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RECENT DEVELOPMENT IN MATHEMATICS

An Edited Book

1st EDITION

Chief Editor

Dr. Deepmala Lohan

Principal, Govt. College Hisar, Haryana, India

Recent Development in Mathematics

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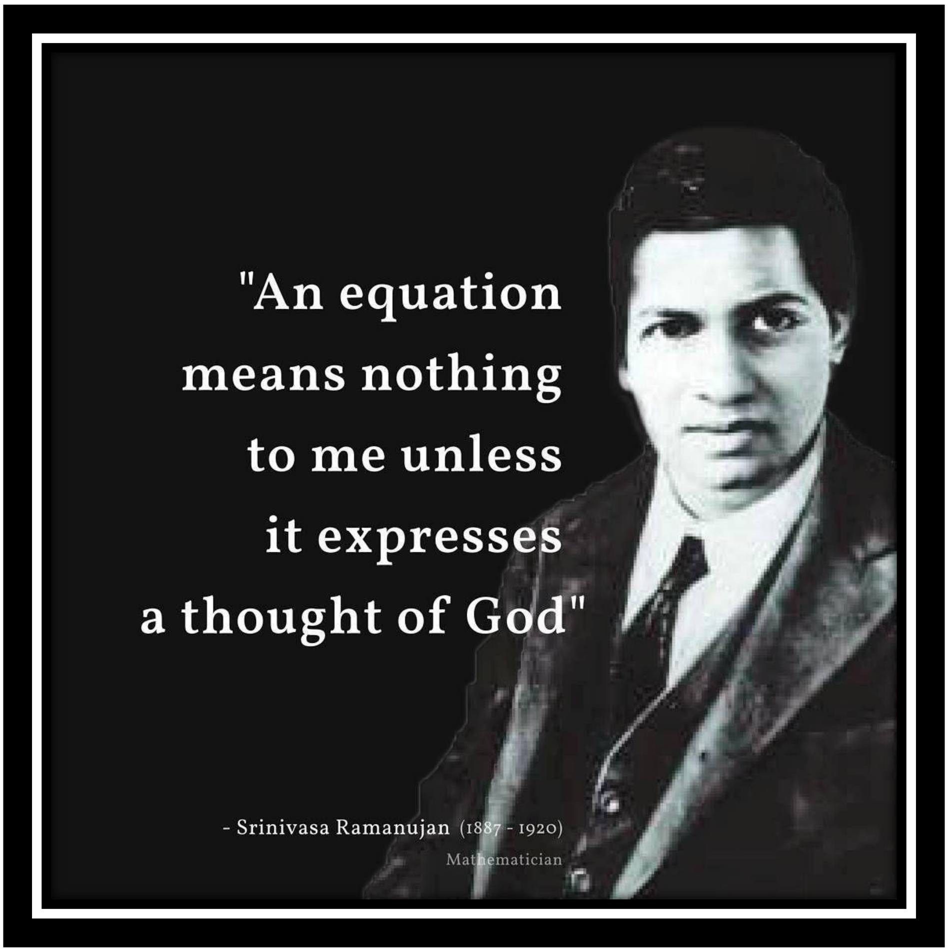

Srinivasa Ramanujan (born as Srinivasa Ramanujan Aiyangar, IPA: [sriːniʋaːsa ɾaːmaːnud͡ʑan ajːaŋgar], 22 December 1887 – 26 April 1920) was an Indian mathematician. Though he had almost no formal training in pure mathematics, he made substantial contributions to mathematical analysis, number theory, infinite series, and continued fractions, including solutions to mathematical problems then considered unsolvable. During his short life, Ramanujan independently compiled nearly 3,900 results (mostly identities and equations). Many were completely novel; his original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and mock theta functions, have opened entire new areas of work and inspired a vast amount of further research. Of his thousands of results, all but a dozen or two have now been proven correct.

Preface

Government College, Hisar is a premier higher education institution of Government of Haryana. It is a prestigious college established in 1950 and has continuously been maintaining its heritage of providing quality education to students since then. It is B+ grade NAAC accredited and is among the Model Colleges of Government of Haryana. It is recognized by UGC under sections 2(f) and 12(B). Presently, it is affiliated to Guru Jambheshwar University of Science & Technology, Hisar. It is running from a sprawling campus of about thirty-four acres. It is situated at the central location of Hisar on Rajgarh Road. It imparts learning to about 5700 students from diverse socio-cultural background in twenty-three courses for twelve undergraduate programmes including five honours and eleven postgraduate programmes. A dedicated, scholarly and hardworking team of about two hundred teachers assisted by committed and industrious sixty members of non teaching staff is the mainstay of the college.

The department of Mathematics in Govt. College, Hisar is a landmark of excellence ever since its establishment in 1950. With the best and most dedicated faculty, the department caters to the need of all the students of B.Sc., BA., B.Com., B.Sc. Honours & M.Sc. courses. The department has active group of teachers conducting research work in different fields of Mathematics. Students of the department have got placement at respectable positions in teaching, research and administrative organizations.

It is a matter of great pleasure for me to welcome all the delegates and resource persons in One Day National Seminar on ‘’Recent Development in Mathematics’’ organize by Department of Mathematics, Government College, Hisar on February 20, 2023. This event is entirely sponsored by Directorate Higher Education, Government of Haryana. The main objective of the seminar is to promote research and innovative ideas in the field of Mathematics. The seminar will provide the platform for the young researchers in the country, to interact with senior researchers and scientists to exchange their views and ideas, also to make possible scientific collaboration with recent development in different areas of their interest. The participants will certainly get benefitted from the seminar as it will provide great opportunities to explore the potential and ideas development. The Seminar deliberations thus aim to focus on latest development of Mathematics and its application in various fields of Physical Sciences, Engineering & Technology. I am sure the Seminar will provide an opportunity to the distinguished delegates from various parts of the country to exchange their views and ideas amongst themselves. Sincere efforts are made by the department for the conduct of technical sessions and comfortable stay, I am confident that the outcome of this Seminar will be fruitful for young researchers and students. I am sure that the seminar will be a great success and I extend my heartily greetings to all the participants.

Dr. Deepmala Lohan

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While every effort has been made to ensure that the book is free of error, it is inevitable that some errors still remain. Please report any errors, suggestions or questions to the authors at the following email address.

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Applications of Fuzzy Logic Computation In Weather Forecasting

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Abstract

Weather plays an important role in crop production as well as human and animal lives. It has a profound influence on the growth, development and yields of a crop, incidence of pests and diseases, water needs and fertilizer requirements in terms of differences in nutrient mobilization due to water stresses and timeliness and effectiveness of prophylactic and cultural operations on crops. Weather forecasting is the prediction weather parameters like rainfall, atmospheric temperature, radiation, fog and wind velocity etc before their actual occurrence. Numerical weather forecasting has its genesis in 1922 when Lewis Fry Richardson made an elemental approach to forecast weather conditions using his hand calculator. Soft computing techniques have opened many opportunities to deal with the complexity and ambiguity that comes with weather forecasting. Presently many computing technologies have been applied to several domains and they have proved to provide more approximate and acceptable results. Fuzzy logic being one of them has been very useful in solving many real-world problems that are inherent for their uncertainty, complexity, impreciseness and a high degree of randomness. Fuzzy logic is now widely used for weather prediction and forecasting studies. Various workers have used fuzzy logic computation techniques to predict the weather parameters like rainfall, temperature, fog, pollution concentration and cyclones in India and other countries. This paper aims to review the application of fuzzy logic in the field of weather forecasting.

Key Words: Fuzzy logic, artificial Intelligence, soft computing, weather forecasting, meteorology

Introduction

Planning has been identified as a roadmap to success as failing to plan implies planning for failure. Information about future weather happenings is instrumental to efficient and effective planning. Weather forecasting is the prediction of the atmospheric weather parameters through application of the principles of physics, supplemented by a variety of statistical and empirical techniques. In addition to predictions of atmospheric phenomena themselves, weather forecasting includes predictions of changes on Earth's surface caused by atmospheric conditions *e.g*., rainfall, snow and ice cover, storm tides, and floods. Weather forecasting is one of the most important and demanding operational responsibilities carried out by meteorological services worldwide. It is a complicated procedure that includes numerous specialized technological fields as all decisions are made within a visage of uncertainty associated with weather systems (Hasan *et al*., 2008). Nasher (2013) reported rainfall and temperature to be important climatic inputs in the context of climate variability.

Though various kinds of models like statistical model have been used for years in the prediction of rainfall, the models have many drawbacks (Cirstea *et al*., 2002). With the rapid evolving technologies in the field of meteorology, it is desirable to merge the experience of many forecasters with algorithms that may aid in difficult forecasting situations (Jim, 2005). Cognitive computing has been an emergent set of problem solving algorithm that attempts to imitate natural problem solving techniques (Jang *et al*., 1997), fuzzy logic being one of them. Fuzzy set theory is a tool for modeling the uncertainty associated with vagueness with a lack of information regarding particular element of the problem in hand (Ross, 1997). In predicting weather conditions, factors in the antecedent and consequent parts that exhibit vagueness and ambiguity are being treated with logic and valid algorithms (Hasan *et al*., 1995). Use of fuzzy set theory has been proved by scientists to be applicable with uncertain, vague and qualitative expressions of the system. The intent of fuzzy set theory was to alleviate problems associated with traditional binary logic, where statements are exclusively true or false (Cirstea *et al*., 2002).

Fuzzy logic model developed by Agboola *et al*. (2013) is made up of two functional components; the knowledge base and the fuzzy reasoning or decision making unit. Two operations were performed on the fuzzy logic model; the fuzzification operation and defuzzification operation. The model that predicted outputs was compared with the actual rainfall data. Fuzzy logic based rainfall prediction method by using the Mamdani fuzzy inference system may be successively used for different environmental problem estimation to mitigate unexpected meteorological problems (Hasan and Rahman, 2019).

Fuzzy Logic

Fuzzy logic was first introduced by Lotfi A Zadeh at the University of California, Berkely in 1964, with the idea of fuzzy sets as an extension of the traditional Boolean / crisp sets. Later in 1974, he first used the term fuzzy logic. The concept was developed as a means to deal with uncertainty, impreciseness, and the vagueness associated with real-world problems.

Fuzzy sets are the collection of objects with the same properties, and in crisp sets the objects either belong to the set or do not. In practice, the characteristic value for an object belonging to the considered set is coded as 1 and if it is outside the set then the coding is 0. In crisp sets, there is no ambiguity or vagueness about each object belongs to the considered set. Otherwise, in daily life humans are always confronted with objects that may be similar to one other with quite different properties. In order to alleviate such situations (Zadeh, 1965) generalized the crisp set membership degree as having any value continuously between 0 and 1. Fuzzy sets are a generalization of conventional set theory. The basic idea of fuzzy sets is easy to grasp. An object with membership function 1 belongs to the set with no doubt and those with 0 membership functions again absolutely do not belong to the set, but objects with intermediate membership functions partially belong to the same set. The greater the membership function, the more the object belongs to the set (Hasan and Zenkai, 1999).

Let X be a universal set. Then A is called a (fuzzy) subset of X if A is a set of ordered pairs.

$$A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0, 1]\}$$

where μ_A is the membership function of A and $\mu_A(x)$ is the grade of the membership of *x* in A. The linguistic expression for the variables and their membership functions are evaluated from the following triangular membership functions and it is defined by a lower limit a, an upper limit b and a value m, where a≤m≤b. The value of the membership function μ(x), ranges from 0 to 1, with 0 denoting no membership, 1 for full membership and values in between has partial membership as shown in Figure 1

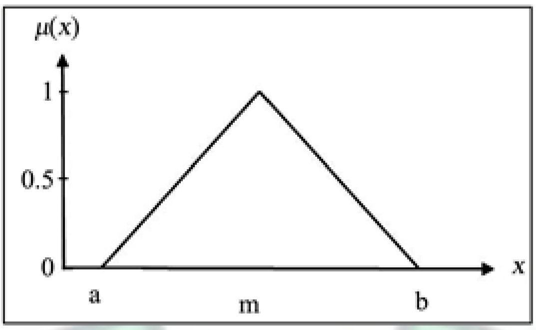

Figure 1: Basic graph of fuzzy membership function (Rahman, 2020)

Fuzzy Logic in Weather Forcasting

Soft computing techniques have opened many opportunities to deal with the complexity and ambiguity that comes with weather forecasting. Numerical weather forecasting has its origin in 1922 when Lewis Fry Richardson, using his hand calculator, made an elemental approach to forecast weather conditions. After the world wars, in 1950 when the computer and information technology started flourishing, Jule Charney and his group made the first successful numerical weather prediction using ENIAC (first computer) at Princeton. Thereafter, many developments have been made from statistical models to soft computing models. These soft computing and hybrid climate models allow for real time data collection and prediction with improved accuracy, less ambiguity, and the ability to deal with vague and noisy environment.

Rainfall Prediction

Amongst all weather parameters, rainfall plays the most crucial role in influencing human life as different civilizations on mother earth depend to a great extent upon the frequency and amount of rainfall to various scales (Agboola *et al*., 2013). Variability of weather and climatic factors, especially those atmospheric parameters will be the major force for daily precipitation event. If variability pattern could be recognized and used for future trajectory, prediction of daily rainfall is feasible (Edvin and Yudha, 2008). Fuzzy Logic helps in true prediction of rainfall conditions in terms of wind speed and temperature variables. Short-term forecasting has also proved to be clear with rainfall predictions using fuzzy logic systems (Singla *et al*. 2019). Accurate or near-accurate rain prediction admits to make better strategies for managing excess water, predicting drought and flood conditions in the future. It has been proved that Fuzzy artificial neural networks (Fuzzy ANN) gives a more reliable prediction on rainfall than the traditional artificial neural networks (ANN) (Lu *et al*., 2014). Fuzzy logic can be used to develop a model for rainfall prediction using the temperature (Jimoh *et al*., 2013). Models built on fuzzy logic and fuzzy inference systems have proved to be more authentic and definitive (Jantakoon, 2016). Fuzzy Membership approach for predicting pre-monsoon weather conditions in Kolkata has demonstrated more accuracy and reliability than the traditionally used statistical method, Linear Discriminant Analysis (Chatterjee *et al*., 2011). A rainfall probability grid developed with fuzzy logic to improve radar precipitation maps gives improved correlation results, transforming location based variables to probabilities (Silver *et al*., 2020).

The accuracy rate of rainfall predictions made using fuzzy inference system in Malaysia is about 72 per cent (Safar *et al*., 2019). Anurag *et al*. (2020) have reported better results with co-active neuro fuzzy inference system (CANFIS) in terms of reliability and accuracy in predicting standardized precipitation index (SPI). This expert system can be used in predicting drought conditions across various time periods. Fuzzy logic is an inevitable and an effective technique for correctly predicting rainfall (Das and Tripathy, 2017).

Atmospheric Temperature

Ghosh *et al*. (2014) used fuzzy set theory in forecasting atmospheric temperature for different cities in India. The variable used here for prediction was rainfall. They used sequence containing 19, 140 sets of daily observations of temperature, rainfall over a period of one year to develop the fuzzy knowledge base. They found that fuzzy systems are highly flexible and could also be applied for other mesoscale situations like stratus formation and dissipation, (non-frontal) thunderstorm development, snow squall situations and ice crystal formation. Fuzzy Inference System was used to forecast atmospheric temperature in the Indian coastal cities by Patel and Christian (2012). The variables used were, Mean Sea level Pressure, Relative Humidity and Temperature. The proposed fuzzy logic prediction model established higher accuracy in predicting the temperature. Fuzzy genetic system was used in creating a model for long-term air temperature prediction based on geographical information by Sadeghi-Niaraki *et al*. (2020).

Radiation Fog

Jim (1995) used fuzzy logic technique to forecast the probability of formation of radiation fog in operational meteorology. Adaptive neuro-fuzzy approach for solar radiation forecasting in cyclone ravaged Indian cities in Eastern part of India have been used by many workers. Several studies have been undertaken with soft computing techniques. Suitable models have been developed based on several inputs and detailed analysis has been performed to show the minimum MSE and maximum regression (R) values in different places of Eastern India after training and testing. After several studies, the ANFIS model seems to be computationally efficient and adaptable in managing different parameters. Consequently, the model is engaged in the estimation of the solar radiation–based data with extensively available meteorological information. **(**Mohanty *et al*. 2022)

Pollution Concentrations

A fuzzy weather forecast was developed to forecast pollution concentrations by collecting data for weather forecasts, meteorological situations and pollution concentrations. The fuzzy logic approach has been used in predicting pollution concentrations in the atmosphere using various parameters (Domanska and Wojtylak, 2010). Historical time series data are employed to derive a set of fuzzy rules, or equivalently a neuro-fuzzy network, for forecasting air pollutant concentrations and environmental factors in the future. The network outputs, derived through the fuzzy inference process, produce the forecast air pollutant concentrations or air quality indices (Lin e al., 2020).

Cyclone Forecasting

Most tropical cyclone(TC) forecasts depend on forecasts like wind radius and velocity. Irawan *et al*., (2019) have developed and validated a TC simulation prediction model that has proved to have lesser error terms. The accuracy of the results with Fuzzy logic algorithms were at 75%.

Besides the above mentioned parameters, fuzzy logic is also used in measuring solar energy/ sunshine du-ration, load forecasting using weather data, thermal maps, sea-breeze events detection, wind generation forecasting etc., Soft computing and hybrid systems have proven to be an established method for more complex and ambiguous data. Meteorology is innate to vague, inaccurate, and uncertain large data accompanied by subjective human conclusions. In such circumstances, fuzzy logic manifests to be the solution to accurate and authentic results for better forecasting and prediction.

Conclusion

Several soft computing technologies have found prominent applications for betterment of mankind. Though fuzzy logic had been almost half a century old, it is being applied to weather forecasting only recently. The results have been great and promising for the future. Fuzzy logic was and is continued to be a basic system of Artificial Intelligence. Its ease of blending with other tools provides an added advantage. Its advantages are plenty and is an easy tool for use. Further applications can be attempted in profound areas of weather forecasting.

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Application of Mathematics in Various Fields: An Overview

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Abstract

Mathematics is the study of space, quality, change and structure. Mathematics history was started with the simple counting. The history of mathematics as old as humanity itself. A person cannot be escape from the learning of the mathematics in one form to another, ranging from the work of the kitchen to the journey from our planet earth to Mars or Moon. The mathematics exists everywhere that can be seen in nature also. Mathematics plays vital role in the foundation of many disciplines. So it is very difficult to make of the outline boundary line for the uses of mathematics. The mathematics play crucial role in the field of economics. The daily human life and the activities are closely related to the mathematics. The existence of the mathematics persists everywhere. The mathematics plays an important role in the fields of real life, finance, banking, computer science, engineering and technology, biology, social science, etc. Mathematics is the basis tool for the solution of various types of problems with accuracy in various fields.

Keywords: Mathematics, engineering, finance etc.

Introduction

Mathematics is the study of space, quality, change and structure. The mathematics of Babylonia developed from the 2000 BC. The mathematics word derived from the Greek word “mathema”, that show the “subject of instruction”. Mathematics history was started with the simple counting. Mathematics is the foundation for the development of many subjects. We are all surrounding a mathematical world. In real life the mathematical formulas, theories and concepts are widely used. The mathematical tools and techniques have importance and understanding its variety of applications. The mathematics plays a very crucial role in the field of economics also. The indispensable linkage has between mathematics and economics. The use of mathematics is growing day by day in different fields like Physics, biology, finance and banking, engineering, in real life etc.

Branches of Mathematics

Mathematics is the language of the universe. There are many branches of mathematics. Some the branches are:-

(i) Number Theory (ii) Topology (iii) Combinatorics

(iv) Calculus (v) Statistics and Probability (vi) Arithmetic

(vii) Trigonometry (viii) Geometry (ix) Algebra

Objective of the Study

1. To be handle real situations through conceptual knowledge of mathematics
2. To know the application of the mathematics in the various fields

The application and role of mathematics is in diverse fields. Some mathematics applications are discussed.

Application of Mathematics in Industry

Industrial mathematics is come under the applied mathematics branch and deals with the issues come back from the industry and business for solutions in an economical way. A person can formulate accurate and precise mathematical models and implement solutions with the help of latest techniques of computer in the increasing complexness environment of business. The solutions can guide the manager, worker and management to take better decisions for the industry. At the present time the demand of mathematical trained persons is increasing in industry and business [1]. A number of issues related to designing solved by using the mathematics tools. With the help of linear programming a business solved the large, real and combination production issues.

Application of mathematics in computer Science

Mathematics is a unique discipline of science and provides a base for various areas of engineering and science. So it is very difficult to make of the outline boundary line for the uses of mathematics. Now these times, many mathematicians studying the computer science in disguise and more computer scientist doing mathematics and their applications in computer science. Some people argue that mathematics play a very little role in computer science and some say mathematics is the base of the computer science [2]. Many subjects of sciences are common and different from each other. Computer science is a subset of mathematical science. Graph theory, discrete mathematics, algebra, binary maths is the mathematical branches and most related to the profession of computer science. The knowledge of these types of math concepts helps to manage databases, algorithms and structures.

Application of Mathematics in Economics

The use of mathematics is in various fields, but plays a major role in the field of economics also. To draw conclusions in the economics, the geometry, calculus, algebra, etc. are used by the economists. Mathematics is the most efficient and universal language in the economics also. A researcher can find the results with accuracy in the field of economics by using the mathematical ideas and formulas. The economics and mathematics are interlinked and the different kinds of advance techniques of mathematics are used for solving the problems in economics suggested [3]. With the help of trend analysis a business can predict the future and also analyzed the risk to avoid the losses and make better decisions.

Application of Mathematics in Finance and Banking

Now this time the use of mathematics is not limited the textbooks also. Mathematics also plays a vital role in the sector of finance and banking. Mathematics plays a major role in the predication of the fall and rise of the stock market. Financial mathematics can be applied to solve the finance related problems. With the help of the various techniques of the mathematics, the stockbroker can earn revenue from the stock market. Banks and other corporate institutions apply the financial mathematics methods for risk management, valuation of securities, manage portfolio. Top rank officer responsibility to make policy formulation that may help the organisation to achieve goals [4]. These tools of mathematics draw results immediately; otherwise it could be difficult for the institutions. The financial decision taken every day affected by the mathematics. The good knowledge of mathematics concept is beneficial in the banking and finance sector. So there is no doubt that mathematics plays key role in the planning and successful financial lives.

Application of Mathematics in Biology

In the modern time mathematical biology is a very exciting application. A large number of mathematical tools are implemented in biology. The area of mathematics like graph theory, algebraic geometry, differential equations, probability theory, coding theory now applied in the field of biology [5]. There are several biostatistics centers in the world. The mathematical modelling is a contribution in the field of biology also. Mathematical biology also includes the evolutionary and ecology. A large number of functioning aspects of tissues and cells can be the matter of mathematical treatment quite successfully [6].

Application of Mathematics in Engineering

Mathematics is the base of various engineering stream. The present growth of engineering is not possible without mathematics. The uses of statistics, numerical analysis, linear algebra, etc. are taught as a subject in various engineering courses. Mathematics is the backbone of mechanical engineering subject and applied in various fields. To solve the problems in the above said field the procedure and concept of mathematics are used [7].

Application of Mathematics in Physics

Mathematics is a very attractive subject and very powerful tools for physic. Math plays a critical role in scientific calculation in the field of physics. Mathematics is very supportive tools for physics and without it physics cannot be done. The mathematical techniques like ordinary and partial equations, topology, infinite series, complex analysis, calculus, group theory, probability and statistics are used in Physics. The calculus is used so frequently in Physics – in all equations of motions. In quantum mechanics, all physical quantities are the most probable values and the wave function itself is probabilistic in nature [8].

Conclusion

The application of theories, ideas and formula in the subject of mathematics has very important in the real life also. To carry out the solution of different problems, everyone tries to understand and learn about the formulas and techniques of mathematics. The mathematical tools help to predication and making policy for future investment.

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Application of statistical methods in Geography after Quantitative Revolution

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Abstract

In 1950s and 1960s a scientific approach has come in Geography with the name of Quantitative Revolution .Quantitative Revolution has given geography a scientific, systematic, precise approach for study. Traditionally Geography was a study of description only. This deals with the study of surface only. But with the passage of time the nature, scope and definition was changed. It has also changed its methodology and techniques for study. The Quantitative approach entrenched with Models, theoretical structures that may be suited geographical aspects. Quantitative Revolution was set up by mathematician and physicists. It has the result of mathematician and physicists upon Geography that converted description approach in to scientific approach. Quantitative Revolution has guided two main approaches for geography analysis. The first approach was spatial analysis which gave statistical framework to geography. The second approach was spatial science which pointed out the role of space as basic effecting factors of society and individuals. After that spatial interaction and spatial structure come in to view as the main features of geography. Thus the main objectives of the Quantitative Revolution were to convert Geography as a scientific subject from descriptive. Secondly to describe and analysis geography with logical manner. Thirdly use of mathematical language for geography concepts. Fourthly use of obvious and precise manner. Fifthly to evaluate and forecast Geographical phenomenon. Lastly location analysis for maximum profit.Thus Quantitative Revolution has changed the frame work of the Geography.Intially Geography was descriptive subject but after this revolution Geography has become scientific, systematic, precise, logical and analytical approach.

Keywords: Quantitative Revolution, Scientific, Logical, Analysis, Descriptive ,Presice etc

Introduction

In the starting time geography was science of description only which deals with knowledge about surface of the earth. But as the time passed the nature, scope, definition and techniques was changed and it also changed the way of concept, phenomena description in geography. Now it becomes scientific systematic, precise description of earth surface and geographical events. The most visible change was come with occurrence of quantitative revolution in geography. Which change the framework of the study. It brings scientific approach and use of statistical methods in geography.In 1950s and 1960s scientific approach has come in geography with the name of quantitative revolution. The quantitative revolution has entrenched with models and mathematical language that may be suited geographical aspects. After second world war empirical study was not accepted by the geographers. But they gave stressed or scientific methods, models, techniques was use for the study of geographical and general aspects. After that geography study has been started mathematical language for geographical concepts. Basically the root of this revolution was lies in positivism. First of all "Burton" describe about quantitative revolution in 1963 in his research paper "The quantitative revolution and theoretical geography"

In 1958 had also told that geography will starting using regression, correlation, varience after some time.

Objectives of Quantitative Revolution

1. Transform geography in to scientific study from descriptive study.
2. Describe and analysis geographical phenomena in to rational manner.
3. Mathematical language use for understanding geographical concepts.
4. For better implementation and management of geographical phenomena.
5. For precise and scientific obvious description.
6. Identification maximum profit from economic activities.

Assumptions

1.Ouantitative revolution assume a man as an optimizer who looks maximum for profit.

2.It also assumes that earth surface is istrofic and same conditions everywhere.

3.There is no space for any emotions like-religion, customs, cultural etc. in scientific study

Quantitative Revolution in Geography

Conventionally, Geography has been their roots in other disciplines. The conviction of environmental determinism has been presented by Sample, Huntington, Griffeth Taylor, and Ratzel . They has been busy with the ideas of a causal association and has been always looking for new "laws". A similar involuntary flavor has been existed in the Quantifiers work. It looks if geography has been re-emerging after it will remain ideographic approach, which will create a space between geography and determinism. It looks in different way; this revolution pull geography direct to environmental determinism. The Quantitative Revolution was strongly opposed the doctrine of environmental determinism .which delayed the process of establishment of the scientific basis that the quantifiers wanted to provide. Still, new methodology has been used. According to Bronowski (1959) “statistics replace the notion of inevitable effect by probable trends”. As the Quantitative revolution developed the use and purpose of statistical techniques of quantification has became more and more in deterministic.

In geography, the revolution has been begun in the late 1940's and peaked from 1957 to 1963. In these years, it gain momentum especially after Ackerman and Schaefer favored in making geography more theoretical and systematic in nature. Ackerman commented, "although the simplified forms of statistical assistance have been part of geographic distribution analysis in the past; discipline is beginning to move towards more complex statistical methods-a completely logical development’. Burton further commented that both Hartshorne and Spate also agreed on the usage of these methods in geographical.

According to Hartshorne (1959) statement, "To raise thinking above the scientific knowledge level, it is important to establish generic concepts, which can be implemented with maximum objectivity and accuracy through quantitative measurements which can be subjected to comparisons through the mathematical logic”.

Spate (1960) in his paper on “Quantity and Quality in Geography’’; published in the Annals of the American Geographers looks somewhat dubious about quantification in geography. The National Research Council Committee on ‘The Science of Geography’ (1965) also discussed the impact of quantitative revolution in geography. They stated that geographers believe that correlation of spatial distributions, considered both statistically and dynamically, maybe the keys to understanding the development of living systems, social structures and environmental changes that occur over the earth surface. In the past progress has been slow as the number of geographers was least interesting. Moreover, the methods of analyzing these multi-variant problems have been meticulous.

Path of the Quantitative Revolution in the Discipline of Geography

The roots of the Quantitative revolution have been seen in the following writing, which has their notable impact on the event and progress of quantification in geography. These were – Neuman and Morgenstern’s “Games and Economic Behavior (1944)” Weiner’s volume on Cybernetics (1948); Human Behaviour and the Principle of Least Effort by Zipf (1949) and Stewart’s paper entitled Empirical Mathematical rules Concerning Distribution and Equilibrium of population’ (1947). Stewart’s paper “needs special mention” they put forward a new way to geographic questions. The impact of quantification has been started immediately in geography. Quantification increased in geography. For example, in 1936, John Ker Rose described in his paper on cultivation and climatic conditions that "the methods of relational analysis would be particularly promising tools for geographical investigation.". Strahler commenced an excellent petition when he striked Davis’s explanatory and descriptive explanation of geomorphology. He has been supported G. K. Gilbert's dynamic-quantitative systems.

