within VR environments.
- **Public Engagement:** Urban planners use AI-enhanced VR to gather feedback by immersing citizens in proposed designs.

## 6. Workplace Collaboration

- **Virtual Meetings:** AI facilitates communication and collaboration in virtual workspaces by providing language translation and meeting summarization.
- **Team Training:** VR simulations with AI-driven scenarios improve teamwork and problem-solving skills.

## Challenges and Ethical Considerations

The integration of AI and VR is not without challenges. Key issues include:

### 1. Technical Challenges

- High computational demands for real-time AI and VR integration.
- Ensuring latency-free interactions to maintain immersion.

### 2. Data Privacy and Security

- Protecting user data collected for AI personalization.
- Preventing misuse of biometric data from VR devices.

### 3. Ethical Concerns

- Addressing bias in AI algorithms to ensure fairness.

- Avoiding over-reliance on virtual environments at the expense of real-world interactions.

#### **4. Accessibility**

- Reducing costs to make AI and VR technologies accessible to a broader audience.
- Ensuring usability for individuals with disabilities.

#### **Future Trends and Opportunities**

The future of AI and VR is poised for remarkable growth, driven by advancements in technology and increasing adoption. Emerging trends include:

##### **1. Hyper-Realistic Simulations**

Advances in AI will enable VR environments that are indistinguishable from reality, offering enhanced training, gaming, and storytelling experiences.

##### **2. Cross-Disciplinary Innovations**

AI and VR will converge with other technologies, such as 5G, IoT, and blockchain, to create integrated ecosystems.

##### **3. AI-Enhanced Avatars**

Highly intelligent and responsive avatars will act as virtual companions, tutors, or collaborators within VR spaces.

##### **4. Democratization of AI and VR**

Open-source frameworks and cost reductions will make AI and VR accessible to startups, educators, and hobbyists, fostering innovation.

#### **Conclusion**

The intersection of AI and VR represents a frontier of innovation that is transforming how we live, work, and play. By combining the analytical power of AI with the immersive potential of VR, these technologies are creating experiences that were once the realm of science fiction. While challenges remain, the opportunities they present far outweigh the obstacles,



promising a future where virtual worlds are seamlessly integrated with intelligent systems to enrich our lives.

This chapter underscores the transformative potential of AI and VR, urging continued exploration and collaboration across industries to harness their full potential.



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