Quantitative Revolution in the branches of Geomorphology and Climatology

Strahler said that Gilbert's paper was better than Davis's work; because it has not accepted as a sign post in geomorphology for future work; rather it had forgotten and neglected from thirty years. The answer was with Strahler himself who thinks that geomorphology was a part of geography. The physical geographers have not adopted these ideas rather they had followed Davis. Some of the prominent followers of Davis include Douglas Johnson, C A. Cotton, N. M Fenneman, and A.K. Lobeck. Strahler finally states these geographers made "excellent contribution to descriptive and regional geomorphology" and has provided a solid foundation for study in "human geography", but did not lay the basis for scientific study within the geographical thinking. This does not mean that prior to Strahler; geographers has not been using quantitative techniques in geomorphology. Quam and Woolridge criticized his views. Quam (1950) states that mathematical formulae and statistical analysis in geomorphology may result in showing an unrealistic picture of reality that might not be accurate and objective. Similarly, Woolridge (1959) critics Strahler's views and states that although there was the prevalence of a 'new' quasi-mathematical geomorphology; it was inadvisable to use mathematics at a higher level as these were not apt in explaining the geomorphologic phenomenon. He further states that whatever the case may be they will continue to regard W. M. Davis as their founder and would criticize all those who has not agreed with the methodology of Davis's interpretations of a different phenomenon occurring over the earth surface.

It was not that geomorphologists who adopt quantification; Strahler found his support in L. King (1962) who wrote that statistical methods were useful for bulk studies and could be well appreciated if used to study complex phenomenon and processes that constitute a large number of variables or indicators. Although a few studies in the branch of geomorphology could apply them, they should be used with great precision so that results were not superficial in nature. Many geographers in addition to Strahler like Chorley, Dury, Mackay and Wolfman, used quantitative methods and it seemed that the practice would spread.

In the case of climatology, there was little dispute about the use of quantification. This branch of geography whole-heartedly embraced these new statistical techniques to interpret various climatic phenomena. Examples can be cited from the works of Thornthwaite, Mather and Green, Bryson who have successfully implemented quantitative techniques to seek answers for climate problems; thus silencing their critics.

Quantitative Revolution in the branches of Human and Economic Geography

So far, the biggest struggle for approval of quantification has been in human and economic geography. It is not surprising that in view of the possibility tradition; it is here that the revolution runs against the ideas of independence and the uncertainty of human behavior. Here comparisons with physical sciences are useful. Physicists who work at a microscopic level, with quanta and energy, face similar problems that social scientists face with people. Such parallels when recognized are a reason for happiness and not for disappointment. In order to make a reputable place in human society, social science must get direct results in the form of a prediction science that does not need any kind of control, restriction or regiment the person. A social science that distinguishes random behaviour at the micro-level and is even able to foresee results at this level is nothing but the consequence of quantitative revolution.

Several works can be cited which used statistical techniques in a positive manner. Most interestingly large number of debates took place between scholars that appeared in the literature (Burton, 1963). Some of these are worth mentioning – Garrison’s and Nelson debate on Service classification of cities; Reynolds – Garrison’s deliberation on the modest use of quantification in geography. The Spate – Berry argument in Economic Geography that ends on the agreement that statistics are half of a filled glass, the other half is understanding and interpretations. The list is endless but some of the other debates that need to be mentioned include the contest between Zobler and Mackay on the use of chi-square in regional geography and the dispute of Lukermann and Berry on ‘geographic’ economic geography.

The deliberations were done through professional magazines, which got them the much-needed attention. The result was the establishment of the Regional Science Association in 1956 that promoted quantification in geography. Moreover, it made quantifiers an essential part of the geographical thinking and giving them appreciation and approving their work part of the geographical academia.

Although most of the literature cites that, the revolution is over, it has remained active in several sub-branches of geography particularly transport, economic, and urban geography. This is evident from the fact that writings with quantitative methods have been regularly published in acclaimed journals in geography, including Annals of the Association of American Geographers, Geographical Analysis, Environment, and Planning A, The Professional Geographer, Journal of Geographical Systems, Urban Geography, and many others (Kwan and Schwanen, 2009). Although quantitative geography is generally "perceived as a relatively static research area," it is actually "a vibrant, intellectually exciting, area in which many new developments are taking place" {Fotheringham, Brunsdon, and Charlton (2000); Clark (2008); Golledge (2008)}.

Interestingly, quantification in geography has changed its course in due course of time. It now an ally of critical geographies - for example, the emphasis has shifted from global generalizations to local levels dealing with local relationships in a spatial framework. It has also become sensitive to variables like gender, race, ethnicity, sexuality, and age; and even pays attention to processes which shape individual's spatial behaviour (Kwan and Weber 2003; Poon 2003; Fotheringham 2006).

Quantitative research is still dominant in the fields of transport, economic, and urban geography in the writings of McLafferty and Preston (1997), Rigby and Essletzbichler (1997), Plummer and Taylor (2001), Schwanen, Kwan, and Ren (2008) and Bergmann, Sheppard, and Plummer (2009). In this regard, Kwan and Schwanen (2009) are of opinion that knowledge in statistical methods is essential for decoding and challenging regressive political agendas; often supported by numbers and quantitative analysis. Quantitative geography, when incorporated with a critical sensibility and used suitably, can be a powerful device for encouraging progressive social and political change.

Major Theoretical and Methodological Developments

- Location Theory: Location theory is basically used for identification of location for economic activities. W.L Garrison (1959) was the first who preferred this location theory. Then 1965 peter Hagget stressed on locational concepts. However, traditionally, in 1826 and Alfred Weber (1909) has been described about location theory.Later on ohlim, losh, Itoover ventured location analysis theories For economic activity. In these location theories mathematical techniques has been used for analysis.

- Central Place Theory: It was a theory which explained about size and distribution of catchment area of facilities in urban system. This theory was given by Walter Christaller in 1933. He used the mathematical concepts like range, threshold for describing good service and economically viable of a urban area. In 1954 Augush Losch was also given theory on urban system. He used Range, threshold hexagonal hinterland etc. mathematical language for his model.
- Diffusion Innovation: Diffusion innovation means the spread of a phenomena over space and time. Ratzel was the first who attempted diffusion innovation model. But it was Hagerstrand who developed the concept. He was the person who developed qualified the concept. In 1953 Pred translated his thesis as 'Innovation Diffusion as a spatial process'. By this, he analysis the diffusion of different and variable innovation among population of a part of central Sweden. In it Hagerstand used interaction matrix for showing mean information and regional system.
- Location Analysis: '1965' peter Hegget who gave an approach named ' Locational analysis in human geography' in which spatial science was the main aspect. In it main emphasis on asking question location order etc.
- System Approach and System Analysis: System concept basically associated with systems concept and structuralism. Ludwig Von Berta langgy (1950) was proposed general system theory. No methods and techniques has been inverted before that. Its main focus on Isomorphism. R.J Choreley was the first geographer who gave general system theory in this paper 'Geomorphology and general system' in 1962. Berry 1964, Wodenberg and Warntz (1973) also emphasis on system approach.

Main Features of Quantitative Revolution

- Usage of statistical mathematical methods.
- Usage of Newton's physics gravity and potential ideas for spatial interaction models

- Usage of German school of location theory for agricultural land use, industrial location, urban location settlement
- Usage of urban sociology for inter metropltian explaintary model
- Usage of Geometry for mathematical study of spatial form
- Usage of computers
- Usage of philosophical justification theory

Advantages of Quantitative Techniques

- Techniques were based on empirical observations and readily verifiable
- Helpful in reducing a multitude of observations in to a manageable number of factors.
- Helpful in the formulation of structured ideas and theories which can be tested in different conditions.
- Helpful in deriving suitable models to understand the interaction of the evolved factors and their process within the models and with reference to observed facts.
- Helpful in identifying tendencies and desired trends, laws and theoretical concepts. 5.

Conclusion

Thus the use of statistical or quantitative techniques is one of the most suitable methods for the development of theory in geography. The revolution can never be over until it is able to seek answers and aid the theoretical development of the discipline. Moreover, theory development and its testing are the only ways of creating new knowledge and subsequently verifying it. Models have just formalized ways of descriptions that an author has visualized and represented through his arguments and justifications. In geography, quantification brought this revolution where the ideographic base was replaced by theory building in a nomothetic approach. Geographers started developing theories and created ‘new’ geography that focused on the philosophy as well as methods. These scholars were of the view that mere description, mere quantification, and mere abstraction were valid to a certain extent; but repeated use of these methods makes one an obscurantist. Theoretical geography got its philosophical base in Bunge’s monograph published in 1962, which identified geometry as the mathematics of space and hence made spatial science the language of new geography. Harvey’s Explanation in Geography (1969) provided a more inclusive channel for the methods and philosophy of new geography. Apart from these scholars, the Department of Geography at Lund University, Sweden became a centre for quantitative and theoretical geography under the leadership of Hagerstrand and Morill. Hagerstrand although based in Seattle provided an academic support to the geographers working in this field at the Lund University. To conclude, whichever method one, the purpose of geography is to seek answers to questions pertaining to problems of quantity and value. Most of our experiences are qualitative in nature and when everything is, reduce to numbers; some essential attributes are lost (Huxley, 1951). Thus one needs to maintain balance as still new worlds are to be conquered and new contributions to be made. Thus quantitative revolution has made Geography more scientific and precise subject.

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Role of Mathematics in Business, Commerce & Management Curriculum

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Abstract

Mathematics is used in almost every domain of human world such as Industry, Commerce, Physics, Chemistry, Economics, Biology, Psychology, Astronomy, Engineering, Medicine and many more, hence application of mathematics is quite extensive.Today, in this rapid changing world, job roles and nature have changed tremendously.Job roles are redefined as analyst, consultant, research associate, data miner, etc. which involve advanced mathematical applications. Business, commerce and management are the most popular disciplines among students. But these subjects lack mathematical applications in its introduction stages. And now in this changed scenario, reputed global institutions offers higher education opportunities for students who are having mathematical background. So the relevance of mathematics is increasing. Hence the objective of this paper is to know the application of mathematics in business, commerce and management as well as role of mathematics in the business. & commerce curriculum.The study is exploratory cum descriptive in nature and secondary sources are used for data collection.

It can be concluded from the above study that the field of business & management from stating to end is heavily depending upon mathematical applications. Mathematics is one of the most important element in building business & management education which is required for self- reliant and self-sufficient country. By including mathematics in commerce & management curriculum the students get *more opportunities for their higher education and most importantly their logical & critical thinking capacity & problem solving analytical skills will be improved.*

Keywords: Mathematics, Curriculum, Business, Commerce

Introduction

Mathematics is considered to be the study of numbers, shapes & patterns and when this discipline is applied in a business, it becomes Business Maths. For a business, it is a vital subject that a student has to deal with. It acts as a tool that helps in solving and controlling various business problems. The basic objective to learn this subject is to adapt the knowledge of various mathematical tools and techniques and models which helps in dealing with real-life business situations. It is also referred to as "commercial mathematics"

In case of curriculum, in India, Business and management is extensively taught in college and university levels. Earlier, the syllabus includes Accounting as only numerical paper and other subjects like management, marketing, entrepreneurship, business studies, auditing etc. Then the role of mathematics and logical thinking came into picture and thus added subjects like business statistics, operations research, and quantitative techniques. Nowadays, to get admission for graduation as well as post-graduation courses in business, commerce and management disciplines, reputed institution made it compulsory to study mathematics as a separate subject in the qualifying examination. So that many commerce and management courses introduces new paper titled Business Mathematics in their curriculum.

Business and management is meaningless without mathematics. Business is concerned with buying and selling of goods and services with an aim to make profit. Here the value of good or service is expressed in monetary term, which is termed as price, and the end result which is either profit or loss also expressed in numerical. That clearly denotes how important mathematics in business is.

On the other hand, management is the art of getting things done through others and decision making. The task of every manager or higher official is to take managerial decision and mathematics aids them for logical solution for the problems concerned.

Decision making is not a single activity, it is process which consist of series of inter related tasks. First step is clearly defining the problem, next step is analyzing the available alternatives in terms of its cost and benefit which is characterized by mathematical model, final step is selection of most suitable model for solution of the problem. Thus, mathematical applications like algorithm, calculus, algebra, linear programming, statistics, probability etc. are needed for the full-fledged functioning of business and management.

Objective

- To know the applications of mathematics in business, commerce and management
- To understand the role of mathematics in various subjects included in commerce and management curriculum
- To know the Career options for commerce students having maths during graduation

Methodology

The study design is exploratory cum descriptive in nature. This paper have a look at secondary facts collected from various online portals, posted research articles, books etc.

Applications of Mathematical Techniques in Commerce & Management

The following table shows the various techniques of mathematics being used in commerce and management

Table 1

Mathematics techniques used in commerce & management

Calculus	• Cost Function i.e **C(x)=F+V(x)** • Demand Function i.e **d=f(p)** • Revenue Function i.e **R(x)=p.x** • Profit Function i.e. **P(x)=R(x)-C(x)** • Break even pointi.e. **P(x)=0or R(x)= C(x)**

Algebra	• Helpful in comparing business cost, profit, revenue and expenses • Helpful in deriving various Ratios like inventory turnover ratio, profitability ratios, debt- equity ratios • Helpful in deriving accounting equation i.e. **A=L+C** • Helpful in taking decision whether manufacture the product or buy it by calculation production cost and buying cost
Operational Research	• Inventory model • Allocation model • Game theory • Network models like **PERT & CPM** • Sequencing model
Statistics &Probability	• Helpful in formulating plans & policies for future • Forecasting of market fluctuations & trends • Forecasting of Sales, scenarios, future returns & risks • Various techniques like correlation, regression, index numbers, time series analysis used in various business section making processes
Linear Programming	• Used for solving Marketing mix Problems • Company restructuring • Routing • Scheduling

Use of Mathematics in Commerce & Management Curriculum

Financial Management

Financial management is one of the core areas in business and commerce. Financial management is concerned with the procurement and effective utilization of funds. Financial management is a more quantitative discipline in which various mathematical applications are there. Some of them are:

- Calculation of rate of return or cost
- Asset valuation

- Portfolio management
- Derivative trading
- Risk management
- Security pricing etc.

Security Analysis and Portfolio Management

One of the important areas in business and commerce is security market. It includes capital market and money market. Various short term and long term securities are traded in these markets. As we know, security market offers huge return at the same time it possess huge risk element too. So before investing money in financial or security markets, it is better to analyze the risk return trade off and select the best possible portfolio which gives maximum return with minimum risk. Portfolio here refers to group of securities. For analysis purpose, various mathematical models are used; they are Capital Asset Pricing Model (CAPM), single index model, fama-french model, treynor ratio ,Jensen's index etc. With these mathematical models, we can make rational investment which in turn benefits the investor.

Statistics & Probability

After realizing the importance of statistics in business field, a separate subject known as business statistics is introduced in the commerce and management curriculum. Statistics assist business in forecasting demand and supply, market fluctuations, trend analysis, ratio analysis, financial statements interpretation etc. As we know future is uncertain, this uncertainty is having major impact in business. There is need to predict the uncertainty and should provide for the future. There comes the need of probability applications (Levine et al., 1998). It is a sub area of statistics and essential for prediction and estimation of risk and return of business investments. It serves as a tool for decision making for the top level management

Accounting

Accounting discipline is further sub divided into financial accounting, cost accounting, corporate accounting and management accounting. In **financialaccounting**, the net result of business is exhibited with the help of Profit and Loss account and Balance sheet. The accounting is based on an equation which is mathematical one. The equation is A= L+C. "A" stands for Assets, "L" stands for Liabilities and "C" stands for Capital. By using mathematical relation, A=L+C, we can calculate total cost and decide whether to make or buy the product. The total cost formula for business is T= a+bx; where “T” is total cost, “a” is fixed cost, “b” is cost per unit produced and “x” is number of units produced. Profit or gain is calculated by subtracting total cost from total revenue and helps in analyzing the financial health of business. Prices are determined by adding some mark-up or profit margin to cost.

In **cost accounting,** calculation of cost pertains to the cost centers, ascertainment of Economic Order Quantity (EOQ), setting of stock levels etc. involve mathematical applications. In corporate accounting, accountings of companies are dealt with. It requires mathematical application in case of business combinations like amalgamation, absorption, external reconstruction, calculation of purchase consideration, computation of value of goodwill, liquidation statement of affairs etc. **Management accountin**g mainly deals with managerial decision making. It involves financial statement analysis, fund flow statements, cash flow statements, budgetary control, ratio analysis etc. ratio equations are derived from mathematical techniques and helpful in business comparison.

Production planning & Pricing

Career options for commerce students having Maths as subject in their curriculum

- Chartered Accountant (CA)
- Company secretary (CS)
- Cost & work accountant (CWA)
- Chartered financial analyst (CFA)
- Marketing manager
- Production manager
- Retail manager
- HR Manager
- Certified public accountant
- Economist
- Banker
- Business analyst
- Hotel manager
- Data scientist

- Event manger
- Statistician
- Actuarial science
- Financial advisor
- Lawyer
- Certified investment banker

Mathematics in curriculum will also very useful in dealing with competitive exams in near future. It also does brain exercise which increases the intellectual level. Mathematics is challenging, rewarding and fun. It is both logical and creative. If mathematics is included in the business, commerce and management curriculum, it will develop skills like:

- Critical thinking
- Problem solving
- Simplification of complex problems
- Analytical thinking ability
- Quantitative aptitude
- Logical reasoning
- Ability to manipulate precise and intricate ideas
- Construct logical arguments and expose illogical arguments
- Decision making
- Flexibility
- Brainstorming ideas

Conclusion

The role played by mathematics in the field of business, commerce and management is remarkable. If we try to extract and set aside mathematical element from these fields, the area becomes meaningless. Linear programming and operations research made complex business problems into simple solutions; calculus and algebra applications of mathematics is essential in profit, revenue and tax computation; Program Evaluation and Review Techniques (PERT) and Critical Path Method (CPM) helps in project scheduling and cost crashing; without mathematical principles and formulas, accounting will be waste of time, product pricing will decide the future of business which is depend on mathematical analysis and forecasting. Thus, in a nutshell, the field of business and management, from its beginning to end is heavily relying upon mathematical applications.By including mathematics in curriculum, the students get more opportunities for their higher education and most importantly, their logical, critical thinking capacity and problem solving analytical skills will be improved.

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Cryptography

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Abstract

With the internet having reached a level that merges with our lives, growing explosively during the last several decades, data security has become a main concern for anyone connected to the web. Data security insures that our data is only accessible by the intended receiver and prevents any modification or alteration of data. In order to achieve this level of security various algorithms and methods have been developed. Cryptography can be defined as techniques that cipher data, depending on specific algorithms that make data unreadable to the human eye unless decrypted by algorithms that are predefined by the sender. Cryptography is a technique to achieve confidentiality of messages. The term has specific meaning in Greek: "secret writing". Now a days, however the privacy of individuals and organizations is provided through cryptography at a high level, making sure that information sent is secure in a way that the authorized receiver can access this information. With historical roots, cryptography can be considered an old technique that is still being developed. Examples reach 2000B.C.,when the ancient Egyptians used "secret hieroglyphics, as well as other evidence in the form of secret writings in ancient Greece or the famous Caesar Cipher of ancient Rome.

Keywords: Cryptography, Security, Algorithm, Cipher, Encryption, Decryption, Data Security

Introduction

Cryptography is a technique to achieve confidentiality of messages. The term has specific meaning in Greek: "secret writing". Now a days, however the privacy of individuals and organizations is provided through cryptography at a high level, making sure that information sent is secure in a way that the authorized receiver can access this information. With historical roots, cryptography can be considered an old technique that is still being developed. Examples reach 2000 B.C., when the ancient

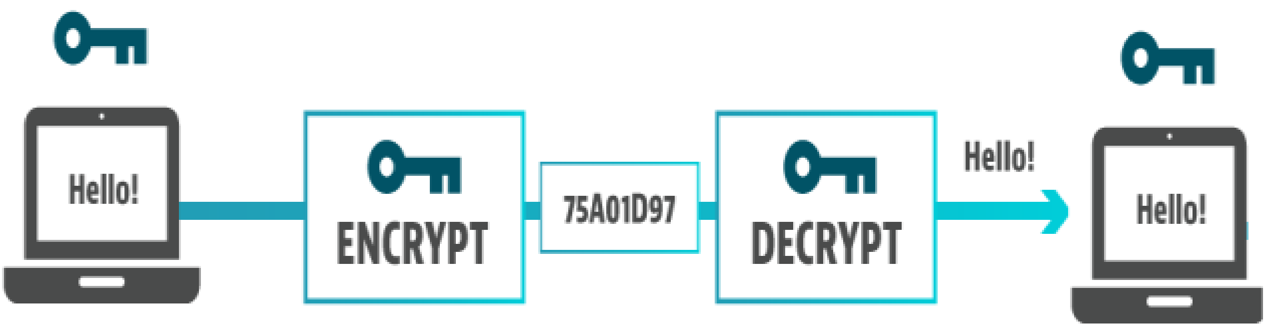
Egyptians used "secret hieroglyphics, as well as other evidence in the form of secret writings in ancient Greece or the famous Caesar Cipher of ancient Rome.

Billions of people around the globe use cryptography on daily basis to protect data and information, although most do not know that they are using it. In addition to being extremely useful, it is also considered highly brittle, as cryptographic systems can become compromised due to a single programming or specification error.

Cryptography Concept

The basic concept of a cryptographic system is to cipher information or data in order to achieve confidentiality of the information in a way that an unauthorized person would be unable to derive its meaning.

In cryptography the concealed information is usually termed **"plain text"**, and the process of disguising the plain text is defined as **"encryption"**; the encrypted text is known as **"cipher text"**. This process is achieved by a number of rules known as **"encryption algorithms".** Usually, the encryption process relies on an **"encryption key"**, which is then give to the encryption algorithm as input along with the information. Using a **"decryption algorithm"**, the receiving side can retrieve the information using the appropriate **"decryption key"**.



Caesar Cipher

This is one of the oldest and earliest examples of cryptography, invented by Julius Caesar, the emperor of Rome, during the Gallic Wars. In this type of algorithm the letters are encrypted by with the letters that come three places ahead of each letter

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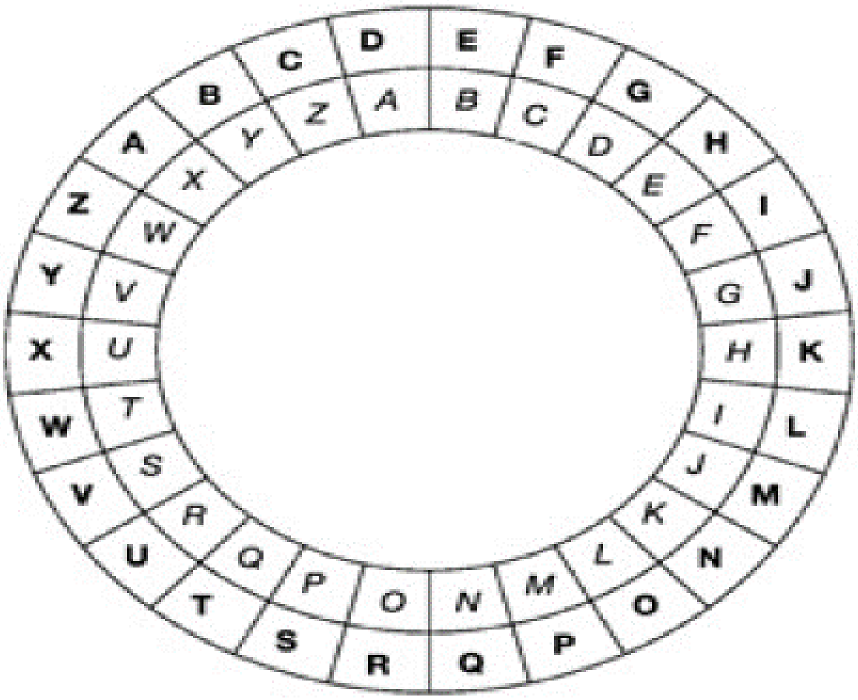
in the alphabet. This means that a “shift” of 3 is used. As the Caesar cipher is one of the simplest examples of cryptography, it is simple to break. In order for the cipher text to be decrypted, the letters that were shifted get shifted three letters back to their previous positions.



Caesar Cipher encryption wheel

Simple Substitution Cipher

Take the Simple Substitutions Cipher, also known as Mono alphabetic Cipher, as an example. In a Simple Substitution Cipher, we take the alphabet letters and place them in random order under the alphabet written correctly, as seen here:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

E R J A U W P X H L C N G D I Q M T B Z S Y K V O F

In the encryption and decryption, the same key is used. The rule of encryption here is that “each letter gets replaced by the letter beneath it”, and the rule of decryption would be the opposite. For instance, the corresponding cipher text for the plaintext MATHEMATICS is **NEBXWNEBHJT.**

Hill Cipher

Hill cipher is the multi letter cipher, developed by the mathematician Lester Hill in 1929.

The Hill Algorithm

This encryption algorithm takes m successive plaintext letters and substitutes for them m cipher text letters. The substitution is determined by m linear equations in which each character is assigned a numerical value (a=0, b=1 ....z=25). For m=3, the system can be described as

$c1 = (k11p1 + k12p2 + k13p3) \bmod 26$

$c2 = (k21p1 + k22p2 + k23p3) \bmod 26$

$c3 = (k31p1 + k32p2 + k33p3) \bmod 26$

$$[\,c1 \quad c2 \quad c3\,] = [p1 \quad p2 \quad p3] \begin{bmatrix} k11 & k12 & k13 \\ k21 & k22 & k23 \\ k31 & k32 & k33 \end{bmatrix} \text{mod}26$$

C = **PK** mod 26.

This can be expressed in terms of row vectors and matrices where **C** and **P** are row vectors of length 3 representing the plaintext and cipher text, and **K** is a matrix representing the encryption key. Operations are performed mod 26.

Transposition Ciphers

All the techniques examined so far involve the substitution of a cipher text symbol for a plaintext symbol. A very different kind of mapping is achieved by performing some sort of permutation on the plaintext letters. This technique is referred to as a transposition cipher. The simplest such cipher is the **rail fence** technique, in which the plaintext is written down as a sequence of diagonals and then read off as a sequence of rows. For

Example , to encipher the message “meet me after the toga party” with a rail fence of depth 2, we write the following:

m e m a t r h t g p r y

e t e f e t e o a a t

The encrypted message is

MEMATRHTGPRYETEFETEOAAT

This sort of thing would be trivial to cryptanalyze.A more complex scheme is to write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns. The order of the columns then becomes the key to the algorithm. For example,

Key: 4 3 1 2 5 6 7

Plaintext: a t t a c k p

o s t p o n e

d u n t i l t

w o a m x y z

Cipher text: TTNAAPTMTSUOAODWCOIXKNLYPETZ

Thus, in this example, the key is 4312567.To encrypt, start with the column that is labelled 1, in this case column 3.Write down all the letters in that column. Proceed to column 4, which is labelled 2, then column 2, then column 1, then columns 5, 6, and 7. A pure transposition cipher is easily recognized because it has the same letter frequencies as the original plaintext. For the type of columnar transposition just shown, cryptanalysis is fairly straightforward and involves laying out the cipher text in a matrix and playing around with column positions. Diagram and trigram frequency tables can be useful. The transposition cipher can be made significantly more secure by performing more than one stage of transposition. The result is a more complex permutation that is not easily reconstructed. Thus, if the foregoing message is re encrypted using the same algorithm,

Key: 4 3 1 2 5 6 7

Input: t t n a a p t

m t s u o a o

d w c o i x k

n l y p e t z

Output: NSCYAUOPTTWLTMDNAOIEPAXTTOKZ

To visualize the result of this double transposition, designate the letters in the original plaintext message by the numbers designating their position. Thus, with 28 letters in the message, the original sequence of letters is

01 02 03 04 05 06 07 08 09 10 11 12 13 14

15 16 17 18 19 20 21 22 23 24 25 26 27 28

After the first transposition, we have

03 10 17 24 04 11 18 25 02 09 16 23 01 08
15 22 05 12 19 26 06 13 20 27 07 14 21 28

Public Key Systems

The invention of public key encryption can be considered a cryptography revolution. It is obvious that even during the 70s and 80s, general cryptography and encryption were solely limited to the military and intelligence fields. It was only through public key systems and techniques that cryptography spread into other areas. Public key encryption gives us the ability to establish communication without depending on private channels, as the public key can be publicized without ever worrying about it. A summary of the public key and its features follows:

1) With the use of public key encryption, key distribution is allowed on public channels in which the system's initial deployment can be potentially simplified, easing the system's maintenance when parties join or leave.

2) Public key encryption limits the need to store many secret keys. Even in a case in which all parties want the ability to establish secure communication, each party can use a secure fashion to store their own private key. The public keys of other parties can be stored in a non-secure fashion or can be obtained when needed.

3) In the case of open environments, public key cryptography is more suitable, especially when parties that have never interacted previously want to communicate securely and interact. For example, a merchant may have the ability to reveal their public key online, and anyone who wants to purchase something can access the public key of the merchant as necessary when they want their credit card information encrypted.

Digital Signatures

Unlike cryptography, digital signatures did not exist before the invention of computers. As computer communications were introduced, the need arose for digital signatures to be discussed, especially in the business environments where multiple parties take place and each must commit to keeping their declarations and/or proposals. The topic of unforgeable signatures was first discussed centuries ago, except those were handwritten signatures. The idea behind digital signatures was first introduced in a paper by Diffie and Hellman titled “New Directions in Cryptography” [22].

Therefore, in a situation where the sender and receiver do not completely trust each other, authentication alone cannot fill the gap between them. Something more is required, i.e. the digital signature, in a way similar to the handwritten signature.

Digital Signature Requirements

The relationship that created the link between signature and encryption came into existence with the “digitalization” era that we are currently witnessing and living in. The requirements for an unforgettable signature schema would be:

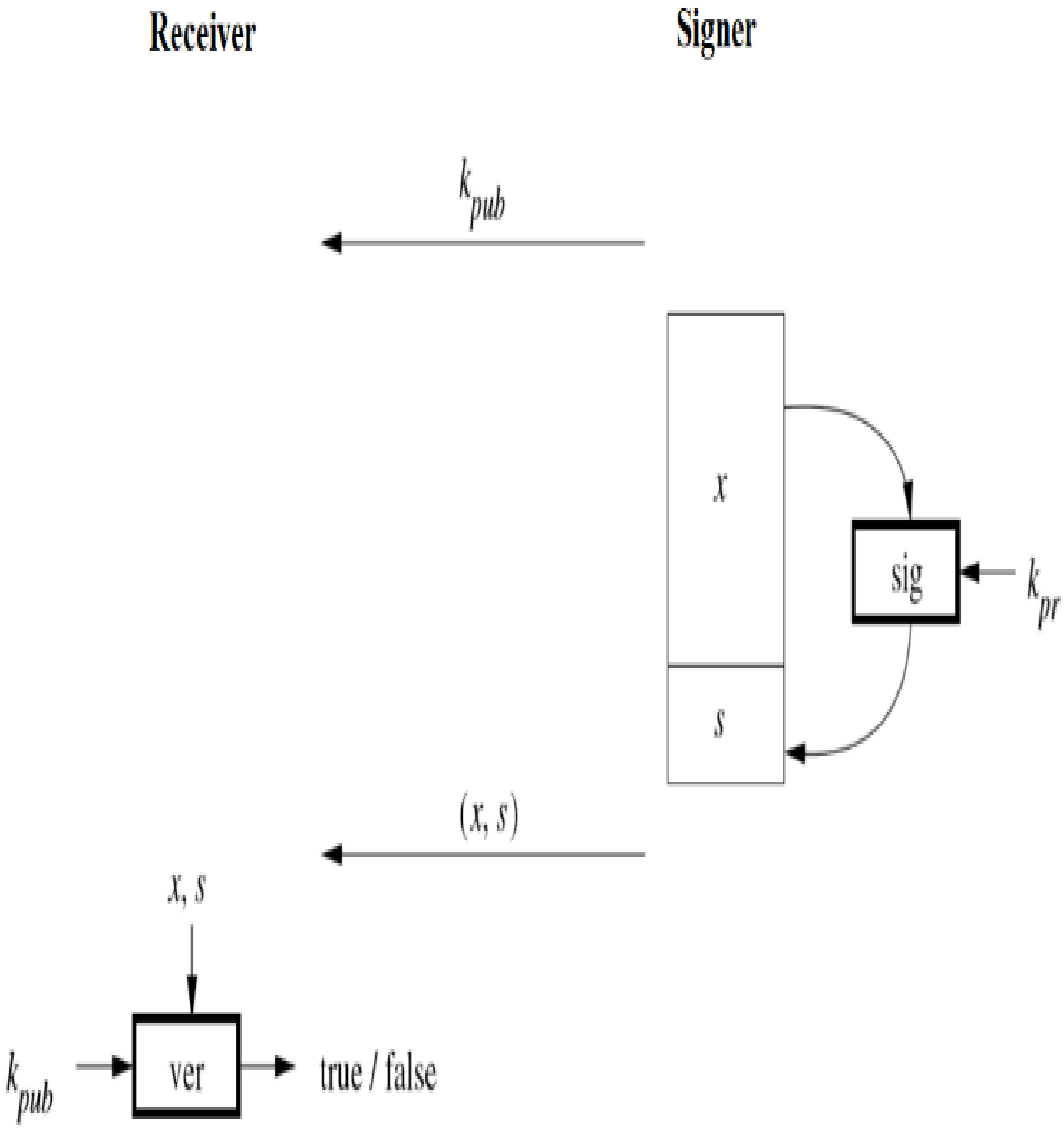
• Each user should have the ability to generate their own signature on any selected document they chose

• Each user should have the ability to efficiently verify whether or not a given string is the signature of another particular user.

• No one should have the ability to generate signatures on documents that the original owner did not sign.

Digital Signature Principles

Being able to prove that a user or individual generated a message is essential both inside and outside the digital domain. In today’s world, this is achieved through use of handwritten signatures. As for generating digital signatures, public-key cryptography is applied, in which the basic idea is that the individual who signs a document or message uses a private key (called private-key), while the individual receiving the message or document must use the matching public-key. The principle of the digital signature scheme is demonstrated in Figure



Digital signature principle

This process starts with the signer, who signs the message x. The algorithm used in the signing process is a function that belongs to the signer's private key (kpr), assuming that the signer will keep the private key secret. Thus, a relation can be created between the message x and the signature algorithm; the message x is also given to the signature algorithm as an input. After the message has been signed, the signature s is attached to the message x, and they are sent to the receiver in the pair of (x, s). It must also be noted that a digital signature is useless without being appended to a certain message, similar to putting a handwritten signature on a check or document.

The digital signature itself has an integer value that is quite large, e.g. a string with 2048 bits. In order for the signature to be verified, a verification function is needed in which both the message x and the signature s are given as inputs to the function. The function will require a public key in order to link the signature to the sender who signed it, and the output of the verification function would be either “true” or “false”. The output would be true in a case in which the message x was signed through the private key that is linked with the other key, i.e. the public verification key. Otherwise, the output of the verification function would be false.

Difference between Digital Signature and Message Authentication

When parties are communicating over an insecure channel, they may wish to add authentication to the messages that they send to the recipient so that the recipient can tell if the message is original or if it has been modified. In message authentication, an authentication tag is generated for a given message being sent; the recipients must verify it after receiving the message and ensure that no external adversary has the ability to generate authentication tags that are not being used by the communicating parties.

Message authentication can be said to be similar to digital signature, in a way, but the difference between them is that in message authentication, it is required that only the second party verify the message. No third party can be involved to verify the message’s validity and whether it was generated by the real sender or not. In digital signature, however, third parties have the ability to check the signature’s validity. Therefore, digital signatures have created a solution for message authentication.

Conclusion

Cryptography plays a vital and critical role in achieving the primary aims of security goals, such as authentication, integrity, confidentiality, and no-repudiation. Cryptographic algorithms are developed in order to achieve these goals. Cryptography has the important purpose of providing reliable, strong, and robust network and data security. In this paper, we demonstrated a review of some of the research that has been conducted in the field of cryptography as well as of how the various algorithms used in cryptography for different security purposes work. Cryptography will continue to emerge with IT and business plans in regard to protecting personal, financial, medical, and ecommerce data and providing a respectable level of privacy.

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Role of Computer in Vedic Mathematics

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Abstract

The findings of Rishis in ancient India can show the path to the world. The knowledge of the Vedas and other ancient texts is an everlasting source of knowledge. Vedic Mathematics is one of such gifts of ancient India. It helps us to solve almost all mathematical problems with less time with only mental calculation. The need for paperwork is very less. In the modern competitive world, every fraction of a second is important in competitive Exams where power tests are used for mathematical and arithmetical aptitude, numerical and nonverbal reasoning. In this present study, the Vedic method of multiplication has been used as an independent variable in order to know the effect on the achievement of students in an experimental setting of 58 students of Class – VI and found that the Experimental group has performed far better than the Control group in Post-test.

Keywords: South Asian, School Level, Elementary Level, Asian level, Asian Study.

Indroduction

The language in which Vedas were composed became too old in the later Vedic period. As a result, the Vedangas have emerged as an auxiliary in the field of Vedic studies. They are considered as sciences that help the people to understand and interpret the Vedas which are written centuries ago. The Vedangas were divided into six subjects such as phonetics (Śikṣā), grammar (Vyākaraṇa), etymology and linguistics (Nirukta), poetic meter (Chandas), rituals and rites of passage (Kalpa), timekeeping and astronomy (Jyotiṣa). The astronomy or Jyotiṣa Shastra is divided into three Skandas (means the big branch of a tree shooting out of the trunk) and Vedic mathematics is a part of that. It is named Vedic mathematics because the system of mathematics is discovered from ancient Vedic literature. The ancient Indian Rishis have mentioned 16 Sutras (Phrases) and thirteen sub-sutras in

Sanskrit which enable us to solve all mathematical problems in easy 2 or 3 steps with less no pen and paperwork.

Though the origin of Vedic Mathematics is controversial but the discovery in the field of Calculation of numbers is very wonderful (Rani, U., 2014). The first printed book on Vedic Mathematics was published in 1965, written by Sri Bharati Krishna Tirtha Maharaja, the Shankaryacharya of the Puri. He passed away in 1960 and this book was published by his disciples posthumously and reprinted fifteen times (Rani, U., 2014). Bharti Krishna Tirtha had good knowledge of Vedas and mathematics as he was a brilliant scholar in all the subjects he studied, such as Sanskrit, Philosophy, English, Mathematics, History and Science. (Katgeri, A.V, 2017). He also admitted that sixteen sutras and thirteen sub-sutras in Sanskrit mention by him were not in parishishtas of Atharvaveda (Sthapathya-subveda) but they occur in his own parishishtas (appendix) and not any others (Shukla, K.S., 1991). Having a good knowledge of Vedas and Mathematics, Tirtha Maharaja has created the sutras and sub-sutras, so the title “Vedic mathematics" is not acceptable (Vasantha Kandasamy, W. B., 2006). The controversy of origin and manning is beyond this paper but the effect of it in teaching mathematics especially at the school level has been studied.

In our Educational system, achievement tests are the keys to measure the progress of any student. Especially at the school level, power tests, are used in which the students are directed to answer the questions within a specific time limit. So time plays an important factor in any examination and if time can be saved in the calculation, it would be used to solve other problems. In the case of mathematics achievement tests, most of the students fail to solve the problem not due to ignorance but due to shortage of time and due to wrong calculation in basic mathematical operations like addition, subtraction, multiplication and division. The study of mathematics has been emphasized by many Education Commission. The education commission (1964-1966) recommended

Vedic Mathematics is an Indian ancient system of mathematical calculations or operations techniques developed in the year of 1957 with 16-word formulae and some sub-formulae. In competitive examinations, students find difficult to solve the aptitude questions effectively with very less or small time durations. Even though students are able to understand the problem, they are not able to speedup calculation process. In this paper some basic mathematical calculations, multiplication, square root, cube root and subtraction of fractional decimal numbers are distributed to a group of 25 students, whom are competitive examination writing students and told to solve questions without and with using Vedic methods techniques. The time taken to complete the calculations are taken in terms of minutes before and after adopting Vedic method's techniques and are analyzed using paired t-test. This paper could able to find that Vedic method significantly improves the speed of calculations while performing some basic mathematical operations. Wish this paper could play an active and supportive role in actual research of Vedic mathematics and techniques to improve the speed of calculations especially while writing any competitive examinations.

The South Asian region has a long history of discovering new ideas, ideologies, and technologies. Since the Vedic period, the land has been known as a fertile place for innovative discoveries. The Vedic technique used by Bharati Krishna Tirthaji is unique among South Asian studies. The focus of this study was mostly on algebraic topics, which are typically taught in our school level. The study also looked at how Vedic Mathematics solves issues of elementary algebra using Vedic techniques such as Paravartya Yojayet, Sunyam Samyasamuccaye, Anurupye Sunyamanyat, Antyayoreva and Lopanasthapanabhyam. The comparison and discussion of the Vedic with the conventional techniques indicate that the Vedic Mathematics and its five unique formulas are more beneficial and realistic to those learners who are experiencing problems with elementary level algebra utilizing conventional methods. Vedic Mathematics is an Indian ancient system of mathematical calculations or operations techniques developed in the year of 1957 with 16 Sutra’s (formulae) and 13 sub-Sutra’s (sub- formulae. In competitive examinations, students find difficult to solve the aptitude questions effectively with very less or small time durations. Even though students are able to understand the problem, they are not able to speedup calculation process. In this paper some basic mathematical calculations, multiplication, square root, cube root and subtraction of fractional decimal numbers are distributed to a group of 26 students, whom are competitive examination writing students and told to solve questions without and with using Vedicmethods techniques. The time taken to complete the calculations are taken in terms of minutes before and after adopting Vedic method's techniques and are analyzed using paired t-test. This paper could able to find that Vedic method significantly improves the speed of calculations while performing some basic mathematical operations. Wish this paper could play an active and supportive role in actual research of Vedic mathematics and techniques to improve the speed of calculations especially while writing any competitive examinations.

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New Innovation In Mathematics

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Abstract

New products and services are not the only examples of innovation. As a result of interactions between teachers and students in a classroom context, it is also a social art. Their perceptions, worries, and aspirations influence the search for novel concepts and the choices that must be made as these concepts are developed into values. Since mathematics is a core subject taught in schools, it requires creative approaches to keep up with the current pace of development and sustainability. Technology has caused a recent exponential surge in educational activities; yet, it can be difficult to keep up with changes and advancements in the new technologies when limited resources are both human and financial. This essay explores creative methods.

Introduction

New products and services are not the only examples of innovation. As a result of interactions between teachers and students in a classroom context, it is also a social art. Their perceptions, worries, and aspirations influence the search for novel concepts and the choices that must be made as these concepts are developed into values. Since mathematics is a core subject taught in schools, it requires creative approaches to keep up with the current pace of development and sustainability. Technology has caused a recent exponential surge in educational activities; yet, it can be difficult to keep up with changes and advancements in the new technologies when limited resources are both human and financial.

This essay explores creative methods. Innovation is a significant phenomena in teaching and learning that may elevate the topic to a whole new level. It is a constant process, thus one must always come up with fresh action plans and immediately begin carrying them out. It also entails novel approaches to task organisation in a formal context, such as a classroom. Innovation has a significant role in setting the product apart from the beginning. In reality, inventive exercises may aid in unravelling the findings and making them stand out more distinctly if the outcomes of arithmetic teaching and learning are not promising.

Simply because it is ongoing, innovation is a process. It can be a result of chance, when nothing was planned or anticipated and the event just happened. It can also be decided upon, planned out, or structured using a certain approach. Mathematics educators may shape their pupils' futures via innovation. Setting the goals and objectives for the planned innovation in such a way that it will specify the reduction in risks to be taken in its applications and its technique of evaluation is a step in the process of effective inventive activities. Innovative teaching strategies, such as the methodology used to introduce math concepts in the classroom, the structuring of arithmetic resources and contents, and reviews of the information covered in math classes, are all pertinent to the mathematics classroom. Innovations in these fields concentrate on a variety of instructional approaches and techniques that are appropriate for the ages and academic levels of students and learners, as well as for their intellectual growth and physical settings. These include problem-based learning, which is especially designed to test students' abilities to find answers to the issues provided to them in relation to the mathematical concepts they have already learned or will study in the next class.

Mathematics Education

Due to the numeracy benefits of mathematics and how it helps people build their mathematical skills, it is taught to students from elementary school through adulthood. Its main focus is on developing, educating, and certifying math instructors in the fields of mathematics education that span the study of science and other academic subjects. For instance, in Nigeria, admittance to the tertiary levels of education requires mathematics as a prerequisite for admission to either engineering, physical, biological, and other life sciences. Some social science courses in higher education, including those in accounting, economics, banking and finance, insurance, management, and many other related fields, also have a prerequisite math requirement.

Due to its practical applications in counting, measuring time, distance, weights, selling, and purchasing goods—tasks that every person performs on a daily basis—

mathematics helps prepare people for a useful life. Without a basic understanding of mathematics, even girls and women who might become housewives might find it difficult to lead happy lives. This is because they will frequently be required to measure, approximate, and estimate the number of items to be used for household tasks, their sizes, make purchases, and quantify services they may occasionally receive or provide. Essentially, mathematics is a tool for problem-solving in all facets of human life.

Importance of Innovations in Mathematics Education and Sustainable Development

Innovation in mathematics education not only helps students perform better, but also helps students become more engaged in the classroom, removing the possibility of math lessons being boring. Math games, which may be played inside or outdoors, are an example of innovation that can be used in math teaching and learning. Such games may be used at the start of a lesson, in the middle of a session, or at the conclusion of a class. As a result, studying math might be entertaining, less tedious, more fascinating, thrilling, and academically rewarding with the use of mathematical games, which could also be used to identify students' learning issues.

When it comes to adopting ideas from other developed nations and borrowing their policies, people in a globalised society might sometimes feel compelled to do so. It might be challenging to successfully deploy innovation. However, necessary elements for effective innovation implementation include things like the relative benefits of the innovation processes, compatibility with the innovation programmed, its validity and dependability, as well as its observability

Classifications of Innovations in Mathematics Education

A separate, but equally legitimate, philosophy that may place more emphasis on whole-class direct instruction, test preparation, or more product-oriented teaching may collide with pedagogical ideals such as learner-centered, communicative, or process-oriented methods. It is essential to place individuals and conflicts at the centre of the creative process and to argue for contextually based approaches to pedagogical advances. Innovations must, however, be created in a way that is sensitive to and respectful of both the local classroom realities and the larger social cultures . Teachers must contribute to both primary and secondary innovations since they are the thought leaders and doers of the work.

Any innovation that is implemented can be successful as long as it adheres to the curriculum and does not deviate from the subject areas' primary objectives, which are to increase student comprehension and boost performance on both internal and external or standardised mathematics tests. When expected changes are presented to students in simpler and clearer ways, it becomes easier for them to understand for the purpose of evaluation because teaching and learning are school activities that are primarily concerned with the development of potentials, which has to do with acquiring new patterns of experiences as a result of changes being exposed to.

Innovation and Change in Mathematics Education

Both instructors and students of mathematics must be prepared and ready to depart from the standard methods in the tracking and learning of mathematics in order to benefit from new activities that might boost productivity in the teaching and learning of mathematics . In actuality, innovation based on any of the mathematical topics to be taught in the classroom only occurs when the mind is open to a change. This leads to creative activities of a nature and, eventually, inventions.

It takes a conscious effort that results from a great lot of thinking, a creative mentality, diligence, planning preparation, decisions, and execution to be innovative as a math teacher. When one's ideas don't provide the results they were hoping for, they must have the mentality to keep trying until they succeed. Bound claims that innovation has a quantitative, research-based, and sequential implementation style. It emphasizes what is already understood and what is yet unlearned. Furthermore, because the new concept is innovative, creative, and subject to difficulties, restrictions, and acceptance, it is not immune to changes made along the way. Before taking further action, a mental image or picture of the desired image or model must first be formed in the thought domain.

Change, which is the changing of the status quo or doing things in new ways than they are now done, is a crucial component required for a result-oriented innovation. When it comes to mathematics teaching and learning, a math teacher must be prepared to change the standard teaching techniques utilized in the classroom, especially when such techniques are not producing the desired results in the pupils. Depending on the subject of the lesson to be given, some of the modifications could take place indoors and others might take place outside. Change is a process, much like invention. The teacher's opinions regarding any mathematical topic to be taught, the best way to teach the topic, appraisal of the teacher's teaching style, and the students' learning experiences as retention of what was learned during the mathematics class must come first before any change can be made. Innovative change may be the consequence of internal factors like pupils' bad performance on prior exams or external factors such social technology advancements that threaten the caliber of mathematics instruction and learning.

Challenges of Innovation in Mathematics Education

There is no question that if a flow of activity is interrupted, there will inevitably be one or more difficulties. Due to the status quo's change, this is unavoidable. Three categories may be used to roughly classify these difficulties: Challenges for teachers: When a teacher doesn't take ownership of or comprehend the innovation, the intended change could not align with the teacher's values and beliefs, which could result in the teacher acting negatively. The teacher's reluctance to feel burdened with the increased effort that the new tasks involved may be one way this is demonstrated.

Student-related Challenges: lack of student cooperation, particularly when they are used to a traditional routine like the teacher-and-board style of information distribution. Students could find it difficult to adjust to the teacher's shift in this situation, especially if the justification for the change has not been given in a convincing manner.

Conclusion

Given the difficulties in implementing innovations, it is a worthy and goal-oriented Endeavour. To provide the desired beneficial consequence, innovative methods in mathematics education must have acceptable time limits and be pertinent to the age and academic level of the children they are intended for. In order to identify where students need aid and where the concept of innovation could be able to help out in fixing students' difficulties, it is crucial to be sensitive to the requirements of the students as early as possible. Institutional-based professional development and support are essential components that must be incorporated into the innovation process for it to be effective, according to a deeper understanding.

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Comparative study of Differential Transform Method and Runge-Kutta method of 4th order

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Abstract

In this paper, we will introduce two methods to obtain the approximate solutions for the ordinary Differential Equation of first order. The first method is differential transformation method (DTM) and the second method is Runge-Kutta method of 4 th order. Moreover, we will make comparison between the solutions obtained by the two methods. The numerical results are presented through table.

Keywords: Differential transformation method, Runge-kutta method, Ordinary Differential Equation.

Introduction

There are many problems in mathematics, physics, medicine, pharmacology etc. which are modeled using Ordinary Differential Equations (Ordinary Differential Equations is a differential equation if it involves derivatives with respect to only one independent variable) But sometimes thecomplexity of these models makes it hard to use analytical approaches to solve them. In such cases, the only possibility lies in using some approximate methods. This research paper deals with Runge- Kutta method of 4th order and Differential Transform method. Firstly Zhou [1] introduce the differential transform method to solve the Euler equidimensional equation. The Adomian decomposition method and the variational iteration method, both employed to solve the moving boundary problem, were compared by Hetmaniok E. et al. [2] in their study. Ganji, R.M et. al. [3] discussed a mathematical model of brain tumor which is an extension of a simple two-dimensional mathematical model of glioma growth and diffusion. This is derived from fractional operator in terms of Caputo which is called the fractional Burgess equations (FBEs). Jafari, H.et. al.[4] studied about the population dynamics model including the predator-prey problem and the logistic equation. The solution of this model is obtained by using three-step Adams-Bashforth scheme. Grzymkowski, R. et al.[5] explained the Taylor transformation to solve the systems of ordinary differential equations, including nonlinear differential equations. Arikoglu, A. [6] implemented a differential Transform method to solve the Bagley–Torvik, Ricatti and composite fractional oscillation equations. Liu, B. et. al. [7] concentrated on the differential transform method (DTM) to solve some delay differential equations (DDEs). Biazar, J.et. al.[8] studied about the differential transform which is employed to solve special kinds of systems of integral equations. Three examples of different forms in this class of functional equations have been prepared. Fausett, L. V. [9] described the use of MATLAB with computational mistakes, encompassing everything from data input/output to various types of computational errors, Techniques include Simpson, Euler, Heun, Runge-kutta, Golden Search, and Nelder-Mead. Butcher, J. C[10] possess a stability property which is a natural extension of the notion ofA-stability for non-linear systems. Mukherjee, S. et. al.[11] described Differential Transform Method (DTM) which is implemented to solve some Riccati differential equations with variable co-efficients. This technique doesn't require any discretization, linearization or small perturbations and therefore it reduces significantly the numerical computation. The results derived by this method are compared with the numerical results by Runge Kutta 4 (RK4) method. Ayaz. F [12] discussed the decomposition method which gives gives the exact solution of linear and non- linear pde. Rashidi, M. M.et.al.[13] studied Differential Transform Method (DTM) for solving different types of differential equations and basic definitions and formulas of DTM and Differential Transform-Padé approximation (DTM-Padé), which are employed to increase the convergence and accuracy of DTM approximations. In the Present Paper, The Differential Transform Method and the RK Method of Fourth Order will be compered by using numerical examples. A table displays the numerical results.

Differential Transformation Method

Recently, there has been a lot of interest in using DTM to solve a various type of problems including boundary value problems, algebraic equations, and partial differential equations . This approach offers the solution as a rapidly convergent series with beautifully and precisely computed components. The method's key benefit is that it may be used to solve a variety of differential and integral equations, including those that are homogeneous and non-homogeneous, linear and nonlinear, with constant and variable coefficients.

The transformation of the kth derivative of a function in one variable is as follows:

$$Y(k)=\frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right] x = x_0 \quad (1)$$

And the inverse transformation is defined as

$$y(x)=\sum_{k=0}^{\infty} Y(K)\,(x - x_0)^k \quad (2)$$

Properties of Differential Transform method

1. If y(x) = g(x) ± h(x), then Y(k) = G(k) ± H(k).

2. If y(x) = cg(x), then Y(k) = cG(k), where c is a constant.

3. If y(x) = $\left[\frac{d^n f(x)}{dx^n}\right]$, then Y(k) =$\frac{(k+n)\,!}{k!}$F(k+n)

4. If y(x) = g(x)h(x), then Y(k)=$\sum_{k_{1=0}}^{k} G(k_1)\,H(k - k_1)$

5. If y(x) = x^n , then Y(k) = $\delta(k-n), where \delta(k-n) = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}$

6. If y(x) = y_1(x) y2(x)...y_n(x), then Y (k) = $\sum_{k_{n-1=0}}^{k} \sum_{k_{n-2=0}}^{k_{n-1}} \ldots\ldots.. \sum_{k_{2=0}}^{k_3} \sum_{k_{1=0}}^{k_2} Y_1(k_1) Y_2(k_2 - k_1) \ldots\ldots\ldots\ldots Y_{n-1}(k_{n-1} - k_{n-2}) Y_n(k - k_{n-1})$

7. If y(x) = $e^{\lambda x}$, then Y (k) = $\frac{\lambda k}{k!}$,where λ is a constant.

8. If y(x) = sin (ωx + α), then Y (k) = $\frac{\omega k}{k!}$ sin(kπ 2 + α), where ω and α constants.

9. If y(x) = cos(ωx + α), then Y (k) = $\frac{\omega k}{k!}$ cos(kπ 2 + α), where ω and α constants.

Properties (1-6) and (7-9) are available in [14]and[15].

Runge-Kutta Method of 4th order

Runge-Kutta 4th order method is a numerical technique used to solve ordinary differential equation of the form

$$\frac{dy}{dx} = f(x, y),\ y(0)=y_0 \qquad (3)$$

The Runge-Kutta 4th order method is

$$Y_{i+1}=Y_i+\frac{1}{6}(k_1 + 2 * k_2 + 2 * k_{3+}k_{4)} \qquad (4)$$

Where

$k_1=h* f(x_{0,}y_0)$

$k_2=h* f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3=h* f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$k_4=h* f(x_0 + h, y_0 + k_3)$

Numerical Example

Consider an ODE of first order for one parameter, say f(x,y).

Let the differential equation be, **y’= x+y (5)**

with initial condition, y(0)=1. Assume h=0.05. Now let us find the value of y(0.2).

From the given, y(0) = 1 →x0=0 ; y0 =1 and h=0.05

By IV ORDER Runge-Kutta method

$k_1=h* f(x_{0,}y_0)$ = 0.0500

$k_2=h* f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$ = 0.0525 k3=h* $f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$ = 0.0526

k4=h* $f(x_0 + h, y_0 + k_3)$ = 0.0551

k=$\frac{1}{6}(k_1 + 2 * k_2 + 2 * k_{3+}k_{4)}$ = 0.0525

y(0.05)=y_1=y_0+k =1.0525

Now, we can find y(0.1) with initial values y1 = 1.0525and x1 = 0.05

k1=h* $f(x_{1,}y_1)$ = 0.0551

k2=h* $f(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$ = 0.0578

k3=h* $f(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$ = 0.0578

k4=h* $f(x_1 + h, y_1 + k_3)$ = 0.0605

k=$\frac{1}{6}(k_1 + 2 * k_2 + 2 * k_{3+}k_{4)}$ = 0.0578

y(0.1)=y_2=y_1+k = 1.1103

Now, we can find y(0.15) with initial values y2 = 1.1103 and x2 = 0.1

k1=h* $f(x_{2,}y_2)$ = 0.0605

k2=h* $f(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2})$ = 0.0633

k3=h* $f(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2})$ = 0.0633

k4=h* $f(x_2 + h, y_2 + k_3)$ = 0.0662

k=$\frac{1}{6}(k_1 + 2 * k_2 + 2 * k_{3+}k_{4)}$ = 0.0633

y(0.15)=y_3=y_2+k = 1.1737

Now, we can find y(0.2) with initial values y3 = 1.1737and x3= 0.15

k_1=h* $f(x_{3,}y_3)$ = 0.0662

k_2=h* $f(x_3+\frac{h}{2},y_3+\frac{k_1}{2})$ = 0.0691

k_3=h* $f(x_3+\frac{h}{2},y_3+\frac{k_2}{2})$ = 0.0692

k_4=h* $f(x_3+h,y_3+k_3)$ = 0.0721

k=$\frac{1}{6}(k_1+2*k_2+2*k_{3+}k_{4)}$ = 0.0691

y(0.2)=y_4=y_3+k = 1.2428

By Differential Transform Method and Inverse Differential Transform Method

(k+1)Y(k+1)=Y(k)+x$\delta(k)$ (6)

y(x)=$\sum_{k=0}^{\infty}Y(k)x^k$ (7)

Using intial condition y (0) = 1 and by substituting the successive values

k ≥0 to the relation (6) and (7), we obtain

y (0.05)= 1.0538

y (0.1)= 1.1157

y (0.15)= 1.1861

y (0.2)= 1.2657

Exact solution

The exact solution of equation (5) is

Y=-x-1+2*exp(x)

Which is obtained as

Equation (5) is of the form $\frac{dy}{dx} + Py = Q$

Where P and Q are function of x .

Integrating factor(I.F)=$e^{\int Pdx}$

Solution is

y.(I.F)=$\int Q(I.F.)dx + c$

where c is constant of integration ,which is obtained from intial condition .

The values obtained from RK method and DTM method are tabulated below:

Table 1 - Comparison of Accuracy Between RK method and DTM method

Method	**RK Method**	**DTM**	**Exact Solution**
Y(0)	**1**	**1**	**1**
Y(0.05)	**1.0525**	**1.0538**	**1.0525**
Y(0.1)	**1.1103**	**1.1157**	**1.1103**
Y(0.15)	**1.1737**	**1.1861**	**1.1737**
Y(0.2)	**1.2428**	**1.2657**	**1.2428**

Conclusion

In the paper, we have presented the comparison of two approaches used for solving ordinary differential equations of first order. The main goal of this elaboration was to indicate and justify the advantages of the differential transformation method (DTM), which is less known and not often applied. The Runge–Kutta method (we decided to use the Runge–Kutta method of order 4 to emphasize better the advantages of the DTM method) .The choice of these methods was dictated by their popularity (considering especially the RK4 method) or their simplicity offers the special routines dedicated for solving the differential equations and their systems, based on the implementation of selected methods and numerical treatment of differential equations. The example describes the problem that is possible to solve by using the two discussed approaches to solve by using the RK4 method and DTM method

.Moreover, the errors of approximate solutions, obtained in the RK4 method, are comparable, or very often even lower, than the errors generated by the other Method. Rk method only works with first order equations, but DTM approach works with higher order derivatives. Table1.shows that the approximate solution for equation(5) obtained by Runge-Kutta 4 Method is much close to the approximate solution obtained by DTM.

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Role of Statistics in Data Science

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Abstract

Statistics is the study of data. Statistics is a branch of mathematics that deals with collection of data, organisation, interpretation, analysis and data presentation. Statistical knowledge helps to choose the proper method of collecting the data and employ those samples in the correct analysis process in order to effectively produce the results.

Data science is the study of data to extract meaningful insights for business. It is a multidisciplinary approach that combines principles and practices from the fields of mathematics, statistics, artificial intelligence, and computer engineering to analyze large amounts of data. Statistics and data science are both dealing with data and utilizing data to produce meaningful results which can be utilised in many fields of life whether it is business, computer science, medical science or other disciplines of sciences and various other fields.

Data Science is about drawing useful conclusions from large and diverse data sets through exploration, prediction, and inference. We can utilize statistical analysis techniques to quantify what we have so instead of sifting through voluminous amounts of data, we can describe it using a few metrics. Advanced machine learning algorithms in data science utilize statistics to identify and convert data patterns into usable evidence. Data scientists use statistics to collect, evaluate, analyze, and draw conclusions from data, as well as to implement quantitative mathematical models for pertinent variables. Having good knowledge of statistics will make tasks of data scientist very easier and ultimately will assist in giving good inferences and decisions.

Use of Statistics in Data Science

Statistics is considered a subset of mathematical science involving the collection, organisation, collation and analysis of data with the intent of deriving meaning, which can then be utilised. Our everyday usage of the internet and various applications across our phones, laptops, and fitness trackers has created an explosion of information that can be grouped into data sets and offer insights through statistical analysis. More than 6 billion searches a day on Google alone means big data analysis.

Nowadays data analytics is heard more than statistics. For data scientists, data analysis is supported by knowledge of statistical methods. Machine learning takes out a lot of the statistical methodology that statisticians would usually use. However, a foundational understanding of some basics in statistics supports strategy in exercises like hypothesis testing. Statistics contribute to technologies like data mining, speech recognition, data compression, network, traffic modelling and artificial intelligence.

Probability is one of the most used statistical testing criteria for data analysis. Being able to predict the likelihood of something happening is important in numerous scenarios, from understanding how a self-driving car should react in a collision to recognising the signs of an upcoming stock market crash. For data-driven companies like Spottily or Netflix, probability can help predict what kind of music you might like to listen to or what film you might enjoy watching next.

Aside from our preferences in entertainment, research has recently been focused on the ability to predict seemingly unpredictable events such as an earthquake, pandemic or an asteroid strike. Because of their rarity, these events have historically been difficult to study through the lens of statistical inference – the sample size can be so small that the variance is pushed towards infinity. However, "black swan theory" could help us navigate unstable conditions in sectors like finance, insurance, healthcare, or agriculture, by knowing when a rare but high-impact event is likely to occur.

The black swan theory was developed by Nassim Nicholas Taleb, who is a critic of the widespread use of the normal distribution model in financial engineering. In finance, the coefficient of variation is often used in investment to assess volatility and risk, which may appeal more to someone looking for a black swan. The normal distributions, standard variation, and z-scores can all be useful to derive meaning and support predictions in computer science.

Some computer science-based methods that overlap with elements of statistical principles include:

- Time series
- Survival models
- Markov processes
- Spatial and cluster processes
- Bayesian statistics
- Statistical distributions
- Goodness-of-fit techniques
- Experimental design
- Analysis of variance (ANOVA)
- A/B and multivariate testing
- Random variables
- Simulation using Markov Chain Monte-Carlo methods
- Imputation techniques
- Cross validation
- Rank statistics, percentiles, outliers detection
- Sampling
- Statistical significance

While statisticians tend to incorporate theory from the outset into solving problems of uncertainty, computer scientists tend to focus on the acquisition of data to solve real-world problems.

As an example, descriptive statistics aims to quantitatively describe or summarise a sample rather than use the data to learn about the population that the data sample represents. A computer scientist may perhaps find this approach to be reductive, but, at the same time, could learn from the clearer consideration of objectives. Equally, a statistician's experience of working on regression and classification could potentially inform the creation of neural networks. Both statisticians and computer scientists can benefit from working together in order to get the most out of their complementary skills.

In creating data visualisations, statistical modelling, such as regression models, is often used. Regression analysis is typically used in determining the strength of predictors, trend forecasting, and forecasting an effect, which can be represented in graphs. Simple linear regression relates two variables (X and Y) with a straight line. Nonlinear regression relates to two variables in a nonlinear relationship, represented by a curve. In data analysis, scatter plots are often used to show various forms of regression. Matplotlib allows you to build scatter plots using Python; Plotly will allow the construction of an interactive version.

Traditionally, statistical analysis has been key in helping us understand demographics through a census – a survey through which citizens of a country offer up information about themselves and their households.

Corona virus has been consistently monitored through statistics since the pandemic began in early 2020. The chi-square test is a statistical method often used in understanding disease because it allows the comparison of two variables in a contingency table to see if they are related. This can show which existing health issues could cause a more life-threatening case of Covid-19, for example.

Observational studies have also been used to understand the effectiveness of vaccines six months after a second dose. These studies have shown that effectiveness wanes. Even more ground-breaking initiatives are seeking to use the technology that most of us hold in our hands every day to support data analysis. The project_EAR asks members of the public to use their mobile phones to record the sound of their coughs, breathing, and voices for analysis. Listening to the breath and coughs to catch an indication of illness is not new – it's what doctors have practised with stethoscopes for decades. What is new is the use of machine learning and artificial intelligence to pick up on what the human ear might miss. There are currently not enough large data sets of the sort needed to train machine learning algorithms for this project. However, as the number of audio files increases, there will hopefully be valuable data and statistical information to share with the world.

Data Science and Statistics Correlation ship

In data science, statistics is at the core of sophisticated machine learning algorithms, capturing and translating data patterns into actionable evidence. Data scientists use statistics to gather, review, analyze, and draw conclusions from data, as well as apply quantified mathematical models to appropriate variables

Data science is a multi-faceted, interdisciplinary field of study. It's not just dominating the digital world. It's integral to some of the most basic functions - internet searches, social media feeds, political campaigns, grocery store stocking, airline routes, hospital appointments, and more. It's everywhere. Among other disciplines, statistics is one of the most important disciplines for data scientists

Josh Wills, a former head of data engineering at Slack, said **"A data scientist is a person who is better at statistics than any programmer and better at programming than any statistician."**

In other words, statistics is an essential component of data science.

Statistics for Data Science

Statistical analysis and probability influence our lives on a daily basis. Statistics is used to predict the weather, restock retail shelves, estimate the condition of the economy, and much more. Used in a variety of professional fields, statistics has the power to derive valuable insights and solve complex problems in business, science, and society. Without hard science, decision making relies on emotions and gut reactions. Statistics and data override intuition, inform decisions, and minimize risk and uncertainty.

In data science, statistics is at the core of sophisticated machine learning algorithms, capturing and translating data patterns into actionable evidence. Data scientists use statistics to gather, review, analyze, and draw conclusions from data, as well as apply quantified mathematical models to appropriate variables. **Data scientists**

work as programmers, researchers and business executives. However, what all of these areas have in common is a basis of statistics. Thus, **statistics in data science is as necessary as understanding programming languages.**

Combining computer science and statistics without business knowledge enables professionals to perform an array of machine learning functions. Computer science and business expertise leads to software development skills. Mathematics and statistics combine result in some of the most talented researchers. It is only with all three areas combined that data scientists can maximize their performance, interpret data, recommend innovative solutions, and create a mechanism to achieve improvements.

Key Statistical Terms for Data Science

Statistical functions are used in data science to analyze raw data, build data models, and infer results.

- **Population**: the source of data to be collected.
- **Sample**: a portion of the population.
- **Variable**: any data item that can be measured or counted.
- **Quantitative analysis (statistical):** collecting and interpreting data with patterns and data visualization.
- **Qualitative analysis (non-statistical):** producing generic information from other non-data forms of media.
- **Descriptive statistics:** characteristics of a population.
- **Inferential statistics:** predictions for a population.
- **Central tendency**: **mean** (average of all values), **median** (central value of a data set), and **mode** (the most recurrent value in a data set).
- Measures of the spread:
    - **Range**: the distance between each value in a data set.

- **Variance**: the distance between a variable and its expected value.
- **Standard deviation**: the dispersion of a data set from the mean.

Statistical Techniques Data Scientists Need to Master

Data scientists go beyond basic data visualization and provide enterprises with information-driven, targeted data. Advanced mathematics in statistics tightens this process and cultivates concrete conclusions. There are a number of statistical techniques that data scientists need to master. When just starting out, it is important to grasp a comprehensive understanding of these principles, as any gaps in knowledge will result in compromised data or false conclusions.

General statistics: The most basic concepts in statistics include bias, variance, mean, median, mode, and percentiles.

Probability distributions: Probability is defined as the chance that something will occur, characterized as a simple "yes" or "no" percentage.

Dimension reduction: Data scientists reduce the number of random variables under consideration through feature selection (choosing a subset of relevant features) and feature extraction (creating new features from functions of the original features). This simplifies data models and streamlines the process of entering data into algorithms.

Over and under sampling: Sampling techniques are implemented when data scientists have too much or too little of a sample size for a classification. Depending on the balance between two sample groups, data scientists will either limit the selection of a majority class or create copies of a minority class in order to maintain equal distribution.

Bayesian statistics: Frequency statistics uses existing data to determine the probability of a future event. Bayesian statistics, however, takes this concept a step further by accounting for factors we predict will be true in the future. For example, imagine trying to predict whether at least 100 customers will visit your coffee shop each Saturday over the next year. Frequency statistics will determine probability by analyzing data from past Saturday visits. But Bayesian statistics will determine probability by also factoring for a nearby art show that will start in the summer and take place every Saturday afternoon. This allows the Bayesian statistical model to provide a much more accurate figure.

Statistical Skills Needed to Perform Data Science Jobs

Data science requires a mixture of technical skills, such as R and Python programming languages, as well as “soft skills,” including communication and attention to detail.

Data manipulation: Using Excel, R, SAS, Stata, and other programs, data scientists have the ability to clean and organize large data sets.

Critical thinking and attention to detail: Using linear regression, data scientists extract and model relationships between dependent and independent variables. Data scientists choose methods with built-in assumptions which are considered during their application. Violating or inappropriately choosing assumptions will lead to flawed results.

Curiosity: The desire to solve complex puzzles drives data scientists to design data plots and explore assumptions. They also discover patterns and sequences by using advanced data visualizations.

Organization: Data scientists are inundated with information from various sources and ongoing project opportunities. With budget and time constraints, data scientists perform efficiently when they are well-versed in statistical functions. In addition, having routinized processes helps ensure data is not compromised.

Innovation and problem solving: Above and beyond pure computations and basic data analysis, data scientists use applied statistics to pair abstract findings to real-world problems. Data scientists also use predictive analytics to determine future courses of action. All of this requires careful consideration, using both logical and innovative approaches to analyze issues and solve problems.

Communication: All of the work a data scientist does must be translated into a captivating story that industry leaders and executives can appreciate. Data scientists

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fill the gap between technology and operations. They translate findings into text and data visualizations that executives and clients can easily understand: an essential skill for a data scientist.

How to Learn Statistics for Data Science

Three popular educational paths are **massive open online courses (MOOCs), bootcamps, or master's programs.** While the data science education options leave employers wondering which path is best, master's degree programs have traditionally been the most valued among the three.

The best education in data science depends upon matching a student's needs with the most appropriate training resources. The process of learning statistics in data science, for instance, will look different depending on a person's educational and professional background. It is reasonable for a data science professional who has already acquired a data science foundation to sharpen their probability techniques through a variety of learning options. However, a recent college graduate, however, will find the deepest comprehensive data science training. Pros and cons of various learning programmes are given in table below.

Learning option	Pros	Cons
Massively Open Online Courses (MOOCs)	• Gain a specialization and resume booster • Learn the latest trends and techniques • Relearn old skills • Learn before committing to a paid program • Generally free	• Learn only targeted material • Don't receive a degree or credential • End relationships after course completion • Lack ongoing career support
Bootcamps	• Gain an introduction to statistics, mathematics, and other data science principles • Receive a certificate for completion • Focus training on immediate job outcomes	• Learn only targeted material • Don't receive a degree or credential • End relationships after course completion • Lack ongoing career support
Master's Degree	• Gain a comprehensive review of statistics, mathematics, and other data science principles. • Gain a comprehensive review of statistics, mathematics, and other data science principles. • Focus training on immediate and long-term career outcomes • Complete a capstone project • Receive internship assistance • Receive lifelong peer, faculty, and career support	• Time investment is longer than a MOOC or a bootcamp program, but material is more in-depth • Search for additional training if specialization isn't offered

Challenges in Data Science

Finding the right data: The first step of any data science project is unsurprisingly to find the data assets needed to start working. The surprising part is that the availability of the "right" data is still the most common challenge of data scientists, directly impacting their ability to build strong models.

The first issue is that most companies collect tremendous volumes of data without determining first whether it is really going to be consumed, and by whom. This is driven by a fear of missing out on key insights that could be derived from it, and the availability of cheap storage.

Getting Access to the Data

Once data scientists locate the right table, the next challenge is accessing the latter. Security and compliance issues are making it harder for data scientists to access datasets. As organizations transition into cloud data management, cyber-attacks have become quite common. This has led to two major issues:

- Confidential data is becoming vulnerable to these attacks
- The response to cyber-attacks has been to tighten regulatory requirements for businesses. As a result, data scientists are struggling to get consent to use the data, which drastically slows down their work. Worse, when they are refused access to a dataset.

Understanding the Data

You would think that once data scientists find and obtain access to a specific table, they can finally work their magic and build powerful predictive models. They usually sit scratching their head for ridiculous amounts of time with questions of the type:

- What does the column name 'FRPT33' even mean?
- Who can I ask this to?
- Why are there so many missing values?

Although these questions are simple, getting an answer isn't. There is no ownership over datasets in organizations, and finding the person that knows the meaning of the column name you are enquiring about is like trying to find a needle in a haystack.

Data Cleaning

Unfortunately, real-life data is nothing like hackathon data or Kaggle data. It is much messier. Data scientists spend most of their time pre-processing data to make it consistent before analyzing it, instead of building meaningful models.

Communicating the Results to Non-Technical Stakeholders

Data scientists' work is meant to be perfectly aligned with business strategy, as the ultimate goal of data science is to guide and improve decision-making in organizations. Hence, one of their biggest challenges is to communicate their results to business executives. In fact, managers and other stakeholders are ignorant of the tools and the works behind models. They have to base their decisions on data scientists' explanations. If the latter can't explain how their model will affect the performance of the organization, their solution is unlikely to be executed. There are two things making this communication to non-technical stakeholders a challenge:

- First, data scientists often have a technical background, making it difficult for them to translate their data findings into clear business insights. But this is something that can be practiced. They can adopt concepts such as **"data storytelling"** to provide a powerful narrative to their analysis and visualizations.
- Second, business terms and KPI's are poorly defined in most companies. For example, everyone knows roughly what the ROI is made of in a company, but there is rarely a common understanding across all departments of how it is computed exactly.

Conclusion

Data Science is about drawing useful conclusions from large and diverse data sets through exploration, prediction, and inference**.** We can utilize statistical analysis techniques to quantify what we have so instead of sifting through voluminous amounts of data, we can describe it using a few metrics. Advanced machine learning algorithms in data science utilize statistics to identify and convert data patterns into usable evidence. Data scientists use statistics to collect, evaluate, analyze, and draw conclusions from data, as well as to implement quantitative mathematical models for pertinent variables. Having good knowledge of statistics will make tasks of data scientist very easier and ultimately will assist in giving good inferences and decisions.

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Radial Basis Function: A Meshless Tool for the Numerical Simulation

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Abstract

In both science and engineering, the study of mathematical models must include partial differential equations (PDEs). Researchers have developed a variety of techniques and methods for solving PDEs. An effective basis function for approximating the solutions of ordinary and partial differential equations is the radial basis function (RBF). In order to solve a number of well-known PDEs, researchers have proposed a series of RBF approaches. It has been designed for approximating the solution using a number of approaches that have resulted in the development of hybrid methods. Additionally popular in statistical analysis and numerical analysis are radial basis function (RBF) techniques. The numerical solutions of PDEs utilising various RBF techniques are examined in this paper. To obtain the ideal value of the numerical solutions, the mathematical formulations of the various RBF algorithms are described along with the available shape parameters.

Keywords: Shape Parameter, Partial Differential Equation, Kansa Collocation method, Radial Basis Function.

Introduction

An effective numerical strategy for solving partial differential equations (PDEs) is to approximate them by radial basis functions (RBFs), which approximate the numerical solution when implemented, making the process computationally efficient. The Radial Basis Function was later developed for finding the solution of interpolation matrices and was then implemented for solutions of partial differential equations. RBF is easy to use and works well by dynamic and irregular domains. In numerical analysis and statistics, RBF approaches have a wide range of applications. Numerical solutions of PDEs, geo modeling, machine learning, price options, neural networks, data mining, and image processing are just a few examples of these applications.

There are several numerical techniques for solving PDEs that are generated as a result of mathematical modelling when the domain is uniformly distributed. In some of the generated PDEs, there is a need to find the solution in the presence of non-uniform data, which is not easy to handle and leads to complexity. To solve this problem, mesh-free methods are used. Radial basis function method is one of useful technique in the mesh-free methods. RBF methods are modern ways to approximate multivariate functions, especially in the absence of grid data. They have been known, tested and analysed for several years now and many positive properties have been identified (Buhmann, 2000).The implementation of RBF techniques in approaching multivariate scattered data has been highly appreciated.

Hardy, (1971) introduced the RBF method in context of the quadric surfaces dealing with the topological approach. Hardy was the first to develop the multi-quadric (MQ) approximation technique. Franke (1982) experimented with scattered data interpolation. He evaluates methods into the form of time, storage, exactness, and ease of implementation, and also considers multi-quadric (which is a type of RBF) to be one of the best. Micchelli (1986) made a step forward in by demonstrating that multi-quadraic surface interpolation is always solvable. The MQ approach has the benefit of obtaining the interpolant using a linear combination of basis functions that are only dependent on the distance from a specific node which is known as the centre. Radial basis functions, especially in their "local" RBF-FD form, have significantly advanced over the past few years from being primarily a curiosity approach for simple PDE "toy problems" to becoming a major contender for very large simulations on refined distributed memory computer systems.RBF-FD discretizations are completely mesh-free and very simple to use, even when local refinements are required (Fornberg, and Flyer (2015).

The paper is arranged as follows: In section 2, the radial basis function is discussed. The third section presented a review of radial basis function methods that are used for finding the solutions of PDEs. In last section, conclusion of the paper is presented followed by the discussion of RBF methods. Wherever possible, we attempted to provide the mathematical formulation of the methods. To the best of the authors' understanding, there is no such investigation that presents all the RBF techniques. Figure 1 and figure 2 depict the evolution of RBF methods.

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Radial Basis Function

A function Φ: $\mathbb{R}^t \rightarrow \mathbb{R}$ is called radial if there exist a function of one variable $: [0, \infty) \rightarrow \mathbb{R}$ such that $\Phi(\mathrm{x}) = \varphi(\|\mathrm{x}\|)$, here Euclidean norm $\|.\|$ is used. $\Phi(r)$ is a univariate continuous real valued radial basis function whose value based upon distance value that is measure from any fixed centre point or the origin. From the definition, it is clear that Φ is a special function which is radially symmetric and only depends on the distance between points. The application of radial basis function to the high dimensional problem is easy as the interpolation problem is insensitive to the space dimension. In all space dimensions, one can work with the function φ that is univariate instead of using a multivariate function Φ. We centring on types of Radial basis functions that are distinguished by the smoothness- piecewise smooth RBFs which are free from shape parameter λ and infinitely differentiable which have parameters called the shape parameterλ.

Types of RBFs

There are various types of RBFs. Some recognized RBFs are as follows:

- Infinitely smooth RBFs – These RBFs are based on the shape parameter λ that controls the shape or outline of the RBF.

I. Gaussian Function (GS) RBFs

$$\varphi(\mathrm{r}) = \mathrm{e}^{-(\lambda \mathrm{r})^2}$$

II. Multiquadric (MQ) RBFs

$$\varphi(\mathrm{r}) = \sqrt{1 + (\lambda \mathrm{r})^2}$$

III. Inverse Multiquadric (IMQ) RBFs,

$$\varphi(\mathrm{r}) = \frac{1}{1+(\lambda \mathrm{r})^2}$$

IV. Inverse quadric (IQ) RBFs

$$\varphi(\mathrm{r}) = \frac{1}{\sqrt{1 + (\lambda \mathrm{r})^2}}$$

- Piecewise smooth RBFs – these RBFs have no shape parameter.

I. Thin Plate Spline (TPS)

$$\varphi(\mathrm{r}) = \mathrm{r}^2 \ln(\mathrm{r})$$

II. Linear radial function (LR)

$$\varphi(r) = r$$

III. Cubic function

$$\varphi(r) = r^3$$

IV. Monomial

$$\varphi(r) = r^{2k-1}$$

Summary of the RBFs presented above and can be shown as Figure 1.

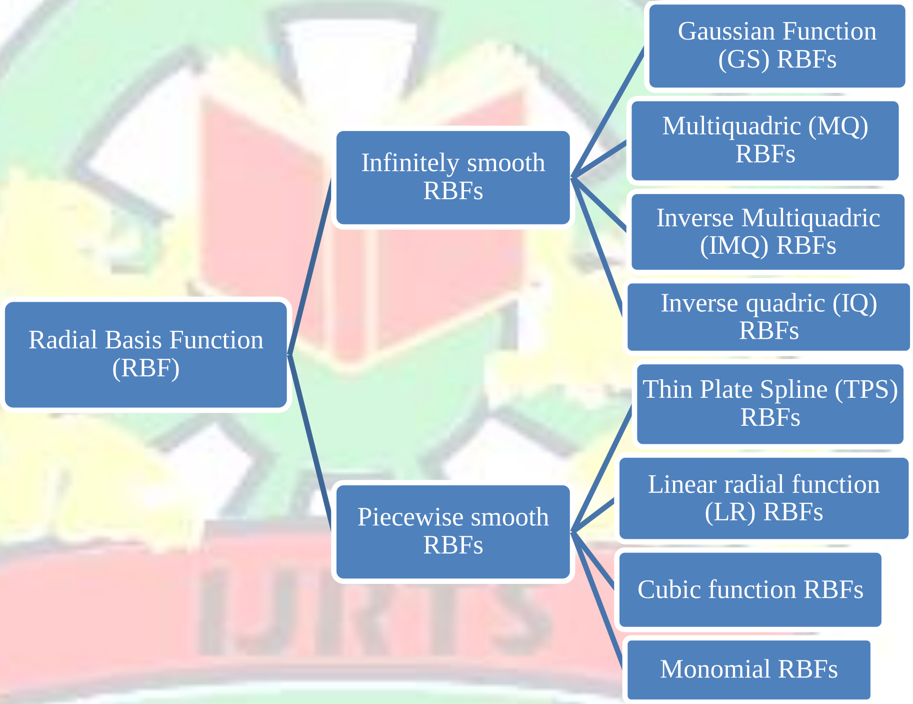

Figure 1.Various types of RBFs

By the analysis of the Figure 2 and Figure 3, that the value of shape parameter results in changing the shape of the radial function when its value is changed on the interval.

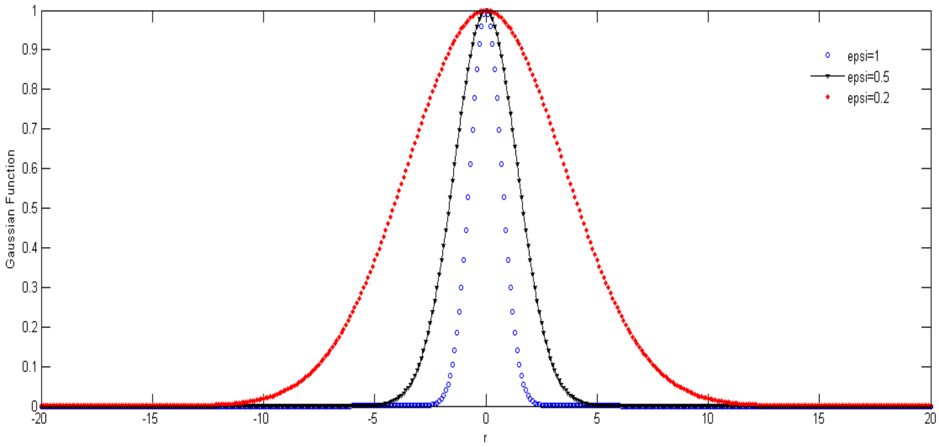

Figure 2. Gaussian RBF with different values of epsilon

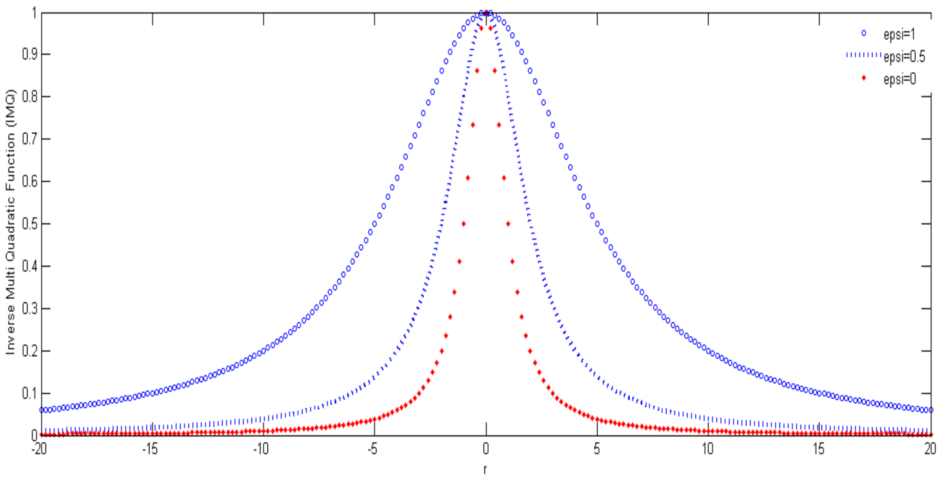

Figure 3. Inverse Multi Quadratic RBF with different values of epsilon

RBF Methods For Solving PDEs

RBF methods are known for their easy way of implementation and simplicity in approximation of multivariate scattered data. For solving partial differential equations, a recent historic and chronologically development strategy of RBF methods has been discussed as follows:

Solutions of PDEs with Kansa-Collocation Method

One of the meshfree approaches is the Kansa method, often known as the RBF collocation method. Compared to mesh methods, meshfree methods have a lot of advantages. They are cost saving since they do not require domain or surface discretization. Kansa (1990) introduced an asymmetric approach in. Kansa technique is an RBF-based approach for solving PDEs.

By mathematically, Consider $x \in \mathbb{R}^d$ and in $\mathbb{R}^d$, consider the norm ‖ ·‖ that is Euclidian norm, the radial basis functions of the form $\emptyset(\|x - x_i\|)$ that supposed to be strictly positive definite. The RBF approximation can be written by the use of nodes that are spotted arbitrary in the domain $\Omega \subset \mathbb{R}^d$ and assign a collection of neighbourhood nodes x_i that are integrated in the supportive domain, to every *x* as:

$$u(x) = \sum_{i=1}^{N} \alpha_i \emptyset(\|x - x_i\|)$$

where α_i is unknown coefficients and N represents the node points. By substituting this solution u(x) in PDE gives the linear system of equations as

$$A\alpha = B \text{ where } \alpha = [\alpha(x_1), \alpha(x_2), \alpha(x_3), \dots \dots \dots \dots \alpha(x_N)]^T.$$

Various problems have been solved by this approach successfully. By using this approach, Zhou et al.(2004) solved shallow water modelling problem, convection diffusion problems solved by Chen et al. (2003), and also solved fractional diffusion equation by using Kansa method, Kovacevic et al. (2003) solved Stefan problem, time dependent heat conduction problems solved by Chantasiriwan (2007), Duan et al. (2009) solved electrostatic problemsusing Kansa method. The Kansa methods have disadvantages due to used in solving various PDEs. Main disadvantage of this method is computational cost that becomes very high due to the unsymmetric interpolation matrix. The accuracy of this method is less in the domain closest to the boundary. To get the better accuracy and hence reduce the errors, the very simplest way is to raise the interpolation points that lead to high condition number matrix. But by increasing the points that are now taken in the entire domain, the resultant matrix turn into ill conditioned. This resulted the need of modification in this method and hence gives rise to following three methods:

Symmetric Collocation Method (SCM)

After modification in the Kansa method, a new method comes in existence which is known as Symmetric collocation method. This method is based upon Hermite interpolation and proposed by Fasshauer (1997) and also invent the RBF expansion for approximating the function. After applying the collocation conditions, there is an *N* X *N* symmetric collocation matrix which is non-singular. Symmetric and non-symmetric techniques had been applied for different applications. These methods are compared by Power & Barraco (2002) and find the result as the symmetric collocation technique is surpassing the non-symmetric (Kansa method) technique due to the lower computational cost. But the implementation of Kansa scheme is unproblematic. The symmetric collocation method is also used by Leitao (2004) to solve 2-dimensionalelastostatic problems.

Modified Collocation Method (MCM)

As the Modified collocation method is the upgraded structure of symmetric method whose resultant is that the interpolation matrix are symmetric. Chen (2002) proposed a method in which Green second identity is used, called modified collocation method.

Ill-conditioned interpolate matrix is the main concern for using Kansa technique to finding the results of the various PDEs. To resolve the issue like Domain Decomposition Method, compactly supported RBF and pre-conditioning, numerous techniques were projected. Process of transformation of a set of linear equation into a new system that is constructive approach for iterative solution is called Pre-conditioning. This transformation produced by a matrix which is known as pre-conditioner. Assessment of the conditioned number is also balanced by pre-conditioning and it also helpful in the improvement of convergence. In the Domain Decomposition method, a problem with huge point of global domain is divided into sub-domains weather these are overlying or uncorrelated. To avoid the ill-conditioned solutions of the problems on sub-domain except the large domain, this process is very effective.

Local Radial Basis Collocation Method

Another method to remove the complex behaviour of the interpolation matrix is local radial basis collocation method (LRBFCM). Local approximation is the main base of this process and depicted by Chen et al. (2014). To see the procedure of this method, taking the Elliptic partial differential equation for mathematical formulation with domain D given by $L[u(x)] = f(x), x \in \mathrm{D}$ with boundary conditions $u(x) = g(x), x \in \partial D$.Let the local approximation $u(x^s)$of the solution and $\{x^s\}_{s=1}^{N} \in D$ then

$$u(x^s) = \sum_{k=1}^{n} \alpha_k^s \emptyset(\|x^s - \mathrm{x}_\mathrm{k}^\mathrm{s}\|)$$

where x^s is collocation point, n is the neighbourhood of the point x^s including itself, and $\emptyset is$radial basis function. Here coefficients α_k^s to be determined. For the distinct values of collocation nodes, non-singularity will becomes necessary condition for resultant matrix.

The above discussed LRBFCM is used for find the solutions of diffusion equations and this method is intended by using local collocation. As the collocation performed on local domain of influence that minimize the size of the collocation matrix. This approach followed by many authors and applied for finding the solutions of large dimensional problems such as fluid flow and heat transfer problems, convective diffusive solid liquid stages change problems, Darcy-flow and also for Transport Phenomena. Further for find the solution of hyperbolic partial differential equation numerically, this method is improved by Siraj. For improving accuracy in this modified approach, Multi-quadric RBF is used with consistent related arrangement. For approximate the time derivative, the finite difference formula of first order is numerically used. While comparing with Kansa collocation method, this method was found to be more stable for numerical problems.

Solution of PDEs with Differential Quadrature Method (RBF-DQ)

Bellmanet al. (1972) proposed an approach-the Differential Quadrature that approximate derivative of the function rather than function itself. In this technique, a smooth function is consider whose partial derivative is estimated on a node seeing as linear summation of the values of function that lies in the domain which is similar to the concept of integral quadrature. The derivative at the node x_i as

$$f^n(x_i) = \sum_{j=1}^{N} a_{ij}^n f(x_j), \ i = 1,2,3,......N$$

Instead of using Lagrange's interpolation, Radial basis function is used by Shu & Wu (2002) in differential quadrature approach for finding the value of weighting coefficients and hence the method is named as RBF-DQ method. The RBF-DQM can be applied in two different ways to solve the PDEs as Global and Local of RBF-DQ method are given by Shu et al. (2003,2004) In Global version, all nodes are used in the whole domain for estimating the derivative at a point. The ill conditioning problem occurs and the computational cost becomes high by using huge set of nodes. And in Local Radial Basis Function Differential Quadrature (LRBFDQ) method local approach is used which takes all the neighbourhood points of the specific point known as supporting nodes.

2-D Navier-Stokes equations solved by Shu *et al.* (2003) by the use of LRBFDQ method and then Shu et al. (2005) apply it for compressible flows. This method is applied for the boundary level problemsby Shen (2010). Two-dimensional transient heat conduction problems are also solved by this method in the work of Soleimani et al. (2010). Integrated radial basis function network used by Shu& Wu (2007) with the concept of differential quadrature named as IRBF-DQmethod. And one-dimensional burger's equation successively solved with this method. By using LRBFDQ, Dehghan & Nikopour (2013) find the solutions of the boundary value problems by using Multiquadric (MQ) radial basis function. Dehghan also applied two different methods- OCSP method and OVSP method for finding the value of λ plays a significant role in RBF.

Solution of PDEs with Partition Of Unity Method (RBF-PUM)

Babuska and Melenk proposed the partition of unity finite element (PUM) method in 1997 for finding the solutions of PDEs. By the proposal of the partition of unity method the region is fractionalized into intersecting local domains. For choosing a family of compactly supported, a continuous function, this approach is important. The RBF-PUM method is a best way to decrease the computational cost with attaining the higher accuracy. The main advantage of this approach in high dimensional problems is to hold the geometrical flexibility, to overcome computation cost and to facilitate adaptive approximation.

In this method, local approximation are defined on sub-domains then merge to structure global approximation by using weight functions which figure out the method of partition of unity. In this method, for local approximation RBF is used. The partition of unity method (PUM) combines with RBF by Wendland (2002) for solving problems on large extent. Consider elliptic problem of partial differential equation for mathematical interpretation on domain D

$$L[u(x)] = f(x), x \in D$$

and with boundary condition

$$u(x) = g(x), x \in \partial D$$

Algorithm for spherical interpolation proposed by Cavoretto & Rossi (2012) for finding the numerical solution of problem using basis function that further projected a method by the use of the partition of unity method. The author use spherical radial basis function mainly in local approximation. In this process many operations can be performed equivalently. Further in the extension of this work, Cavoretto & Rossi (2014) intended an algorithm of partition of unity method in which domain is partitioning into nodes or cell. This procedure principally based upon cell search. Also the author extended this 2-dimensional algorithm to 3-dimensional by using cube partition searching procedure. Applications of partition of unity method investigate by Safdari et al. (2014) for the solutions of parabolic partial differential equation. For this work 2-D diffusion equations was considered and pseudo-spectral and finite difference methods are compared with RBF-PUM. After comparison, researcher initiate that RBF-PUM gives more exact solution than that of pseudo-spectral method. This method can be applied to irregular shaped domains due to their restricted nature. The constancy of this method was proved by the assistance of abstract and investigational methods. Further improvement in partition of unity method is done by Heryudono et al. (2015). The resultant matrix of this method is ill conditioned, asymmetrical but efficient pre-conditioner is required. Distinct pre-conditioning approaches based on LU factorization are compared and discussed by researcher.

RBF Methods with Shape Parameters

In RBF research field, optimising the shape parameter λ is continuously a major area. In this regard, a number of studies have been conducted**.** There are some methods for finding best shape parameter λ listed in the **Table 1.**

Table 1: Approaches of finding best shape parameter λ

<table>
<tr><td rowspan="2">1. Trial and Error
String of experiments by different values of λ based upon interpolation.</td><td colspan="2">Rolland L. Hardy</td><td colspan="2">Richard Franke</td><td colspan="2">G. E. Fasshauer</td></tr>
<tr><td colspan="2">$\frac{1}{0.815}$</td><td colspan="2">$\frac{0.8\sqrt{N}}{D}$</td><td colspan="2">$2\sqrt{N}$</td></tr>
<tr><td rowspan="3">2. The Power Function
Estimation of the interpolating error that dissociate it into autonomous factor and a component dependent upon the function.</td><td colspan="3">For Gaussians 1-Dim</td><td colspan="3">For Gaussians 2-Dim</td></tr>
<tr><td>N</td><td colspan="2">Shape Parameter</td><td colspan="2">N</td><td>Shape Parameter</td></tr>
<tr><td>9</td><td colspan="2">1.72</td><td colspan="2">9</td><td>0.16</td></tr>
<tr><td>3. Leave-One-Out Cross-</td><td colspan="6">Optimal value of λ is approximated near to</td></tr>
</table>

Validation (LOOCV) Based upon a cross validation approach	the value that is obtain by trial and error approach.

Huang et al. (2007) used arbitrary precision computing to determine the relation among the value of λ and the exactness of the solutions in their investigation. According to their research, method of finding the solution of radial basis functions by 100-digit precision arithmetic is used to avoid the singularity due to round-off error occurs in regular 16-digit precision arithmetic when the parameter value is small. They devised error formulations with respect to the value of λ and grid spacing based on the numerical data obtained. Guo & Jung (2017, 2017a) calculated the best value of λ for discretization approach by Taylor series. The higher-order derivative components that arose in the ideal form parameter that was optimised were calculated using a polynomial reconstruction approach. Homayoon et al. (2013) used RBF-based differential quadrature method (RBF-DQ) for finding the results of shallow water and long wave's problems. Here, leave-one-out cross validation (LOOCV) approach being implemented for getting the optimal value of λ.

Table 2.A chronological scheme of Leave One Out Cross Validation (LOOCV) technique

Researcher	Year	Findings
D. M. Allen	1974	For ridge regression
Peter Craven and Grace Wahba	1979	For smoothing splines
S. Rippa	1999	Optimised RBF's shape parameter λ.
Gregory E. Fasshauer and Jack G. Zhang	2007	Extensions of LOOCV approach

There is no specific technique or process to controlling the shape parameter λ for RBF kernel methods. The shape parameter can be chosen through numerical evaluations of RBFs to stabilise the solution, that is exceedingly hard and time consuming. Timesli & Saffah (2021) build an algorithm for determining the optimal value of shape parameter λ rapidlyand instantly determines the appropriate value of λ . The strategy for determining the best possible value of λ is depending upon the idea of combining the RBF method, numerical continuation approach and high order algorithm Taylor expansion. In this algorithm, author use the description of $\lambda = \alpha ds$, here any coefficient α is used and *ds* =domain. So

$$\mathrm{ds} = \frac{1}{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{d_i} \;;$$

where distance is calculated by $\mathrm{d_i}$ among i[th]-point & neighbourhood points. In this work Timesli and Saffah aim to reduce the inaccuracy at order 1 of the higher order mesh-free algorithm. Marko Urleb proposed a strategy for finding an optimised value of shape parameter λ for unknown results of PDEs with initial and boundary conditions. In this procedure, Gershgorin's theorem, multi-quadric RBF and the Newton method are implemented for optimal values of diffusion equations and then made a comparison with the results of Finite Element Method. The goal of this work is to find an optimal value λ using Gershgorin's theorem (regarding eigen values of matrix), MQ RBF) and the Newton method for evaluating the zeros of a function.

As in the findings of the finite element method, other methods those have or have not λ gives the exactness of obtained optimal method. The main purpose is to validate the exactness and strength of the procedure comparative to others. In this paper, an iteration algorithm is given by calculating a matrix which is obtained by the multi-quadric functions outer side of the time loop. In this algorithm, the value of the operator *L* on the MQ basis function is defined by matrix W over domain $\Omega \backslash \partial\Omega$ and the boundary operator *B* on the MQ basis function on $\partial\Omega$.

A novel higher-order RBF-FD schema with optimal variable shape parameter was proposed by Nga Y. L. et al. (2019) for the numerical results of various PDEs. For the solutions of partial differential equations, RBFs with multi-quadric kernel have been used generally. A user-friendly shape parameter is used in the MQ kernel and exactness of result depends upon the shape parameter value. In this approach, RBF finite difference method based on MQ is calculated in polynomial structure i.e, The RBF finite difference (RBF-FD) method used for approximating the second derivative that contains shape parameter which affects the accuracy of the PDEs solution. The best value of shape parameter is found by removing the RBF-FD scheme's leading error term, which improves solution accuracy and speeds up convergence. Combined compact differencing and finite difference techniques are used to determine the ideal shape parameter. The best shape parameter is discovered to vary across the domain, according to the analysis. As a result, comparing with the RBF-FD method that uses value of shape parameter, the accuracy of the solution of

PDEs is high when employing the localised shape parameter. Generally the solutions derived by employing the shape parameter calculated from combined compact differencing (CCD) scheme gives more accuracy, but it comes with a result of high computational cost. However, as the number of iterations of shape parameter is restricted to two, the current RBF finite differencing (FD) with shape parameter by combined compact differencing technique is as effective as applying FD scheme, according to the cost-effectiveness analysis.

When the RBFs are going to almost flat and the selection of the value of shape parameter is done correctly, RBF approximation is capable to produce an appropriate estimation for huge collection of data points that provides smooth result for specified tangled points. In research work of Kazeem et al. (2020), the inverse multi-quadric (IMQ) RBF function was included for writing and implementing a technique for the solution of partial differential equations. Preference is given to the selection of shape parameters, which must be made carefully. The approach as an algorithm that runs a series of interpolation tests while adjusting the range of the shape parameters, and then chooses the optimal shape parameter with the resultant as the smallest root mean square error (RMSE). Matlab was used for all of the computational work. The selected problems of interpolantion and it root mean square errors (RMSEs) are tabularised and diagrammed.

Conclusion

For solving interpolation matrices, Radial Basis Function (RBF) was created and later its extension upto solve PDEs. RBF is easy to implement, accurate and works well with dynamic or irregular domain. This article provides an overview of the approaches that are based on the construction of radial basis functions. We make an attempt to highlight some of the current developments. To make the approach understandable to the readers, the methods are supplied using mathematical formulation. This review is intended to familiarise the reader with RBF approaches. As RBF collocation methods produce completely populated matrices, which raise computing costs and be unsuccessful to perform well for vast applications, by this there is a requirement to discover alternatives. Because of their local adaptivity, the local collocation RBF approaches have remained popular among all of these methods. The matrices' bad conditioning which occurs in global approximation, is evade due to the use of the local approximation. To tackle the challenges in the Kansa collocation approach, other hybrid methods were introduced. To get the most out of RBFs, hybrid methods combine the RBFs with conventional methods like Finite Difference. These strategies aid in the lowering of computational costs and are particularly useful in the solution of large-scale problems. The minimal shape parameter leads to good accuracy for smooth RBFs, while the near flat radial basis leads to poor conditioning of the interpolation matrices. To overcome this issue, several algorithms were proposed that enable stable approach for all values of the shape parameter. The given approaches can be further improved by investigating the optimal value of the shape parameter for the better accuracy and steadiness of radial basis function-approximations. The efficiency of RBF techniques for solving higher-order PDEs are still being investigated.

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Mathematics And Real World- Modern Approach

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Abstract

Without mathematics, there is nothing you can accomplish. Over the past two decades, new branches that were generated from the oldest areas of mathematics have been added to our contemporary world. By describing all of mathematics' branches and the various fields in which they are applied, it disproved the notion that it is dull. Math and science are working together to eliminate the myth that math solely involves difficult calculations and concepts that are rarely applied in practice. Contrary to popular belief, math is the universal language and is employed in almost all facets of living. Whether we are creating music or playing video games, math is crucial for fostering our creativity and accomplishing our goals. It doesn't take much investigation to see that arithmetic permeates our daily lives and is everywhere. This essay focuses on the connections between mathematics and other disciplines, including technology. The most current developments in mathematics are also included. The applications of mathematics to gambling, tomography, epidemic analysis, cryptography, satellite navigation, and other fields are also covered.

Keywords: Technology and Mathematics, gambling, music and video games , cryptography, tomography, epidemics analysis

Introduction

Maths has influenced our lives to a great extent. Maths is a language of technology and science. The patterns in nature, social dynamics, and economic systems are only a few examples of the deeper complexities of the universe that mathematics can help us grasp. Math is a potent tool for examining complex social topics like political division and wealth inequality. Math is fundamentally a problem-solving and analytical subject. It teaches us how to evaluate information, spot trends, and come up with original answers to challenging issues. These abilities are in great demand across numerous industries, including technology, finance, and healthcare, in today's fast-paced, data-driven environment.

For instance, math is what has brought order to communities all across the world and averted turmoil and natural disasters. Mathematical theories foster and enhance many of our innate human traits, including spatial awareness, problem-solving abilities, the capacity for reason (which incorporates calculating thought), as well as creativity and communication.

Mathematical content must assist students in achieving global competency, which includes understanding various viewpoints and global conditions, realizing that global concerns are interconnected, as well as communicating responding in suitable ways. This entails rethinking the standard material in math in unusual ways and demonstrating to pupils how the universe is made up of problems, occurrences, and phenomena that can be resolved mathematically.

Maths and real world People use maths in their life directly or indirectly. Many applications of mathematics are discovered till now and many more to come. Many examples can be seen like algebra is used to explain the extent of contamination of water and predict the number of people in an area become ill by drinking that contaminated water, understanding Mayan and Babylonian systems (with base 20 and 60), statistics and probability is used to estimate deaths due to natural disasters, Geometry and shapes can be used in Islamic tessellations to tessellate a plane, prime numbers in cryptography etc.

Cryptography Cryptography is a science of secrecy and there is mathematics behind it. Now a day it has become a part of our daily life. Whenever we use an email or online shopping via debit, credit cards or online banking this technique is used to keep our data safe and secure. Secret data must be transmitted between your computer and a web server every time you use your credit card or send an email online. This information can be encrypted using mathematics to prevent unauthorized parties from reading and using it.

The receiving computer generates two extraordinarily big prime numbers—typically with more than 100 digits—and publishes the product. This product is used by the sending computer to encrypt the communication before sending it to the recipient. You need to be familiar with both the product and the two initial primes in order to go backward and understand the message, though. Given how difficult it is to factor large numbers, you must already be familiar with the original primes in order to understand the message. This method is called RSA and is widely used in sending messages via cell phones or PCs and in online shopping and banking.

This RSA padlock consists of two numbers- one is encryption number e and second is modulus N. before encrypting a message it is converted to binary digital code and then for secure encryption of the message it is changed into a different number C called ciphertext which is calculated by the formula $c = m^e \; mod \; N$ where m is the digital message. For example let us take e=3 and N=55 and message m=14. Then c is calculated as

$$C = m^e \; mod \; N$$

$$C = 14^3 \; mod \; 55$$

$$C = 2744 \; mod \; 55$$

$$C = 49 \; mod \; 55$$

Hence ciphertext obtained is 49. This ciphertext can be decrypted using a key d. In this case d=27 and decryption can be done by the following calculation

$$C^d \; mod \; N = \; 49^{27} \; mod \; 55$$

$$= 14 \; mod \; 55 = m \; mod \; 55.$$

After decryption original message m = 14 is obtained.

It can be now written as: $m = C^d \; mod \; N$

$$= (m^e)^d \; mod \; N$$

$$= m^{ed} mod \; N$$

It means that RSA works out on Finding numbers e, N, and d such that raising any message m to the power of ed modulo N is similar to raising a number m to the power of 1 in our everyday arithmetic. The RSA scheme offers a means to calculate these numbers so that the key d can only be found with the padlock e and N if you are aware of the factors of N. Here is where the system's security is located. The choice of the padlock ensures that N is the result of two very large prime numbers.

The information age and the revolution in e-commerce have been made possible by the mathematics used in cryptography.

Music Waves, tiny vibrations in the instruments, loudspeakers, air molecules, and our ear make up music and sound. By capturing samples frequently, typically 44100 times per second, these waves can be saved as a string of numbers on a CD. This is a big number, which explains why CDs can be up to 700MB in size. Several waves of various frequencies combine to form sound waves. All frequencies are recorded on a CD, but only a specific range is audible to the human ear. A complicated sound wave can be divided into numerous smaller sine waves with various frequencies using the mathematical Fourier transform. Just keeping the frequencies that can be heard by humans, audio formats like MP3 greatly reduce the size of the files.

With Huffman Codes, which analyses the digital content of music and ensures that common parts are encoded using less space than rare parts, the file size can be reduced further. There wouldn't be any iPods, Spotify, or iTunes without maths. Digital music editing, such as equalization, reverb, noise reduction, and mixing, uses even more mathematics. Sine waves, the regular waves that engineers produce on oscilloscopes, can be used to disassemble any wave type, including a sound wave. A series of integers can be used to indicate the precise arrangement of sine waves that come together to create a specific sound wave. It is possible to calculate those numbers from the original wave using a mathematical technique called a Fourier Transform. Sound can be converted into numbers in this way. A series of numbers is produced using a technique called sampling. The most popular technique is known as pulse code modulation (PCM). The data on CDs and DVDs is stored as a succession of minute "hills" and "valleys" carved in the surface of the discs. The hills and valleys on the disk's surface can be destroyed, along with the data they encode, by scratches and dust. This should imply that a DVD or CD won't play at all or won't play properly. This issue can be resolved mathematically because Reed-Solomon algorithms are used to encrypt the data saved on CDs. They are made so that even if some of the data is wrong or missing, computers can still use the remaining information to fill in the gaps and discover the problems. You can't play a CD that is fully scratched because this only functions if a specific percentage of the data is accurate. Cross-interleaved Polynomials over finite fields serve as the foundation for the Reed-Solomon codes used for CDs and DVDs.

Gambling Probabilities and likelihoods are present all around us, in everything from games, insurance, and political polls to weather forecasts. Yet, probability is actually a relatively new concept in the history of mathematics. While Greek mathematicians studied numbers and geometry more than 2500 years ago, the ideas of probability didn't begin to take hold until the 17th and 18th centuries. Gambling or betting is based on the mathematical concept probability, game theory and expected payoff.

Tomography One of the most significant applications of mathematics to the issues of maintaining your life is tomography. By collecting a large number of two-dimensional "snapshots" from various angles, MRI scanners may produce three-dimensional representations of the human body. The process of reconstructing the original 3-dimensional model from these images,, is called tomography and it is not possible without sophisticated mathematics like Radon Transforms and measure theory. Earlier surgeons had to convert these 2D snapshots into 3D images of the body and it was a difficult task and also not accurate. Now several mathematical algorithms are used in this field.

A CT scan image is made up of individual "voxels," which are essentially 3D counterparts of the small individual pixels that make up 2D images. It is likely that a voxel with a high value, say 300, represents a volume with a high density, like bone. But this also varied from person to person. To overcome this problem all the voxel values are plotted on a histogram using a software; if there is a cluster of values near the higher end of the histogram, the software recognizes this as an area of bone and compute a threshold value for that specific patient. To find tumors or large organs like the kidneys, a similar procedure is performed.

The development of X-rays at the beginning of the 20th century was the first step in the widespread use of imaging techniques in modern medicine. These imaging techniques essentially come in two forms. The radiation source used in X-ray and ultrasound that lies outside the body. After the radiation has travelled through the body, it is detected, and an image is created based on how it has been absorbed. When X-rays are used this procedure is known as computerized axial tomography or CAT. The patient enters the CAT scanner through a hole in the middle while lying on a bed. An X-ray source is housed in this hole, and it revolves around the patient. These X-rays penetrate the patient and are picked up on the opposite side. Accurate measurements of the X-intensity ray's can be made, and the results can be processed.

Conclusion

Mathematics has become an essential part of life. It is associated with every field medical, agricultural, navigation and even literature also. There is no field in which maths is not used. It is used to develop the rest of science and interpret its theories, especially physics, chemistry, astronomy, geography, etc. Cryptography ensures the safety of our secret data whereas tomography saves lives. So maths is not only a subject full of complex calculations but a boon of science.

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Applications of Mathematical Concepts and Techniques in Different Aspects of Media Industry

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Abstract

Mathematics has played a crucial role in the development and growth of the media industry. From data compression and digital signal processing to cryptography and data analytics, mathematical concepts and techniques have been instrumental in shaping the media landscape. This research paper aims to explore the role of Mathematics in the field of media and the various applications of mathematical concepts and techniques in different aspects of media production, distribution, and consumption. The paper also highlights the challenges and opportunities that lie ahead in the field of media and Mathematics and the potential for further research and development.

Key words Media Industry, Mathematical techniques, Mathematical tools

Introduction

The field of media has been rapidly evolving in recent years, and the use of Mathematics has been growing in tandem. The amount of data that is generated and transmitted in the field of media is enormous, and Mathematics has been instrumental in addressing the challenges that arise from this data explosion. From data compression to image and video processing, Digital Signal Processing, cryptography, and data analytics Mathematics has been playing a critical role in the field of media.

Data Compression

Data compression in media refers to the process of reducing the size of digital media files, such as images, audio, and video, by removing or encoding redundant or unnecessary information. The goal of data compression is to reduce the amount of storage space needed for these files, making it easier to store and transmit them over digital networks.

Data compression is an essential aspect of digital media. The amount of data that is generated and transmitted in the field of media is enormous, and without data compression, the transmission of such large amounts of data would require an enormous amount of bandwidth, which is often not available. Mathematics plays a crucial role in the development of data compression algorithm. Data compression is achieved using mathematical algorithms and techniques, such as statistical modeling, linear algebra, and signal processing. These algorithms analyse the data to identify patterns and redundancies, which are then removed or encoded using mathematical representations. The resulting compressed data can be decompressed back into its original format when needed.

For example, Huffman coding is a mathematical algorithm that is used to compress data by assigning a unique code to each data symbol based on its frequency of occurrence. This algorithm reduces the size of the data and ensures that the most frequently occurring symbols are assigned the shortest code length. Similarly, other algorithms such as Arithmetic coding, LZ77, and LZW are used in data compression and involve mathematical concepts such as entropy, probability, and information theory.

As another example, an hour-long television show requires several gigabytes of data to store. The transmission of such large amounts of data would require an enormous amount of bandwidth, which is often not available. To overcome this challenge, data compression algorithms are used to reduce the size of data without affecting its quality.

Image and Video Processing

Mathematics plays a crucial role in image and video processing by providing the foundation for many of the algorithms and techniques used in these fields. Some of the key mathematical concepts and techniques used in image and video processing include:

Linear Algebra: Linear algebra is used in image and video processing to transform, manipulate, and analyze image and video data. For example, matrix operations can be used to apply filters to an image or video, such as blurring or sharpening.

Signal Processing: Signal processing is used to analyze and transform image and video data, including filtering, noise reduction, and feature extraction. Techniques such as Fourier analysis and wavelet transforms are used to represent image and video data in a way that is more amenable to processing.

Probability and Statistics: Probability and statistics are used to model and analyze image and video data, including image and video quality, compression efficiency, and feature extraction. These techniques help to identify patterns and trends in the data, which can be used to improve image and video processing algorithms.

Optimization: Optimization techniques are used to improve the performance and efficiency of image and video processing algorithms. For example, these techniques can be used to minimize noise or reduce image or video artifacts during compression.

Also image filtering algorithms use mathematical concepts such as convolution, Fourier transforms, and wavelets to remove noise, sharpen edges, and improve the overall quality of images. Similarly, in video processing, mathematical algorithms are used to interpolate and extrapolate frames, reduce flickering, and improve the overall quality of the video.

The role of mathematics in image and video processing is essential in creating advanced and effective processing algorithms that can be used to improve the quality and efficiency of digital media.

Digital Signal Processing

Digital Signal Processing (DSP) is a branch of signal processing that involves the manipulation of signals, which are represented as a sequence of numbers, using mathematical algorithms and techniques implemented on digital computers. DSP is used in various fields such as audio signal processing, image processing, and speech processing.

DSP is used in a wide range of applications, such as audio and video compression, speech recognition, noise reduction, and equalization. The goal of DSP is to improve the quality of digital signals by removing noise, distortion, and other unwanted artifacts. DSP is a critical component of many modern digital technologies, including smartphones, audio and video recording devices, and wireless communication systems.

Mathematics plays a central role in digital signal processing (DSP) by providing the foundation for many of the techniques and algorithms used in this field. In DSP, analog signals are converted into a digital form using analog-to-digital converters (ADCs). Once the signals are in digital form, they can be processed using various mathematical algorithms such as filtering, Fourier analysis, and statistical analysis. The processed signals are then converted back into an analog form using digital-to-analog converters (DACs).

Some of the key mathematical concepts used in DSP include:

Fourier Analysis: Fourier analysis is used to transform signals from the time domain into the frequency domain. This allows for the identification of the frequency components of a signal, which is useful in filtering and compression.

Linear Algebra: Linear algebra is used in DSP to represent signals as vectors and matrices, which can be manipulated using various operations such as addition, multiplication, and inversion.

Probability Theory: Probability theory is used in DSP to model and analyse the statistical properties of signals, such as noise and random variation.

Overall, the role of mathematics in DSP is essential in creating advanced and effective signal processing algorithms that can be used to improve the quality of digital signals. The mathematical concepts and techniques used in DSP help to extract useful information from signals, reduce noise and distortion, and enhance the performance of digital systems.

Cryptography

Cryptography is the practice of secure communication in the presence of third parties who may attempt to intercept or alter the communication. In the field of media, cryptography is used to secure digital content, such as digital music and video files, from unauthorized access and tampering.

Mathematics plays a critical role in cryptography, the practice of secure communication in the presence of adversaries. Cryptography uses mathematical algorithms and techniques to secure digital data and protect it from unauthorized access or interception.

Some of the key mathematical concepts used in cryptography include:

Number Theory: Number theory provides the foundation for many cryptographic algorithms, including public key cryptography. Concepts such as prime numbers, modular arithmetic, and discrete logarithms are used to create cryptographic keys, which are used to encrypt and decrypt digital data.

Cryptographic Hash Functions: Cryptographic hash functions are mathematical functions that take an input of arbitrary length and produce a fixed-length output. These functions are used in various cryptographic applications, such as creating digital signatures and verifying the integrity of data.

Symmetric and Asymmetric Cryptography: Symmetric cryptography uses the same key for both encryption and decryption, while asymmetric cryptography uses different keys for encryption and decryption. Both types of cryptography use mathematical algorithms to scramble and unscramble digital data.

Probability and Information Theory: Probability and information theory are used in cryptography to quantify the amount of uncertainty in data and to measure the amount of information contained in a message.

Also, the RSA algorithm is a mathematical algorithm that is used to secure digital communications by encrypting and decrypting messages. This algorithm uses large prime numbers and modular arithmetic to ensure that only the intended recipient can read the message. Similarly, other algorithms, such as AES, DES, and Blowfish, use mathematical concepts such as substitution, permutation, and modular arithmetic to secure digital content.

Overall, mathematics is fundamental to the design and implementation of secure cryptographic systems. By using advanced mathematical techniques and algorithms, cryptography ensures that digital data remains private, secure, and protected from unauthorized access.

Data Analytics

Data analytics is the process of examining and interpreting data to uncover patterns and insights. In the field of media, data analytics is used to analyse consumer behaviour, assess the effectiveness of marketing campaigns, and track the performance of media content.

Mathematics plays a crucial role in data analysis, which involves extracting useful information and insights from large and complex data sets. There are many ways in which mathematics is used in data analysis, including:

Descriptive Statistics: Descriptive statistics are used to summarize and describe the key characteristics of data sets. This includes measures such as mean, median, mode, standard deviation, and variance, which can help to identify trends and patterns in the data.

Probability and Statistical Inference: Probability theory and statistical inference are used to model and analyse the likelihood of different outcomes and to estimate the parameters of statistical models. This is particularly useful in making predictions and drawing conclusions from data sets.

Regression Analysis: Regression analysis is used to establish relationships between variables and to make predictions based on those relationships. This is particularly useful in identifying cause-and-effect relationships in data sets.

Machine Learning: Machine learning uses mathematical algorithms to learn from data and to make predictions or decisions based on that learning. This includes techniques such as clustering, classification, and prediction.

Data Visualization: Data visualization uses mathematical techniques to transform data into visual representations, such as graphs, charts, and maps. This helps to identify patterns and relationships in the data and to communicate those findings to others.

Overall, mathematics is critical to the field of data analysis, as it provides the tools and techniques needed to understand, analyse, and interpret complex data sets. By using advanced mathematical methods, data analysts can identify patterns, make predictions, and draw meaningful insights from large and complex data sets.

Conclusion

Mathematics plays a significant role in media by providing the foundation for many of the algorithms and processes used in various media technologies. In digital media, for instance, mathematics is crucial in the fields of digital signal processing, image processing, video processing, and data compression. The mathematical concepts and techniques used in these fields help media technologies function efficiently and effectively. Additionally, mathematics provides the means to model and analyse the behaviour of media consumers, which can aid in the development of new technologies and improved user experiences. Overall, the role of mathematics in media is essential in creating innovative and reliable media technologies that serve and engage audiences.

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Use of Mathematics in Geography: A Temporal Analysis

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Abstract

Geography studies places and relationships between people and their surroundings. Besides this, geography is also the study of anything that happens on or above the Earth's surface, including how people live and use this surface. It is also a scientific and mathematical description of our Earth in its universe. It is said that mathematics is an intellectual game that often has many important applications. Like all other revolutions we may have read in the history of the world, mathematics has its own kind of revolution. Geography is called the mother of sciences. Geography has subject matters from sciences as well as social science but we gave the spatial perspective to it. The early geographer uses many calculation techniques in their work of geography. even the father of geography Eratosthenes use mathematics calculation for calculating the circumference of the earth which was close to reality. Geographer also use geometry in surveying small areas in the field, while spherical geometry and trigonometry are required in the construction of map projections, both traditional elements of mathematical geography. The research work in geography is totally incomplete without mathematical technique or statistical techniques. The latest branch of geography. Remote Sensing and Geographical Information System (GIS) use many calculations to complete the concept. Economic Geography and its sub branches such as Agricultural Geography. Industrial Geography. Commercial Geography etc. uses mathematical methods and techniques in one way or the other. The modern history of geography also has a major paradigm named "Quantitative Revolution in Geography". This is totally related to the mathematics. Even we can't think about theory and models in geography without use of mathematics. The major branch of geography named Cartography uses calculation techniques for the construction and drawing of maps and diagrams. In this paper I underline some of the numerous applications of mathematics in geography. There are a number of ways in which mathematics is used in geography.

Keywords Mathematics, aspects, Geography, Statistical Techniques, Quantitative Revolution, Surveying etc.

Objectives Following are the objectives of the present research paper *"Use of Mathematics in Geography: A Temporal Analysis"*.

1. To show the relationship of Mathematics and Geography.
2. To analyse the use of mathematics in the different aspects of geography.
3. To emphasize the necessity of use of mathematical phenomenon in various academic and research work in geography.

Introduction

A man without the knowledge of mathematics is like an injured person in the field of other sciences also. As he is unable to analyse the situation in the mathematical perspectives. All These leads to the wrong conclusions and results. All the subjects are handicap without the knowledge of mathematics. Geography is also not an exception, as it also requires mathematical tools and techniques not only for its development but also for its existence. A person related to the any discipline if he is ignorant of mathematics cannot know the other sciences or the things of the world." Generally, it is accepted, that mathematics is the most difficult division of geography, but it is becoming a very indispensable discipline now a days.

A passive knowledge of mathematics is not enough, it must be activated and brought up to date and in accordance with the modern contemporary scientific concepts of the physical world. The knowledge of mathematics is needed by the geographer to help in fixing the coordinates of various places in the world. Surveying is the main technique of gaining knowledge of geography in ancient as well as in the modern period. Anyone cannot explore the world without the surveying methods. The empirical approach is one of the approaches most trusted approach of acquiring the knowledge of various places of the world. Without the technique of survey, we unable to know about any part or area of the earth.

Mathematics in Ancient Time in Geography

In the ancient period Eratosthenes used mathematics in calculating circumference of the earth. In spite of having limited technology calculations of Eratosthenes were near to the actual value measured by using modern techniques. Ptolemy another prominent geographer used mathematics with open heart in his writings. Von Richthofan Humboldt, Carl Ritter, Strabo, Vidal-De-La-Blasch, Ellen Churchil Sampul, Carl 'o' Sover, Demangia, El Edrisi, Ibn Batuta, Ibn Khaldun, L.Dudley Stamp and many more geographers of Greek, Roman, French and Arabian school of geography were fluently used mathematical calculations, formulas and equations in their books. Plane Euclidean geometry a technique of mathematics is used in surveying small areas in the field. The trigonometrical techniques are necessary for the construction of map projection. Without the map projection we can't even think about the making of maps. Anyone can't make the assumption of geography without maps. Spatial network analyses are one of the newer branches of geography which is nothing without topology. To study and analyse the dynamic processes of geomorphology needed deferential equations of mathematics. Statistical techniques, such as trend surface analysis, factor analysis, cluster analysis and multiple discriminant analysis, can be applied to the description and analysis of the data of regional geography and reginal planning and development. Geography has gained a great deal in quantitative value and precision in adopting mathematical techniques (Cuchlaine A.M. King, 2006).

Mathematical and statistical concepts and methods approaching geographical related problems in the study of geography are frequently used. All researches in the subject of geography are not completed without mathematical and statistical techniques. Everyone has to adopt one or another technique of mathematics and statistics to complete their research work. Drawing and fixing of points, lines, areas, and volumes, which have intuitive interpretations in the spatial representation are only possible due to mathematics. Observe, describe, analyse and measure essentially constitute scientific actions, which in its synthesis is a mathematization of reality.

Quantitative Revolution

Some Trends in Geography towards Mathematics Quantitative practices in geography date back at least to Greek attempts to measure the circumference of the earth, but the term —quantitative geography was coined in the 1960s. The power of this moment in Anglo-American geography, both in its own terms and also in generating a concern for theory and for philosophical foundations that remains with us today, is no doubt one reason why the term quantitative geography still resonates with what was practiced then. The quantitative revolution brought a change not only in disciplinary language, but also in worldview, each reinforcing the other. In the history of geography, the quantitative revolution (QR) was one of the four major turning-points of modern geography – the other three being environmental determinism, regional geography and critical geography. The quantitative revolution occurred during the 1950s and 1960s and marked a rapid change in the method behind geographical research, from regional geography into a spatial science. The main change of the quantitative revolution is that it led to a shift from descriptive (idiographic) geography to an empirical law making (nomothetic) geography. The Quantitative Revolution began in the universities of Europe with the support of geographers and statisticians in both Europe and the United States. First emerging in the late 1950s and early 1960s, the Quantitative Revolution responded to the rising regional geography paradigm. Some of the techniques that epitomize the quantitative revolution include:

- ***Descriptive statistics;***
- ***Inferential statistics;***
- ***Basic mathematical equations and models such as Gravity model of social physics, or the Coulomb equation;***
- ***Stochastic models using concepts of probability,***
- ***Deterministic models, e.g., Von Thünen's and Weber's location models***

Cartography and Mathematics

Spaces and mappings One of the most important effects of the increasing use and importance of mathematics in contemporary geography. It is the emphasis it has placed upon the most ancient and particular tool of the geographer. Maps have always been used as important illustrative and analytical devices, and in a very simple but real sense they represent models of the world. At the same time, the geographer's view of the map and the act of mapping has been enlarged and extended to bring it closer to more powerful mathematical definitions. To the extent that any cartographic illustration represents a highly selected simplification, abstraction and compression of the real world, it forms a homomorphic, or many-to-one mapping or models. Peter Gould a filter (Hammer, 1969), which allows us to see that maps, and many apparently disparate mathematical models and approaches, are really only different superficial aspects of a deeper underlying idea, perhaps closely linked to human cognitive processes (Downs and Stea, 1973). The parallel cartographic revolution, characterized by genuine cartographic research, rather than traditional map making and atlas compilation.

Conclusion

It becomes crystal clear that knowledge of mathematics occupies a key place in the study of geographers. We can even say that mathematics for geographical concepts is like that golden chain that firmly fixes many pieces of different pearls to form a beautiful necklace. A student of geography is required to have sufficient knowledge of drawing and understanding the maps. The study of mathematics can undoubtedly help them in this task. They can locate and describe the position of a particular place on the world map. They can also understand and calculate the local, standard and international time with the help of their knowledge of mathematics.

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Understanding the different aspects of Chemistry with the use of Mathematical tools

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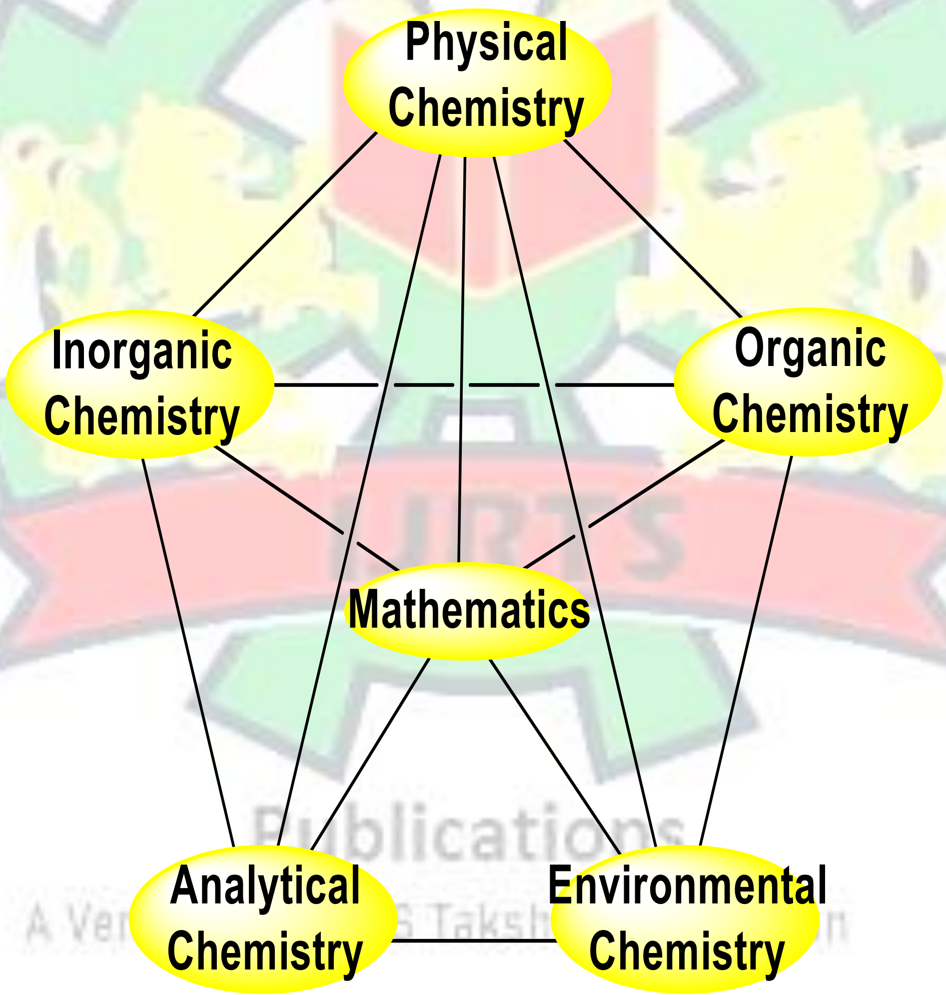
Abstract

Mathematics is an essential component of all scientific disciplines. Mathematics is extensively used and is absolutely necessary to explore different concepts of chemistry. Like every other investigator, the chemist also seeks to categorize his data and make generalizations from it in the form of theories and laws which he can express mathematically as the dependence of one thing on another. However, knowledge and skills in the areas of basic mathematics, calculus, and 3-dimensional geometry can be useful as a prerequisite or co-requisite to general chemistry. Performing dimensional analysis, taking measurements, converting units to other units, determining temperature, density, and so on are all things that cannot be done without the use of mathematical tools. A student will not excel in general chemistry without a solid understanding and facility of the following elementary topics like unit conversions, significant figures, proportions and concentrations, basic trigonometry and algebra including graphing, differential equations etc. Operators are generally used in chemistry to perform various calculations. They are even more important in Quantum mechanics as each observable in Classical mechanics has an operator associated with it in Quantum mechanics. Applied mathematics is essential in both organic and physical chemistry and it extends far beyond general chemistry. Mathematics as a subject is very important to the daily life of every individual as it aids the development of knowledge and the required skills in problem solving situations. Mathematics is seen as science of structure, order and relation that evolves from counting, measuring and describing the shapes of object. It deals with logical reasoning and quantitative calculations. Mathematics nurtures the power of reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability and even effective communication skills. It is the bedrock of scientific and technological development of any society.

Introduction

Mathematics is an essential component of all scientific disciplines, and its applications extend beyond the realm of mathematics. The relationship between mathematics and chemistry is well documented. In some cases, students studying chemistry face mathematically related difficulties. Although chemistry is difficult with mathematics, it is impossible without it. The most obvious reason for these difficulties in learning chemistry arises because of insufficient mathematics preparation. This is especially true for physical chemistry, inorganic chemistry, organic chemistry, analytical chemistry and environmental chemistry.



In chemistry we basically study about the transformation of matter and the energy changes happening during such transformations. A chemist also have to learn about the the physical properties (solubility's, melting points, boiling points, etc.) and chemical properties (reactivity, transformations, etc.) of substances. Taking measurements, performing dimensional analysis, determining temperature, density, and so on are all things that cannot be done without mathematics. During his investigation a chemist has to collect a large numbers of widely dispersed facts. He has to classify this data and make generalizations from it in the form of theories and laws. A chemist always wants to make his theories a law by expressing it mathematically as the dependence of one thing on another.

Problem solving ability and basic knowledge of mathematical concepts such as number concepts, logarithms and indices, algebraic manipulations, graphing, calculus and 3-dimensional geometry are probably the most important factors in studying general chemistry. Let's now look at how different mathematics concepts are used within chemistry.

The basic mathematical concepts of addition, subtraction, multiplication, and division are used extensively in chemistry. These calculations are necessary for calculating the molecular mass of components, volume, and other concepts. It is unlikely that a student will excel in general chemistry without a solid understanding of and familiarity with the following elementary topics:

- Unit conversions
- Significant figures
- Concentrations and proportions
- Basic trigonometry and algebra
- Expressions involving exponents and logarithms
- Basic probability and statistics

Some examples of mathematical concepts and their area(s) of applications to chemistry are given below in tabular form:

Mathematical concept	Chemistry context
Ratios	Used in mixing solutions with certain molarities, making dilutions
Proportional reasoning	Used in analysis of molecular structure

Algebra and graphs	Used in analysis of experimental plots of reaction rates and gas laws
Calculus	Used in predicting and measuring rates of reaction
Units of measurements	Making sense of real, complicated measurements
Vectors	Used in understanding crystal structure
Logarithms	Used in understanding pH
Probability	Used in drawing general conclusions from trials
Single and multivariable calculus	Used in physical chemistry
Linear algebra	Used in physical chemistry
Differential equations	Used in chemical kinetics and thermodynamics
Applied statistics	Used in almost all areas of chemistry
Analysis, advanced calculus, or Fourier analysis	Used in physical chemistry and quantum mechanics
Abstract algebra or group theory	Used in inorganic chemistry and crystallography
Partial differential equations	Used in chemical reaction kinetics, mass transport and thermodynamics
Numerical analysis	Used in computational chemistry
Introduction to programming or computer science	Used in physical chemistry

In fact mathematics has such a vast role in the field of chemistry that a separate branch of chemistry called mathematical chemistry has come into existence. One of the primary goals of mathematical chemistry is to develop mathematical models that can simulate and analyze chemical processes at different levels of detail, ranging from molecular interactions to the behaviour of large chemical systems. These models are based on fundamental laws of physics and chemistry, such as the laws of thermodynamics, quantum mechanics, and statistical mechanics. The models can be used to calculate the physical and chemical properties of molecules and reactions, predict the outcome of chemical reactions, and study the mechanism of chemical processes. For example, in the field of quantum chemistry, mathematical techniques are used to study the electronic structure of molecules and predict their properties, such as their reactivity, stability, and spectroscopic behaviour. In molecular dynamics simulations, mathematical models are used to predict the motion of individual molecules and their interactions over time. This allows us to understand the behaviour of chemical systems under different conditions, such as temperature, pressure, and solvent effects.

Thus application of mathematics plays a crucial role in advancing our understanding of chemical phenomena and the behaviour of molecules and chemical reactions. The application of mathematical techniques provides a quantitative and predictive approach to the study of chemistry, which is otherwise limited by the complexity of chemical systems and their interactions.

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Role of Mathematics in Economics: An Analysis

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Abstract

Each and Every Practical subject is incomplete without the mathematics because calculation is important part of every practical subject. Mathematics plays very important role in the Subject Economics. Business mathematics is the part of Economics and Commerce. Mathematics is used in Economics for the calculation of various concepts i.e. revenue, demand, profit etc. Mathematics is used at the undergraduate and Postgraduate level in different parts of Economics. Mathematical Economics is used for clarifying the various concept of economics. Many concepts of Mathematics like Differentiation, Set theory, Metrics, integration, Linear programming, game theory, Difference Equation, functions, limits are very useful in different economics concepts. This paper shows the uses of concepts of Mathematics in Economics. It is a descriptive study which shows the relation of Mathematics and Economics.

Key words Economics, Mathematics, Mathematical Concepts

Objective To Study the use of mathematics concepts in Economics

Methodology

The use of different mathematical tools in economic analyses are explained with the help of books and journals.

Introduction

Mathematics helps the systematic study in Economics. Economists uses the mathematics to determine the risk or outcome of an investment and production. For example, a investor calculate the returns through the rate of interest and marginal efficiency of capital and both are mathematical concepts can be calculated through the mathematical formulas. Economists use their math skills to find ways to save money, even in counter-intuitive ways. Economists also use math to determine a business & risk, long-term success, even when some factors are unpredictable. For instance, an economist working for an airline uses statistical forecasting to determine the price of fuel two months from now. The company uses this data to lock in fuel prices, or to hedge fuel.

The application of mathematical techniques to the analyses of economic problems is a methodology possibility. This technique called Mathematical Economics. The Mathematical Economics is the application of mathematical methods to represent theories and analytical problems in economics.

Economists uses numbers and graphs, and will be using equations to solve for either of the variables. Mathematics is starting to mingle with the economic concepts and helps us actually understand better what the theory states. So you need the fundamentals in algebra, geometry, and calculus all brushed up for starters, and then linear programming and matrices, vectors, and sets for others.

The simple linear equation (since it is a straight line) for the demand curve is $q=a-bp$, where q is quantity, p is price, and a and b are constants. The relation between quantity demanded at various prices being an inverse one implies the line has a negative slope. It can be proved through mathematics.

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As you move to further related topics, say market demand curves (summation of individual demand curves) or change in demand or calculating the elasticity of demand, each concept is corroborated with mathematical examples. One definitely needs clarity on solving for those to grasp these fundamental economic concepts.

Probably if you are fairly confident about your knowledge of statistics and statistical tools, that too will help a lot in studying as well as applying Economics. Whether it is microeconomics, production systems, economics growth, macroeconomics, it is hard to explain as well as understand the theory without the use of mathematics.

Applications of Mathematics in Economics

Differentiation – The Concept of derivative is very useful in economics. If we want to calculate the effect of independent variable on dependent variable then the concept of derivative is uses in economics. Derivative is used for find the slope of demand curve, slope of supply curve, elasticity of demand, elasticity of supply, revenue function, cost function, partial elasticity of demand, slope of indifference curve, slope of iso-quant, solve the problem of maximization and minimisation. The concept of derivative is used in all these concepts. for example if the TR, TC given then the MR and MC can be calculated with the help of derivatives.

Functions

The concept of function has specially significance of economics and business. Some of the important functions widely used in economics are as Demand function- Demand functions shows the relationship between demand and price the demand function states that other things being equal the demand of a commodity is inversely related to its price it may be written as D= F(P). Supply function shows the relationship between quantities supplied of a commodity and its price. Total cost function shows the relationship between cost and output production function.

Integration

In integration the rate of change is given then we find the function. In economics the concept of integration is used in various concepts like to find out producer surplus, consumer surplus, the producer surplus and consumer surplus can be calculated through integration. If the concept of marginal revenue and marginal cost is given then the total revenue and total cost can be calculated through integration. The definite integration is used to find out the area under the curve. So it is clear that integration plays an important role to solve the problems in economics.

Linear Programming

Linear programming is a technique of decision making mostly used in business industry and in various other fields. Some of the applications of LP as

1. Diet problem - To determine the minimum requirement of Nutrition subject to availability of fruits and their prices, 2. Manufacturing problem - To find the number of items of each type that should be manufactured so as to maximize the profit subject to production district imposed by limitations of the use of machinery and labour. 3. Transport problem- To find the list costly way of transporting shipments from the warehouse is to customers. 4. Blending problems- To determine the optimum amount of several cost to use in producing a set top products while determining the optimum quantity of is product is produced assembling problem to have the list combination of basic component to produce goods according to certain specification and many other uses in production to solve production problem.

Difference Equations

An equestion that shows the relation between the independent variable and the dependent variable and its finite differences called difference equation. Difference equation are widely used in economics to determine the conditions of dynamic stability in economic models such as Harrod-Domar model and income determination model. the multiplier-accelerator interaction model of trade cycle given by samuelson and hicks can be better understood with the help of difference equations.

Metrics

Matrices is a important concept of mathematics and is very useful in economics to solve the problem of input-output. W.W.Leontief gave the concept of input-output analysis and used the concept of matrix to solve the various problems in two sector or three sector economy.

Mathematicians and Economists

Analysis and study in economics help explain the interdependent relation between different variables. They try to explain what causes a rise in prices or unemployment or inflation. Mathematical functions are modes through which these real-life phenomena are made more understandable and logical.

Conclusion

It is proved from the above discussion that mathematics plays a vital role in economic concepts. Mathematical Economics plays an important role to solve the economic problems. Business Mathematics is used in economics and commerce. So it can be concluded that Mathematics plays an important role in Economics. It is interesting to know that a number of economists have been awarded the Nobel Prize for their application of mathematics to economics, including the first one awarded in 1969 to Ragnar Frisch and Jan Tinbergen. Leonid Kantorovich won a Nobel prize in 1975 in economics, and he was a mathematician.

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Mathematics and Personality Development: A Study

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Abstract

Mathematics can contribute to personality development in several ways. The research paper deals with how Mathematics helps to develop personality traits like critical thinking and problem-solving skills, build confidence and develop perseverance and patience. When working on mathematical problems, individuals must think logically and systematically to come up with a solution. These skills are not only valuable in mathematical contexts, but also in many real-world situations. Mathematics can help to build confidence and self-esteem. When someone succeeds in solving a difficult mathematical problem, they gain a sense of achievement and can feel proud of their accomplishment. This can increase their confidence and motivation to tackle other challenges, both inside and outside of Mathematics. Mathematics can also help individuals develop perseverance and patience. Mathematics often requires persistence and determination to solve complex problems. When someone keeps working at a problem and finally finds the solution, they can see the value of perseverance and understand the importance of not giving up in the face of difficulty.

Key words: Personality development, Critical thinking, Problem solving skills, Personality traits

Introduction

Mathematics is a fundamental subject that has been taught in schools and universities for centuries. While its primary aim is to provide students with the skills to solve numerical and abstract problems, the subject can also have a profound impact on a person's overall development. In this paper, we will explore the ways in which Mathematics can contribute to personality development and examine the evidence to support this claim.

Critical Thinking and Problem-Solving Skills

Mathematics is a subject that is widely recognized for its ability to develop critical thinking and problem-solving skills. Here are some ways in which Mathematics helps to develop these skills:

Analysing and interpreting information: Mathematics involves working with data, equations, and formulas. To effectively work with this information, one must be able to analyse and interpret it critically. This requires the ability to identify patterns, extract meaning, and draw conclusions based on evidence.

Identifying and defining problems: Mathematics often involves identifying and defining complex problems, which is the first step in solving them. To do this, one must be able to break down complex problems into smaller, more manageable parts and identify the underlying issues that need to be addressed.

Applying problem-solving strategies: Mathematics requires the use of a variety of problem-solving strategies, such as trial and error, pattern recognition, and deductive reasoning. These strategies can be applied to a wide range of problems in other areas of life

Thus, one of the primary ways that Mathematics can contribute to personality development is through the development of critical thinking and problem-solving skills. Mathematics involves working through logical and systematic processes to arrive at a solution, and these skills are not only valuable in mathematical contexts, but also in many real-world situations. When working on mathematical problems, individuals must analyse the information provided, consider different possible solutions, and make decisions based on their understanding of the problem. These skills can be applied to a range of problems and situations, from decision-making at work to problem-solving in relationships.

Perseverance and Patience

Mathematics can help individuals develop perseverance and patience. Mathematics often requires persistence and determination to solve complex problems, and when someone keeps working at a problem and finally finds the solution, they can see the

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value of perseverance and understand the importance of not giving up in the face of difficulty. This type of experience can translate into other areas of life and help individuals develop a more resilient and persistent approach to challenges and obstacles.

Another study by the University of Illinois found that students who took advanced Mathematics courses reported higher levels of perseverance and patience compared to students who had not taken these courses. The study found that these students were more likely to persist in the face of difficult tasks and were more likely to approach problems with a long-term perspective.

Smart Thinking

Mathematics can promote smart thinking and help individuals develop a more analytical and systematic approach to problem-solving. By working through mathematical problems, individuals must analyse information, consider different possible solutions, and make decisions based on their understanding of the problem. These skills can be applied to a range of problems and situations, from decision-making at work to problem-solving in relationships. Furthermore, by constantly challenging one's mind to think logically and systematically, Mathematics can help individuals develop cognitive skills that can improve their overall ability to think critically and solve problems.

Additionally, Mathematics can also improve reasoning skills and help individuals develop a more rigorous and logical approach to thinking. Through exposure to mathematical concepts such as proofs and deductive reasoning, individuals can develop a better understanding of how to make logical conclusions based on evidence and argument. These skills can be applied to a wide range of areas and can help individuals make more informed and effective decisions in their personal and professional lives.

In conclusion, Mathematics can play a significant role in promoting smart thinking and helping individuals develop a more analytical and systematic approach to problem-solving. Through exposure to mathematical concepts and problem-solving exercises, individuals can develop valuable critical thinking skills that can benefit them in many aspects of their lives

Logical thinking

Mathematics is widely recognized as a subject that requires and develops logical thinking skills. Here are a few ways in which Mathematics helps to develop logical thinking:

Precise definitions and concepts: Mathematics is built on a foundation of precise definitions and concepts. Understanding these concepts requires a high degree of clarity and logical thinking, as one must be able to reason through complex ideas and understand the relationships between different concepts

Use of logic and reasoning: Mathematics often requires the use of logical and deductive reasoning. In order to solve mathematical problems, one must be able to identify patterns, make connections between different ideas, and use logical reasoning to draw conclusions.

Problem-solving skills: Mathematics requires strong problem-solving skills, which often involve the use of logical thinking to identify and eliminate incorrect solutions. The process of working through complex problems can help individuals develop a logical approach to problem-solving that can be applied to other areas of life.

Abstract thinking: Mathematics often involves working with abstract concepts and structures that do not exist in the physical world. This requires a high degree of abstract thinking and the ability to reason through complex ideas and visualize relationships between different concepts.

Use of mathematical language: Mathematics has its own unique language and symbols that are used to represent concepts and ideas. This language requires a high degree of logical thinking and precision, as one must be able to use these symbols to accurately and concisely represent complex mathematical ideas.

Overall, Mathematics provides a rich environment for developing logical thinking skills. By requiring the use of logic and reasoning, problem-solving skills, abstract thinking, and mathematical language, Mathematics can help individuals develop a logical approach to problem-solving that can be applied to many other areas of life..

Building Confidence and Self-Esteem

By successfully solving mathematical problems, individuals can develop a sense of accomplishment and increase their self-confidence. This can help them approach new challenges with a more positive attitude and improve their overall well-being.

Mathematics can develop confidence as a personality trait in several ways. Here are a few examples:

Mastery of a challenging subject: Mathematics is a notoriously difficult subject, and mastery of it requires a great deal of hard work and dedication. Successfully mastering challenging mathematical concepts can lead to a sense of accomplishment and boost self-confidence.

Problem-solving skills: Mathematics involves solving complex problems that require critical thinking, persistence, and creativity. Developing strong problem-solving skills can translate into greater confidence in other areas of life, as individuals become better equipped to tackle challenges and overcome obstacles.

Clear understanding of concepts: Mathematics often requires a clear and precise understanding of concepts and definitions. Developing a deep understanding of mathematical concepts can help individuals feel more confident in their knowledge and abilities, as they are able to articulate complex ideas and explanations.

Practice and repetition: Mathematics requires a great deal of practice and repetition in order to become proficient. The process of working through problems and practicing mathematical techniques can help build confidence and mastery of the subject.

Feedback and evaluation: Mathematics often involves clear feedback and evaluation, such as the correct or incorrect answers to problems. This can provide individuals with a clear sense of progress and achievement, leading to greater confidence in their abilities.

Overall, Mathematics can be an excellent tool for developing confidence as a personality trait. By providing opportunities for mastery, problem-solving, and clear feedback, Mathematics can help individuals build confidence in their abilities and increase their sense of self-efficacy.

Creativity

Mathematics is often thought of as a rigid and structured field that is focused on precise rules and formulas. However, Mathematics can actually be a highly creative discipline that fosters creativity as a personality trait in several ways.

Firstly, Mathematics involves solving complex problems and discovering new insights and patterns. This requires a great deal of creativity and ingenuity, as mathematicians must come up with novel approaches to tackle difficult problems and find new connections between seemingly unrelated ideas.

Secondly, Mathematics often involves exploring abstract concepts that require imagination and visual thinking. Mathematicians may work with shapes and geometries that do not exist in the physical world, or develop models and simulations to explore complex systems. This requires the ability to think outside the box and imagine new possibilities.

Finally, Mathematics can also foster creativity by encouraging experimentation and risk-taking. Mathematicians are often encouraged to explore new ideas and take chances in their work, even if the outcome is uncertain. This willingness to take risks and try new things can help to develop a creative mind-set that is open to new possibilities and unafraid of failure.

Conclusion

In conclusion, Mathematics can play an important role in personality development by helping individuals develop critical thinking and problem-solving skills, build confidence and self-esteem, and develop perseverance and patience. While the primary aim of Mathematics education is to provide students with the skills to solve numerical and abstract problems, the subject can also have a profound impact on a person's overall development. Further research is needed to fully understand the relationship between Mathematics and personality development, but the evidence suggests that Mathematics can be a valuable tool in promoting a person's overall growth and development.

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A Critical Look at the Implication of PLS-SEM in Existing Literature

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Abstract

In behavioural research studies, Partial Least Square SEM plays a vital role and is gaining momentum continuously. The present study aims to understand the concept of PLS-SEM and its implications. For this purpose, reviews of the available literature related to PLS-SEM has been done to understand areas of application of PLS-SEM. The findings revealed that PLS- SEM is mostly useful for non-normal data, small sample size and reflective or formative to analyse behavioural studies.

Keywords: Structural Equation Modelling, PLS-SEM, Partial Least Square, Behavioural studies

Introduction

Applications of Structural Equation modelling, that is, SEM is in existence since many years back. SEM helps to understand relationship between constructs and its latent variables in a structural path model. It includes analysis of measurement properties as well as structural model, simultaneously (Muthusamy, 2011). For this, it involves use of exploratory factor analysis as well as analysis of structural path model. Earlier, main Co-variance based Structural Equation Modelling (CB-SEM) was the main focus of researchers. However, CB-SEM losing its popularity due to some assumptions like normality of data, unsuitable for small sample size. Partial Least Square is substitute for the same. With minimum requirements regarding sample size, non-normality of data, use of PLS SEM is gaining momentum day by day in various fields of study such as business, marketing, engineering, psychology etc. It is a multivariate data analysis technique that helps to identify cause and effect model or behavioural relationship in the observed and latent variables in the structural path model of the study.

The main aim of this is to understand the concept of PLS-SEM and its implications. First section deals with introduction. Section two provides detailed understanding by reviewing available literature which is, lastly, followed by conclusion.

Literature Review

PLS-SEM helps to analyse complex structural path model and understand the concept of observed and latent variable. This section reviews the existing literature available in the field of structural equation modelling.

Selection of PLS-SEM as Data Analysis Technique

Lowry and Gaskin (2014) discussed two generations of data analysis techniques. The first generation (1G) techniques analysis the casual relationship using correlation, regression and hypothesis test (such as z-test, f-test, t-test). It identifies only the casual relationship among variables of the study. In other words, it studies the change in dependent variable due to change in independent variable. Moreover, it fails in the field of behavioural research. Therefore, use of second generation (2G) techniques, namely, SEM have proved its superiority over the former (Chin, 1998). Here, various paths exist between exogenous and endogenous constructs and each path depicts a different proposed relationship of the latent variables based on theoretical assumptions. CB-SEM and PLS-SEM are two forms of SEM (Lowry and Gaskin, 2014). CB-SEM is based on some assumptions related to normal distribution of data, reflective constructs and large sample size. Further, it is confirmatory in nature and requires pre theoretical background to support model (Lowry & Gaskin, 2014). However, PLS-SEM acts as a substitute when any of the assumptions of CB-SEM is not fulfilled (Hair, Sarstedt, & Ringle, 2012). PLS-SEM was developed by Herman Wold in the 1996 (Chin, 1998). It is component based structural equation modeling and preferred over CB-SEM due to minimum requirement regarding measurement scale and sample size (Monecke & Leisch, 2012; Kummer, 2013). It can be used with both formative as well as reflective indicators (Lowry & Gaskin, 2014). It aims at maximising the explained variance. There is no need of empirical support for testing a theory thus is preferred to explore theoretical relationship among the constructs (Peng & Lai, 2012). On the same track, Wetzels, Odekerken-Schroder, and Oppen (2009) commented that it is more applicable for prediction based study and new approach which lacks of strong theoretical framework. Moreover, it can be used for confirmatory studies also or to confirm a theory

(Barroso, Carri´on, & Rold´an, 2010). Further, it has been successfully used study of behavioural intention by previous research namely, Jayasingh and Eze (2009); Muthusamy (2011).

Sample Size Requirement

PLS-SEM does not require large sample size. But on the other hand, researchers should consider that analysis with very small sample would lead to over estimation of outer loading and under estimation of structural paths. So, sample size needs to be determined carefully. In this context, 'the ten times rule method' has been widely used by previous research studies i.e. ten times of the maximum number of path pointing at any latent construct in inner or outer model (Kock & Hadaya, 2016). Chin (1998) suggested that sample size should be ten times of the largest of the following:

a) largest number of formative indicators (i.e. largest measurement equation)

b) largest number of independent latent variable predicting a particular dependent variable.

Muthusamy (2011) has used the same rule to determine sample size for PLS-SEM analysis. Moreover, Chin, Marcolin, & Newsted (1996) opined the requirement of sample size of 100 approximately in a study consisting of six to eight indicators.

Model Fitness

PLS-SEM includes analysis of two models, namely, structural model (inner model) and measurement model (outer model). There is not global index for evaluation of PLS model (Monecke & Leisch, 2012; Garson, 2016). So, it is necessary to assess fitness of both the models separately. The correctness of both the measurement and structural models results in fitness of the final research model and provide better estimation of model parameters (Chin, 1998). Assessment of measurement model ensures validity and reliability of the proposed research model whereas structural model aims to assess significance of path coefficient and explanatory power of the exogenous constructs.

Assessment of Measurement Model

Measurement model measures whether variables are representative of related constructs or not. It is outer model and shows relationship between observed item(s) and respective latent variable(s) (Chin, 1998). One observed item must relate with single latent variable only otherwise assessment of structural relationship will be of no use. Study of measurement model depends upon nature of model either reflective or formative. In reflective construct model, observed indicators are the effect of the latent variable (Lowry & Gaskin, 2014) and arrow goes from latent to manifest variable. Whereas, in formative construct model, the relationship is reversed i.e. latent variable is considered as the effect of manifest variable, resulting arrow goes from manifest variable to latent variable.

Convergent Validity

It states that items related to a construct should be high loaded on the related construct instead other. item reliability, internal consistency and average variance extracted (AVE) are used to check convergent validity, namely, (Muthusamy, 2011). Among these techniques, item reliability includes investigation of individual item loadings (Kummer, 2013). Item loading shows the amount of variance in indicators explained by latent construct (Chin, 1998) and poor loading depicts unreliable nature of the item or excessive influence of different factors on the item. Most of the researchers, namely, Peng and Lai (2012); Barroso et al. (2010) have described minimum item loading value of 0.7. Whereas, Neil (2008); Matsunaga (2010); Hamid, Sami, and Sidek (2017) stated a minimum threshold value of 0.4 for item loading to retain an item. Further, Hair et al. (1998) stated that loading value of 0.3 is also acceptable.

Further, Internal consistency measures the consistency of the construct in measurement of same results every time. Poor consistency represents multidimensionality of the factor. Composite reliability or Cronbach's Alpha may be used to evaluate internal consistency. Composite reliability is based on the actual factor loadings whereas Cronbach's alpha adopts equal weighing approach (Chin, 1998). Composite reliability overcomes deficiencies of Cronbach's alpha and is improved form of it (Hair et al., 2012; Muthusamy, 2011). Previous research studies opined that composite reliability value of more than 0.7 shows good level of internal consistency (Hair, Sarstedt, Hopkins, & Kuppelwieser, 2014).

AVE, the third indicator to ensure convergent validity, shows the amount of variance in the item which is explained by related construct. The thumb rule specified for AVE is 0.5 by previous research studies, namely, Fornell and Larcker (1981); Hair et al. (2014); Chin (1998) and Gye-Soo (2016). If any construct has lower AVE value then related item should be removed according to item loading as lowest loading item has been removed first.

Discriminant Validity

After confirmation of convergent validity, one must ensure presence of discriminant validity. It measures the extent to which one construct differs from each other. In other words, one item should be highly related with its own construct in comparison with other. It ensures that results of hypothesised relationships in the research model are real. In order to check discriminant validity, Fornell-Larcker criterion and cross-loading need to be evaluated.

Fornell-Larcker criterion states that a construct shares more variance with its related construct rather any other latent variable (Urbach & Ahlemann, 2010). In the analysis table of Fornell-Larcker, diagonal elements show square root of AVE and lower diagonal elements represent correlation among different latent variables. To ensure discriminant validity, square root of AVE should be more than lower diagonal elements (i.e. correlation with other latent variable should not be high).

The second criterion to ensure discriminant validity is assessment of cross loading. It is also called item level discriminant validity (Henseler, Ringle, & Sarstedt, 2015). It indicates that an item should carry high loading value with that construct only which it purports to measure and cross loading should be less with other construct. Moreover, there should be minimum difference of 0.2 in the loading of item with its related and other construct.

Structural Model Evaluation

After validation of measurement model, there is need to assess structural model in order to assess hypothesised relationship. Structure model is inner model which depicts path among latent variables. It measures predictive ability and relationship among constructs (Duarte & Raposo, 2010). The structural model validity can be assessed by using coefficient of determination (r^2), effect size (f^2), Stone Geisser's Q^2 and path coefficients and the related level of significance. Coefficient of determination shows the explanatory power of exogenous constructs i.e. proportion of variance of endogenous construct that is accounted by exogenous construct. R^2 ranges from 0 to 1. The more it is closer to 1 the more it depicts predictive power of independent variable. However, higher value does not ensure presence of real causal impact (Moksony, 1990). Further, effect size (f^2) measures changes in r^2 in order to understand practical impact of IDV over DV. In interpretation of f^2 for structural model, it has been suggested that when f^2 value is 0.35 then effect size is large, effect size is medium with f^2 value =0.15 and small if f^2 = 0.02 (Chin, 1998). Further, Stone and Geisser's Q^2 measures predictive relevance of endogenous construct by exogenous construct. It can be computed in two ways either construct cross validated and construct cross validated redundancy. Hair et al. (2011) suggests use of construct cross validated redundancy.

Conclusion

In the present, detailed analysis of PLS-SEM has been done. Discussion about sampling requirement, model fitness and process of doing structural analysis using partial least square approach has been discussed. No doubt application of PLS-SEM is continuously increasing, present would guide future researchers to understand the concept of PLS based SEM. It would contribute significantly to existing literature in the area of structural equation modelling. It would provide reference to new researchers in the area of structural equation analysis regarding development and application of theory. As the present study reviews available literature across different nations which gives a multinational context and makes it universal.

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RPT: The Technique for Behavioural Analysis Of Industries

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Abstract

This paper discusses the technique known as Regenerative Point Technique ***(RPT)****, which is used for Behavioral Analysis of various industries having Single Unit and multiple Unit connected in series and parallel configuration.The analysis of various Industrial systems using the 'Regenerative Point Technique* ***(RPT)****' is for determining the parameters such as the Mean Time to System Failure(MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement etc. (under steady state conditions). The complete process is Mathematically formulated using the system has a single unit with priority repair and various parameters such as the Mean Time to System Failure (MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement are evaluated for further discussing the profitability of industry under discussion.*

Key words:-Reliability, Availability, Priority Maintainance, Regenerative point, Regenerative Point Technique **(RPT)**.

Introduction The researchers have discussed important reliability feathers such as steady-state transition probabilities, mean sojourn time, mean time to system failure (MTSF.), reliability, point-wise and steady state availability and expected down time of the system, expected busy period of the expert repairman and profit analysis etc. are obtained by using regenerative point technique and Semi-Markov processes.

Reliability Variables

MTSF

The probabilistic expected life time in which a system reaches a failed state is known as MTSF. Sometimes, it is also known as MTFF. If we consider a failed state as absorbing are then we can calculate time for which once the system enters into the failed state(s) it remains there forever. Let *T* be the survival time of a system and m*f*(t) and M*F*(t) be the pdf and cdf of *T* respectively, then

$$\text{MTSF} \quad = \quad \int_0^\infty t\,\text{mf (t)dt}$$

But

$$\text{m f (t)} \quad = \quad \frac{d}{dt} MF(t) = -\frac{d}{dt}\rho(t)$$

Hence

$$\text{MTSF} \quad = \quad -\int t\,d\rho\,(t)$$

$$= \quad [-t\rho\,(t)]_0^\infty + \int_0^\infty \rho\,(t)\,dt$$

$$= \quad \int_0^\infty \rho\,(t)\,dt$$

Where ρ(t) is the reliability function of the unit at time t

Availability

Availability is an important aspect which includes both reliability as well as maintainability aspects. This measure is defined as the possibility that the unit will be able to work within the capacity at a given time epoch t, subject to the condition that the repair / replacement is permitted.

$$X(t) \quad = \begin{cases} 1 & \textit{If system is operating within tolerance at time } t \\ 0 & \textit{Otherwise} \end{cases}$$

Symbolically point wise availability

$$A(t) = \quad \Pr[X(t) = 1]$$

Mean – Sojourn Time

The possible life time taken by the system in one state before changing to any other state is known as *mean sojourn time*. Let Ti be the time in state Sti then the mean sojourn time in this state is given by

$$\mu_i \quad = \quad \int_0^\infty P_r(T_i > t)\,dt$$

Markov Process A Markov process is a stochastic process whose dynamic behaviour is such that the probability distribution for its future development depends only on the present state and not on how the process arrived in the present state. If the state space is discrete (finite and countably infinite), then the Markov process is called a Markov chain. The parameter of the Markov chain may be discrete or continuous. If the parameter space(Index set) is also discrete then the chain is called discrete parameter Markov chain.

In a Markov-process with state space I={0,1,2,3,….}:

(i) The duration of stay in any state i is exponential.

(ii) Given that it is in state i at a certain epoch of time 't', the next transition will be to a state j, depends only on i and j and not on the history of the process that how the state i was arrived at, prior to time 't'.

Semi-Markov process Many problems in queuing, reliability and inventory theories may be approached through Markov renewal and semi-Markov processes. A semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and the time interval between two successive transitions is a random variable, whose distribution may depend on the state from which the transition takes place as well as on the state to which the next transition take place. In other words, a semi-Markov process is one that changes its state according to a Markov chain but takes a random amount of time between changes and it does not possess the Markovian property that given the present state the future is independent of the past.

Regenerative Points The epochs of time $\{t_n\}$ where a given renewal process {N(t), t≥0} probabilistically starts afresh, forgetting (independent of) past history,are called regenerative points, i.e. {N(t), t≥ t_n} is a probabilistic replica of {N(t), t≥0}. The epochs of time at which the renewals occur, are called regenerative points and the event which is renewed is called regenerative state of the system.

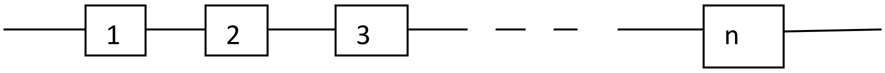
Classification and Configuration of Systems

On the basis of the location/position and arrangement of the components/sub-systems, a system can be categorized in the following ways:

Series-System

It is a system which fails if any of its components fails. The system is operative only if all the components are operative. The components need not be physically connected in series for the system to be called a Series-System. For example, the four tires of an automobile car are in series for the purpose of reliability calculations because the failure of any one makes the car inoperative.

Let a system having n components/units connected in series then a series system may be shown as in following figure



Series-System
Figure 1

If ρ_k (t) denotes the reliability of the k-th component (k = 1, 2, 3… n), then the reliability of the series-system is

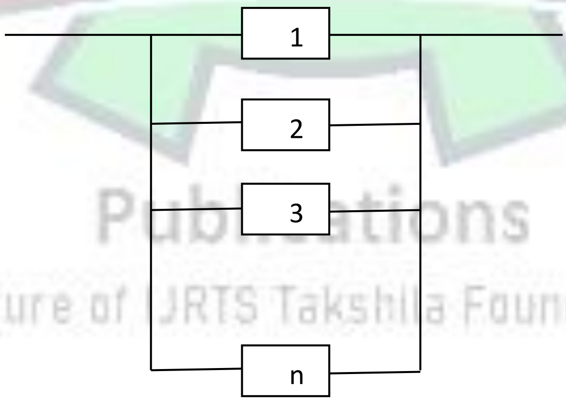
$$\rho_s(t) = \prod_{k=1}^{n} \rho_k(t)$$

A series system is also called a non-redundant system.

Parallel System

It is a system which fails only if all of its units fail. The system is operative even if one of the units is operative. This system is also called a Rope Model, since the system fails when all fibers (units) break. Parallel configuration is often referred to as an active-redundancy. A parallel system having n components may be shown as in Figure 1.2. If ρ_k (t) denotes the reliability of the k-th unit (k=1, 2, 3… n), the reliability of the parallel-system is

$$\rho_p(t) = 1 - \prod_{k=1}^{n} \{1-\rho_i(t)\}$$



Parallel System
Figure 2

Series-Parallel System

It is a system in which there are number of units (called stages) in series configuration and each of these units (stages) is further composed of many units with parallel-configuration. It is a case of low level redundancy. A series-parallel system consisting of m units (stages) in series-configuration (where the i-th stage consists of - identical components each of reliability and are put in the parallel configuration).The reliability (assuming that all components independent) of the series-parallel system is

$$R_{sp}(t) = \prod_{i=1}^{m}[1 - \{1 - R_i(t)\}^{n_i}]$$

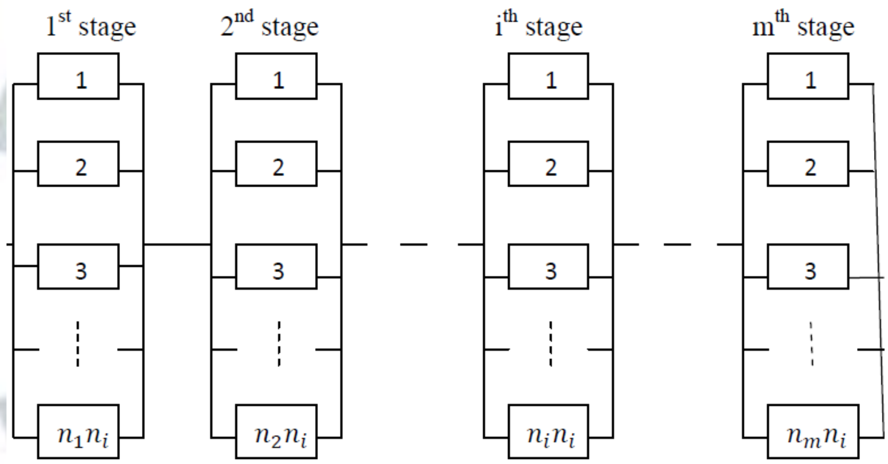


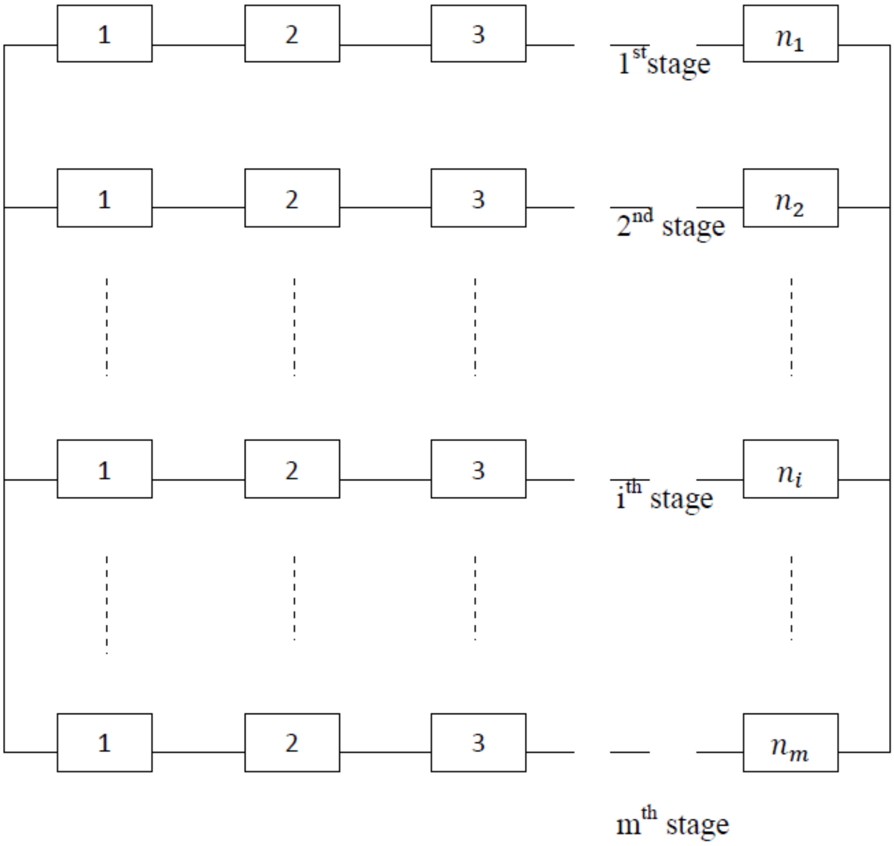
Figure: 3

Parallel-Series System

It is a system in which there are a number of units(branches) in parallel configuration and each of these units (branches) is further composed of many units with series-configuration. It is a case of high level redundancy.

A parallel-series system n components (branches) in parallel-configuration (where i-th unit consists of n_i - identical components each of reliability Ri (t); i= 1, 2, 3… n and are put in the series-configuration) and is shown as in Figure. 3. The reliability of this type of system is

$$R_{ps}(t) = 1 - \prod_{i=1}^{m}[1 - \{R_i(t)\}^{n_i}]$$



Parallel-Series System

The reliability and availability of many stochastic systems and process industries by using very cumbersome and time-consuming techniques. The general formulae in the closed form are not yet developed to determine the key parameters for a semi Markov renewal process like mean time to system failure (MTSF), availability of the system and busy periods of the servers doing different jobs, the number of server's visits and the number of replacements of the components/sub-systems. Tuteja, R.K., Malik, S.C. and many others, have analyzed and discussed various systems under steady state conditions, using the following formulae of the regenerative point technique to find the key parameters of a stochastic system:

a) MTSF $= \lim_{s \to 0} \frac{1-\phi_0^{\sim}(s)}{s}$

b) Availability $= \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.A_0^*(s)$

c) Busy period of the server $= \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s.B_0^*(s)$

d) Expected number of server's visits/replacements $= \lim_{t\to\infty} \frac{V_0(t)}{t} = \lim_{s\to 0} s.V_0^*(s)$

Here, We take an example of the Dairy Plant, which is divided into two unit 'A' and 'B' in which unit 'A' is milk producing unit and other subsidiary products like milk water, Cream, Ghee etc are produced by unit 'B'. As milk is the main demand of market so unit 'A' is the main unit of system, so we try to keep unit 'A' in working as far as possible, therefore we give priority in repair to unit 'A' over unit 'B'. The system is in working state if at least one unit is in working state and fails when both units fail. Nothing can fail when the system is in failed state. When both units are working the system is good otherwise it may be working in reduced state or failed state. If any unit of the system fails the system works in reduced capacity and the failed unit is immediately put under repair. Repairs are perfect i.e. repair does not damage to any part of the system. The repaired unit works like a new one. Further if both 'A' and 'B' units of system are in failed system, then repair to unit 'A' is given priority in repair over 'B'. The distributions of the failure times and repair times are exponential and general respectively and also different for units 'A' and 'B'. These are also assumed to be independent of each other.

Assumptions and Notations

- The system consists of two non – identical units 'A' and 'B'. 'A' is main unit and 'B' unit is subsidiary.
- A single repair facility is available for both units 'A' and 'B'.
- The distributions of failure times and repair times are exponential and general respectively and also different for units 'A' and 'B'. Failures and repairs are statistically independent.
- Repair is perfect i.e. it does not damage any units during repair.
- Repaired units to be good.
- When both units fail then the system is in failed state.
- Nothing can fail when the system is in failed state. After the failure of any one unit the system works in reduced state.

- When both units 'A' and 'B' are in failed state then repairman repair unit 'A' on priority basis.
- The system is discussed for steady-state conditions.
- Upon failure, if main unit 'A' is under repair and unit 'B' also fails it joins the queue of the failed unit.
- Both the units cannot fail simultaneously.

$\lambda/\ \lambda_1$: Constant failure rate of units A/ of unit B.

g(t)/G(t) / $\bar{G}$(t) : Probability density function/ Cumulative distribution function / complement of the repair – time of unit A.

h(t)/H(t) / $\bar{H}$(t) : Probability density function / Cumulative distribution function / complement of the repair time of the unit B.

A/a : Main unit in the operative state / failed state.

B/b : Operative state / failed state.

Transition Diagram:

Considering the above assumptions and notations the Diagram of the system is

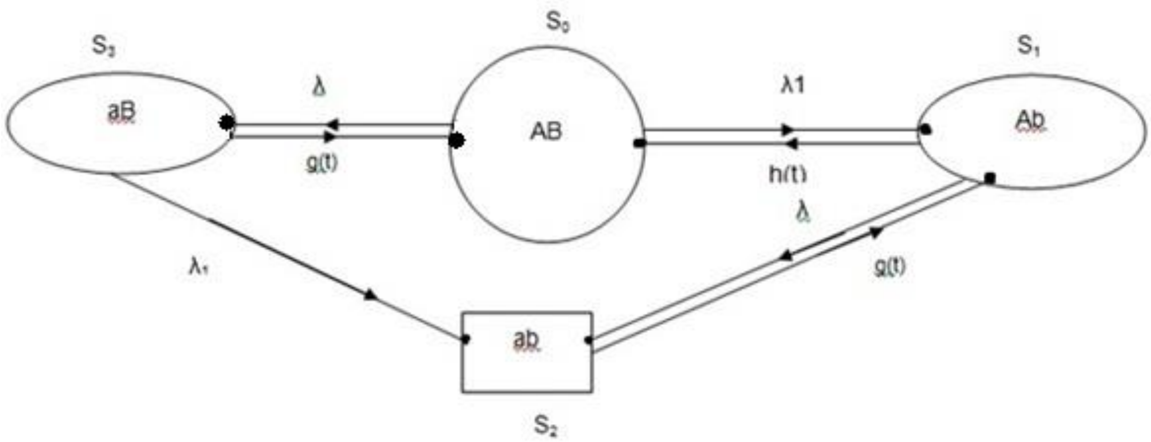


Figure – 4

State	Symbol	Model
Regenerative State/Point	⊙	0-3
Up-state	○	0
Failed State	▭	2
Reduced State	⬭	1,3

Table 1

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Evaluation of Parameters

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying *Regenerative Point Technique**(RPT)*** :

MTSF(π_0): From Fig.4, The mean time for system failure **(π_0)** at a point of time t after the unit entered the regenerative position at t = 0. The recursive relations for □$_0$ (t) are as follows:

$$□_0 (t) = dQ_{01} (t) \ominus □_1 (t)$$

$$□_1 (t) = dQ_{10} (t) \ominus □_0 (t) + dQ_{12} (t)$$

$$□_3 (t) = dQ_{32} (t) + dQ_{30} (t) \ominus □_0 (t)$$

The system of equations can be written in non-homogeneous system of equations and solved by using the Cramer Rule of determinants as explained below:

$$□_0 (t) - dQ_{01} (t) \ominus □_1 (t) = 0$$

$$-dQ_{10} (t) \ominus □_0 (t) + □_1 (t) = dQ_{12} (t)$$

$$-dQ_{30} (t) \ominus □_0 (t) + □_3 (t) = dQ_{32} (t)$$

$$\begin{pmatrix} 1 & -q_{01} & 0 \\ -q_{10} & 1 & 0 \\ -q_{30} & 0 & 1 \end{pmatrix} \begin{pmatrix} □_0 (t) \\ □_2 (t) \\ □_5 (t) \end{pmatrix} = \begin{pmatrix} 0 \\ q_{12} \\ q_{32} \end{pmatrix}$$

$$D(s) = \begin{pmatrix} 1 & -q_{01} & 0 \\ -q_{10} & 1 & 0 \\ -q_{30} & 0 & 1 \end{pmatrix}$$

$$= 1 - q_{01}q_{10}$$

$$D(0) = 1 - p_{01}p_{10}$$

$$N(s) = \begin{pmatrix} 0 & -q_{02} & 0 \\ q_{12} & 1 & 0 \\ q_{32} & 0 & 1 \end{pmatrix}$$

$$= q_{12}q_{02}$$

$$N(0) = p_{12}p_{02}$$

Taking Laplacian Stieltjes Transforms for □$_0^{**}$(s), we get

$$□_0^{**}(s) = \frac{N(s)}{D(s)}$$

$$\text{MTSF} = □_0 = \lim_{s \to 0} \frac{1 - \pi_0^{**}(s)}{s} = \frac{D'(0) - N'(0)}{D(0)}$$

$$= N \div D$$

$$= \frac{\mu_0 + p_{01}\mu_1}{1 - p_{01}p_{10}}$$

Where,

$$D'(s) = -q_{01}q'_{10} - q_{10}q'_{01}$$

$$D'(0) = p_{01}m_{10} + p_{10}m_{01}$$

$$N'(s) = q_{01}q'_{12} + q_{12}q'_{01}$$

$$N'(0) = -p_{01}m_{12} - p_{12}m_{01}$$

Availability of the system: From Fig.4, Let Av_i (t) be the probability that a system is available at a point of time t after the unit entered the regenerative position at t = 0. The recursive relations for Av_i (t) are as follows:

$$Av_0(t) = R_0(t) + Q_{01}(t) © Av_1(t)$$

$$Av_1(t) = R_1(t) + Q_{10}(t) © Av_0(t) + Q_{12}(t) © Av_2(t)$$

$$Av_2(t) = Q_{23}(t) © Av_3(t)$$

$$Av_3\,(t) = R_3\,(t) + Q_{30}\,(t) \circledcirc Av_0\,(t) + Q_{32}\,(t) \circledcirc Av_2\,(t)$$

The system of equations can be written in non-homogeneous system of equations and solved by using the Cramer Rule of determinants as explained below:

$$Av_0\,(t) - Q_{01}\,(t) \circledcirc Av_1\,(t) = R_0\,(t)$$

$$-Q_{10}\,(t) \circledcirc Av_0\,(t) + Av_1\,(t) - Q_{12}\,(t) \circledcirc Av_2\,(t) = R_1(t)$$

$$Av_2\,(t) - Q_{23}\,(t) \circledcirc Av_3\,(t) = 0$$

$$-\,Q_{30}\,(t) \circledcirc Av_0\,(t) - Q_{32}\,(t) \circledcirc Av_2\,(t) + Av_3\,(t) = R_3\,(t)$$

$$\begin{pmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{pmatrix} \begin{pmatrix} Av_0 \\ Av_1 \\ Av_2 \\ Av_3 \end{pmatrix} = \begin{pmatrix} R_0 \\ R_1 \\ 0 \\ R_3 \end{pmatrix}$$

$$D_1(s) = \begin{pmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{pmatrix}$$

$$N_1(s) = \begin{pmatrix} R_0 & -q_{01} & 0 & 0 \\ R_1 & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ R_3 & 0 & -q_{32} & 1 \end{pmatrix}$$

$$D_1(s) = (1- q_{10}\, q_{01})(1- q_{23}\, q_{32})- q_{01}\, q_{12}\, q_{23}\, q_{30}$$

$D_1(0) = (1- p_{10}\, p_{01})(1- p_{23}\, p_{32}) - p_{01}\, p_{12}\, p_{23}\, p_{30} = 0$

Now taking Laplacian Transforms for $Av_0^*(s)$, we get

$$Av_0^*(s) \quad = \quad \frac{N_1(s)}{D_1(s)}$$

Availability in the long run is as follows;

$$Av_0 \quad = \quad \lim_{s \to 0}(sAv_0^*(s)) = \frac{N_1(0)}{D_1'(0)}$$

Here,

$N_1(s) = (R_0 + R_1 q_{10})(1-q_{23}q_{32}) + q_{01}R_3q_{23}q_{12}$

$N_1(0) = (\mu_0 + \mu_1 p_{10})(1-p_{23}p_{32}) + p_{01}\mu_3 p_{23}p_{12}$

$D_1'(s) = (1-q_{10})(-q'_{32} - q'_{23}q_{32}) + (1-q_{32})(-q'_{10} - q'_{01}q_{10}) + q_{12}\, q_{30}(-q'_{01} - q'_{23}) - q_{30}\, q'_{12} - q_{12}\, q'_{30}$

$D_1'(0) = (1-p_{10})(m_{32} + m_{23}p_{32}) + (1-p_{32})(m_{10} + m_{01}p_{10}) + p_{12}\, p_{30}(m_{01} + m_{23}) + p_{30}m_{12} + p_{12}m_{30}$

$N_1(0) = [(1- p_{32})(\mu_0 + \mu_1) + p_{12}\mu_3]$

$D'_1(0) = [(1-p_{32})(\mu_0+\mu_1) + p_{12}(\mu_2 + \mu_3)]$

Busy period of the Server: From Fig.4, Let $Bp_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state at t=0. The recursive relations for $Bp_i(t)$ is as follows;

$Bp_0(t) = q_{01}(t) © Bp_1(t)$

$Bp_1(t) = W_1(t) + q_{10}(t) © Bp_0(t) + q_{12}(t) © Bp_2(t)$

$Bp_2(t) = W_2(t) + q_{23}(t) © Bp_3(t)$

$Bp_3(t) = W_3(t) + q_{30}(t) © Bp_0(t) + q_{32}(t) © Bp_2(t)$

Here,

$W_i(t) = R_i(t)$

The system of equations can be written in non-homogeneous system of equations and solved by using the Cramer Rule of determinants as explained below:

$$Bp_0\,(t) - Q_{01}\,(t) © Bp_1\,(t) = 0$$

$$-Q_{10}\,(t) © Bp_0\,(t) + Bp_1\,(t) - Q_{12}\,(t) © Bp_2\,(t) = W_1(t)$$

$$Bp_2\,(t) - Q_{23}\,(t) © Bp_3\,(t) = W_2\,(t)$$

$$- Q_{30}\,(t) © Bp_0\,(t) - Q_{32}\,(t) © Bp_2\,(t) + Bp_3\,(t) = W_3\,(t)$$

$$\begin{pmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{pmatrix} \begin{pmatrix} Bp_0 \\ Bp_1 \\ Bp_2 \\ Bp_3 \end{pmatrix} = \begin{pmatrix} 0 \\ W_1 \\ W_2 \\ W_3 \end{pmatrix}$$

$$D_1(s) = \begin{pmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{pmatrix}$$

$$N_1(s) = \begin{pmatrix} 0 & -q_{01} & 0 & 0 \\ W_1 & 1 & -q_{12} & 0 \\ W_2 & 0 & 1 & -q_{23} \\ W_3 & 0 & -q_{32} & 1 \end{pmatrix}$$

$$D_1(s) = (1- q_{10}\, q_{01})(1- q_{23}\, q_{32})- q_{01}\, q_{12}\, q_{23}\, q_{30}$$

$$D_1(0) = (1- p_{10}\, p_{01})(1- p_{23}\, p_{32})- p_{01}\, p_{12}\, p_{23}\, p_{30} = 0$$

Taking Laplacian Transforms for $Bp_0^*(s)$, we have

$$Bp_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

In the steady state

$$Bp_0 = \lim_{s\to 0} sBp^*(s) = \lim_{s\to 0}\frac{sN_2(s)}{D_1(s)} = \frac{N_2(0)}{D_1'(0)}$$

Here

$N_2(s) = q_{01}\{W_1(1-q_{23}q_{32}) + q_{12}(W_2 + W_3q_{23})\}$

$N_2(0) = p_{01}\{\mu_1(1-p_{23}p_{32}) + p_{12}(\mu_2 + \mu_3p_{23})\}$

$D_1'(s) = (1-q_{10})(-q'_{32} - q'_{23}q_{32}) + (1-q_{32})(-q'_{10} - q'_{01}q_{10}) + q_{12}q_{30}(-q'_{01} - q'_{23}) - q_{30}q'_{12} - q_{12}q'_{30}$

$D_1'(0) = (1-p_{10})(m_{32} + m_{23}p_{32}) + (1-p_{32})(m_{10} + m_{01}p_{10}) + p_{12}p_{30}(m_{01} + m_{23}) + p_{30}m_{12} + p_{12}m_{30}$

$N_2(0) = [(1-p_{32})\mu_1 + p_{12}(\mu_2 + \mu_3)]$

$D'_1(0) = [(1-p_{32})(\mu_0+\mu_1) + p_{12}(\mu_2 + \mu_3)]$

Expected number of Server's visits: Let $V_0(t)$ be the expected number of visits of the server starting at a point at any instant 't' given that the system entered regenerative state at t=0. The recursive relations for $V_0(t)$ is as follows;

$V_0(t) = q_{01}(t) © V_1(t)$

$V_1(t) = X_1(t) + q_{10}(t) © V_0(t) + q_{12}(t) © V_2(t)$

$V_2(t) = q_{23}(t) © V_3(t)$

$V_3(t) = q_{30}(t) © V_0(t) + q_{32}(t) © V_2(t)$

The system of equations can be written in non-homogeneous system of equations and solved by using the Cramer Rule of determinants as explained below:

$V_0(t) - Q_{01}(t) © V_1(t) = 0$

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$-Q_{10}(t) © V_0(t) + V_1(t) - Q_{12}(t) © V_2(t) = X_1(t)$

$V_2(t) - Q_{23}(t) © V_3(t) = 0$

$-Q_{30}(t) © V_0(t) - Q_{32}(t) © V_2(t) + V_3(t) = 0$

$$\begin{pmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ X_1 \\ 0 \\ 0 \end{pmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -q_{01} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ -q_{30} & 0 & -q_{32} & 1 \end{vmatrix}$$

$$N_1(s) = \begin{vmatrix} 0 & -q_{01} & 0 & 0 \\ X_1 & 1 & -q_{12} & 0 \\ 0 & 0 & 1 & -q_{23} \\ 0 & 0 & -q_{32} & 1 \end{vmatrix}$$

$D_1(s) = (1- q_{10}\, q_{01})(1- q_{23}\, q_{32})- q_{01}\, q_{12}\, q_{23}\, q_{30}$

$D_1(0) = (1- p_{10}\, p_{01})(1- p_{23}\, p_{32})- p_{01}\, p_{12}\, p_{23}\, p_{30} = 0$

Now taking Laplacian Transforms for $V_0^*(s)$, we get

$$V_0^*(s) = \frac{N_3(s)}{D'_1(s)}$$

The expected number of visits by the server per unit time is given by

$$V_0 = \lim_{s\to 0} sV_0^*(s) = \frac{N_3(0)}{D'_1(0)}$$

Here

$N_3(s) = q_{01} X_1 (1-q_{23}q_{32})$

$N_3(0) = p_{01}(1-p_{23}p_{32})$

$D_1'(s) = (1-q_{10})(-q'_{32} - q'_{23}q_{32}) + (1-q_{32})(-q'_{10} - q'_{01}q_{10}) +$

$q_{12}q_{30}(-q'_{01} - q'_{23}) - q_{30}q'_{12} - q_{12}q'_{30}$

$D_1'(0) = (1-p_{10})(m_{32} + m_{23}p_{32}) + (1-p_{32})(m_{10} + m_{01}p_{10}) + p_{12}p_{30}(m_{01} + m_{23}) +$

$p_{30}m_{12} + p_{12}m_{30}$

$N_3(0) = (1-p_{32})$

$D'_1(0) = [(1-p_{32})(\mu_0+\mu_1) + p_{12}(\mu_2+\mu_3)]$

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Basic Study of Cryptography

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Abstract

Cryptography is actually the science of writing secret code. It is an ancient art. In the present day context, it refers to the tools and techniques used to make messages secure for communication between the sender and the receiver. For private communication through public network, Cryptography plays a very crucial role.

The message sent by the sender is known as Plaintext. Cryptography contains two main categories called Encryption – encoding of the message and Decryption-decoding of the message. Encrypted message is called cipher text, which is received at the other end of the medium and decrypted to get back the original Plain text message. The key used for encryption and decryption is known as Secret Key.

Cryptography is of two types- Symmetric Key Cryptography and Public Key Cryptography.

In Symmetric key cryptography, same key is used for both encryption and decryption of the message while in Public Key Cryptography, each user uses different secret keys for encryption and decryption.

So, basically Cryptography is the science to communicate securely over an insecure public channel. Now a days, Cryptography is widely used in Whats App, Digital Signatures and Http Secure for secure communication and transaction.

Key words: Cryptography, Plain Text, Encryption, Decryption

Introduction

Message secrecy can be achieved through the use of cryptography. Its translation from Greek is "Hidden Writing." The confidentiality of people and organisations is now very successfully achieved through the use of cryptography.

Cryptography is used by billions of people worldwide to secure data and information Network and computer security is a young and developing field of technology. Security courses mostly focus on encryption and hashing techniques. The primary goal of cryptography is to ensure security using various encryption techniques .

Concept of Cryptography

The basic concept of cryptography is to achieve security of the information .The concealed information is usually named "plaintext", and the process of revealing the plaintext is defined as "encryption" ; the encrypted plaintext is known as "ciphertext" . This process is gained by using "encryption algorithms". The possible attacks depend on the actual resources of the attackers. They are usually classified as follows:

Ciphertext-Only Attack

Ciphertexts are accessible to the attacker. This is probably true in all circumstances involving encryption. One must presume that an attacker may obtain encrypted messages even if he cannot carry out the more complex techniques outlined below. An encryption technique is entirely unsafe if it cannot withstand a ciphertext-only assault.

Known-Plaintext Attack

The attacker has access to pairings of plaintext and ciphertext. He tries to decrypt a ciphertext for which he doesn't have the plaintext by using the details from these pairs. It might seems at first that an attacker would not typically have access to this kind of information. Communications could be sent in forms that the attacker is familiar with.

Chosen-Plaintext Attack

The attacker has the option of obtaining ciphertexts for specific plaintexts. Then, without having access to the plaintext, he tries to decrypt a ciphertext. He confidently provides some fascinating facts to his chosen victim. He will send out after encrypting. This kind of attack presupposes that the attacker must first get whatever plaintext-ciphertext combinations he desires before performing his analysis independently. He will only require access to the encryption equipment once as a result.

Adaptively Chosen Plaintext Attack

The only difference between this assault and the one before it is that the attacker may now analyse the plaintext-ciphertext pairs to obtain additional pairs. As often as he chooses, he can alternate between collecting pairs and carrying out the analysis.

Chosen and Adaptively Chosen Ciphertext Attack

These two attacks resemble the plaintext assaults mentioned previously. The plaintexts for any chosen ciphertexts are provided to the attacker. He has usage of the decryption tool.

Objectives of Cryptography

Providing secrecy is not the only objective of cryptography. Cryptography is also used to provide solutions for other problems:

Data Integrity

The receiver of a message should be able to check whether the message was modified during transmission, either accidentally or deliberately. No one should be able to substitute a false message for the original message or for parts of it.

Authentication

The receiver of a message should be able to verify its origin. No one should be able to send a message to X and pretend to be Y (data origin authentication). When initiating a communication, X and Y should be able to identify each other (entity authentication).

Non-repudiation:

The sender should not be able to later deny that she sent a message.

Cryptography Terminology

While Cryptography is the science of securing data , Cryptanalysis is the science of analyzing secure communication. Cryptology comprises of both cryptography and cryptanalysis. A cryptosystem is an implementation of cryptographic techniques and their accompanying infrastructure to provide information security services. A cryptosystem is also referred to as a cipher system. The various components of a basic cryptosystem are as follows ——

. Plaintext
. Encryption Algorithm
. Ciphertext
. Decryption Algorithm
. Encryption Key
. Decryption Key

History Of Cryptography

Hieroglyph

The roots of Cryptography are found in Roman and Egyptian civilizations. The first known evidence of cryptography is the use of hieroglyph used by Egyptians to communicate messages.

Ceaser Cipher

The Ceaser Cipher is named after Julius Ceaser , who used it with a shift of three letters to protect messages of military significance. In Ceaser Cipher , the message was encoded by shifting the letters by an agreed number usually called key, then the recipient of this message would shift the letters back by using same key for decoding.

Types of Cryptography

- Symmetric Key Cryptography
- Asymmetric Key Cryptography

Symmetric Key Cryptography

Also known as Secret Key Cryptography or Conventional Cryptography, Symmetric Key Cryptography is an encryption system in which the sender and receiver of a message share a single, common key that is used to encrypt and decrypt the message. The Algorithm used is also known as a secret key algorithm or sometimes called a symmetric algorithm .

Asymmetric Key Cryptography

Asymmetric Key cryptography , also known as Public-key cryptography, refers to a cryptographic algorithm which requires two separate keys, one of which is private and other one is public. The public key is used to encrypt the message and the private one is used to decrypt the message.

Algorithms

Stream Ciphers

The pseudorandom bits used by stream ciphers are produced using the key. Each bit is encrypted separately. Stream Cipher is now more secure to use with Bluetooth, connections, communications, mobile 4G, and other technologies. There are two categories of stream ciphers: synchronous and asynchronous. Whereas the ciphertext depends on the key stream in an asynchronous stream cipher, the key stream is dependent on the key in a synchronous stream cipher.

Block Ciphers

Block Ciphers consist of algorithms for both encryption and decryption. A block cipher refers to the size of the block and the size of the key. Here, the security depends on the value of both.

Hash Functions

A cryptographic hash function is a hash function that accepts an arbitrary block of data and outputs a fixed-size bit string known as the cryptographic hash value. This means that any modification to the data, whether deliberate or unintentional, will alter the hash value. The hash value is frequently referred to as the message and the encoded data as the message, or as the digest.

Digital Signatures

A digital signature is a mathematical method for verifying the legitimacy of a communication, piece of software, or digital document. The foundation of digital signatures is public key cryptography. Public-key techniques are required for digital signatures. They are meant to give authentication and non-repudiation, just like traditional handwritten signatures. Digital signatures are generated solely by the signer using his secret key and are dependent on it. On the other hand, anyone can verify a signature's validity by using a publicly available verification technique that is dependent on the signer's public key.

Conclusion

Cryptography plays a vital and critical role in achieving the primary aims of security goals such as authentication, integrity, security and non-repudiation.

The primary goal of cryptography is to keep the plaintext secret from snoops trying to get some information about the plaintext. Attackers may also be active and try to modify the message. Then, cryptography is expected to guarantee the integrity of the messages. Attackers are assumed to have complete access to the communication channel.

Cryptanalysis is the science of studying attacks against cryptographic schemes. Successful attacks may, for example, recover the plaintext (or parts of the plaintext) from the ciphertext, substitute parts of the original message, or forge digital signatures.

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Response of Thermoelastic Material With Voids Under The Temperature Dependent Properties In Generalized Theories.

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Abstract

In the present investigation the two dimensional plane strain problem in homogeneous, isotropic thermoelastic half-space with voids with variable modulus of elasticity and thermal conductivity subjected to thermal boundary condition has been investigated. The formulation is applied to the coupled as well as five generalized theories: Lord-Shulman theory (with one relaxation time), The Green-Lindsay theory (with two relaxation times), Green-Nagdhi theories (type II and III) and Chandrasekhariah-Tzou theory (with dual phase leg). The Laplace and Fourier transforms techniques are used to solve the problem. The concentrated thermal source has been taken to illustrate the utility of the approach. The components of displacement, stress, change in volume fraction field and temperature distribution are obtained in the transformed domain. Various special cases of interest have been deduced from the present investigation.

Key words: Thermoelastic, Elasticity, Fration field.

Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms, second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. Biot [1] formulated the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both the theories share the second shortcoming since the heat equation for the coupled theory is also parabolic.

Hetnarski and Iganczak in their survey article [2] examined five generalizations to the coupled theory and obtained a number of important analytical results.

The first is due to Lord and Shulman(L-S) [3], in which, in comparison to the classical theory, the Fourier law of heat conduction is replaced by Maxwell-Cattaneo law that generalizes the Fourier law and introduces a single relaxation time into consideration. Since the heat equation of this theory is of wave type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory namely the equation of motion and constitutive relations remain the same as those for the coupled and uncoupled theories.

The second generalization is given by Green and Lindsay [4] (G-L Theory), in which, in comparison to the classical theory, the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times into consideration. This theory is also known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity.

The third generalization to the coupled theory is known as the low-temperature thermoelasticity, introduced by Hetnarski and Iganczak [5]. This model in characterized by a system of non-linear field equations.

The fourth generalization to the coupled theory is know as the thermoelasticity without energy dissipation, proposed by Green and Naghdi [6] (G-N theory of type (II)), in which, in comparison to the classical theory the Fourier law is replaced by a heat flux rate temperature gradient relation. The heat equation in this model does not contain the temperature rate; therefore the solution represents an undamped thermoelastic wave. The so called Green-Naghdi theory of type III, can be derived from Green and Naghdi [7-8].

The fifth generalization to the coupled theory is known as the dual-phase lag thermoelasticity, proposed by Chandrasekhariah and Tzou [9-10] (C-T-Theory), in which the Fourier law is replaced by an approximation to a modification of the Fourier law with two different translations for the heat flux and the temperature gradient [11-12].

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical use for investigating various types of geological and biological materials for which elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume is included among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A non-linear theory of elastic materials with voids was developed by Nunziato and Cowin [13]. Later, Cowin and Nunziato [14] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behaviour of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Considerable amount of work has been done in the linear theory of elastic materials with voids [15-21].

Iesan [22] developed the theory of the thermoelastic material with voids. Recently, Kumar and Leena [23-28] investigated various problems in the linear theory of thermoelastic materials with voids.

Most of the investigation was done under the assumption of temperature-independent material properties which limit the application of the solutions obtained to certain ranges of temprature.

Modern structural elements are often subjected to temperature change of such magnitude that their material properties may be longer be regarded as having constant values even in an approximate sense. At high temperature the materials characteristics such as the modulus of elasticity, the Poisson ratio, the coefficient of linear thermal expansion and the thermal conductivity are no longer constants. The thermal and mechanical properties of the material vary with temperature, so the temperature-dependent of the materials properties must be taken into consideration in the thermal stress analysis of these elements.

Tanigawa (29), Tanigawa et al (30), investigated thermoelastic problems for non-homogeneous structural material. Ezzat et al [31-32] investigated the dependence of modulus of elasticity on reference temperature in generalized thermoelasticity and obtained interesting results. Youssef [33] used the equation of generalized thermoelasticity with one relaxation time with variable modulus of elasticity and the thermal conductivity to solve a problem of an infinite material with spherical cavity. Othman [34] investigated the two dimensional problem in thermoelastic solid with one relaxation time under the dependence of modulus of elasticity on the reference temperature.

In the present investigation the equations of thermoelastic with void with the dependence of modulus of elasticity and thermal conductivity on the reference temperature are used to obtain the components of displacement, stress, change in volume fraction field and temperature distribution due to thermal source.

Basic Equations

Following Cowin and Nunziato [14] and El-Karamany [35] the field equations and constitutive relations in thermoelastic body with voids without body forces, heat sources and extrinsic equilibrated body force can be written as:

$$(\lambda+2\mu)\nabla(\nabla.\vec{u})-\mu(\nabla\times\nabla\times\vec{u})+b\nabla\phi-\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

$$\mathrm{K}\left(\mathrm{n}^*+\mathrm{t}_1\frac{\partial}{\partial \mathrm{t}}\right)\nabla^2\mathrm{T}-\beta\mathrm{T}_o\left(\mathrm{n}_1\frac{\partial}{\partial \mathrm{t}}+\mathrm{n}_o\tau_o\frac{\partial^2}{\partial \mathrm{t}^2}+\mathrm{t}_2^2\frac{\partial^3}{\partial \mathrm{t}^3}\right)\nabla.\vec{\mathrm{u}}-\mathrm{m}\mathrm{T}_o\frac{\partial\phi}{\partial \mathrm{t}}$$

$$=\rho C_e\left(n_1\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}+t_2^2\frac{\partial^3}{\partial t^3}\right)T\ , \quad (2)$$

$$\alpha\nabla^2\phi-\mathrm{b}\nabla.\vec{\mathrm{u}}-\xi_1\phi-\omega_o\dot{\phi}+\mathrm{mT}=\rho\psi\frac{\partial^2\phi}{\partial \mathrm{t}^2}, \quad (3)$$

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+b\phi\delta_{ij}-\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \quad (4)$$

where λ, μ - Lame's constant, $\alpha, b, \xi_1, m, \omega_o, \psi$ - material constants due to presence of voids, $t_1, t_2, n^*, n_1 n_o$ -constants, τ_o, τ_1 -the relaxation times,T_o-uniform temperature, ρ, C_e-density and specific heat respectively, t_{ij}-component of stress tensor, K-thermal conductivity, $\vec{u}$-displacement vector, ϕ-change in volume fraction field,T-the temperatur distribution $\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial y}$, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$,

$\beta=(3\lambda+2\mu)\alpha_t$, α_t -coefficient of linear thermal expansion .

Formulation of The Problem and Solution

We consider a homogeneous, isotropic, generalized thermoelastic half space with voids in the undeformed temperature T_o. The **r**ectangular Cartesian co-ordinate system (x,y,z) having origin on the surface z=0 with z – axis pointing normally in to the medium is introduced.A concentrated thermal source, is assumed to be acting at the origin of the **r**ectangular cartesian co-ordinates.

For two dimensional problem, we assume

$$\vec{u} = (u, 0, w) \tag{*}$$

With the help of (*),equations (1)-(4) reduced in the form

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial e}{\partial x} + b\frac{\partial \varphi}{\partial x} - \beta\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} + b\frac{\partial \varphi}{\partial z} - \beta\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{6}$$

$$\alpha\nabla^2\phi - be - \xi_1\phi - \omega_o\dot{\phi} + mT = \rho\psi\frac{\partial^2\phi}{\partial t^2}, \tag{7}$$

$$K\left(n^* + t_1\frac{\partial}{\partial t}\right)\nabla^2 T - \beta T_o\left(n_1\frac{\partial}{\partial t} + n_o\tau_o\frac{\partial^2}{\partial t^2} + t_2^2\frac{\partial^3}{\partial t^3}\right)e - mT_o\frac{\partial\phi}{\partial t}$$

$$= \rho C_e\left(n_1\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2} + t_2^2\frac{\partial^3}{\partial t^3}\right)T, \tag{8}$$

where

$$e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Our aim is to investigate the effect of temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume

$$\lambda = \lambda_o\left(1-\alpha^* T_o\right), \mu = \mu_o\left(1-\alpha^* T_o\right), \beta = \beta_o\left(1-\alpha^* T_o\right), \xi_1 = \xi_{10}\left(1-\alpha^* T_o\right),$$
$$m = m_o\left(1-\alpha^* T_o\right), b = b_o\left(1-\alpha^* T_o\right), \alpha = \alpha_o\left(1-\alpha^* T_o\right), \psi = \psi_o\left(1-\alpha^* T_o\right)$$
$$K = K_o\left(1-\alpha^* T_o\right), \omega_o = \omega_{1o}\left(1-\alpha^* T_o\right) \qquad (9)$$

where $\lambda_o, \mu_o, \beta_o, \xi_{10}, m_o, b_o, \alpha_o, \psi_o, \omega_{1o}$ are considered constants, α^* is called empirical material constant, incase of the reference temperature independent of modulus of elasticity, then $\alpha^*=0$.

Equations (5)–(8) with the help of (9) take the form

$$\mu_o \nabla^2 u + (\lambda_o + \mu_o)\frac{\partial e}{\partial x} + b_o \frac{\partial \varphi}{\partial x} - \beta_o\left(1+\tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \frac{\rho}{1-\alpha^* T_o}\frac{\partial^2 u}{\partial t^2}, \qquad (10)$$

$$\mu_o \nabla^2 w + (\lambda_o + \mu_o)\frac{\partial e}{\partial z} + b_o \frac{\partial \varphi}{\partial z} - \beta_o\left(1+\tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \frac{\rho}{1-\alpha^* T_o}\frac{\partial^2 w}{\partial t^2}, \qquad (11)$$

$$\alpha_o \nabla^2 \phi - b_o e - \xi_{1o}\phi - \omega_{1o}\frac{\partial \varphi}{\partial t} + m_o T = \rho \psi_o \frac{\partial^2 \phi}{\partial t^2}, \qquad (12)$$

$$K_o\left(n^* + t_1 \frac{\partial}{\partial t}\right)\nabla^2 T - \beta_o T_o\left(n_1 \frac{\partial}{\partial t} + n_o \tau_o \frac{\partial^2}{\partial t^2} + t_2^2 \frac{\partial^3}{\partial t^3}\right)e - m_o T_o \frac{\partial \phi}{\partial t}$$

$$= \frac{K_o}{\kappa}\left(n_1 \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2} + t_2{}^2 \frac{\partial^3}{\partial t^3}\right), \qquad (13)$$

where

$\rho C_e = \dfrac{K}{\kappa}$ and κ is the diffusivity.

The initial and regularity conditions are given by:

$$u(x,z,0) = 0 = \dot{u}(x,z,0),$$
$$w(x,z,0) = 0 = \dot{w}(x,z,0), \qquad (14)$$
$$\phi(x,z,0) = 0 = \dot{\phi}(x,z,0),$$
$$T(x,z,0) = 0 = \dot{T}(x,z,0) \quad \text{for } z \geq 0, \quad -\infty < x < \infty,$$

$$u(x,z,0)=w(x,z,0)=\phi(x,z,0)=T(x,z,0)=0 \quad \text{for } t>0, when \quad z\to\infty. \qquad (15)$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x'=\frac{\omega_1^* x}{c_2}, z'=\frac{\omega_1^* z}{c_2}, u'=\frac{\omega_1^* u}{c_2}, w'=\frac{\omega_1^* w}{c_2},\ t'=\omega_1^* t,\ t'_{zz}=\frac{t_{zz}}{\mu_o}, t'_{zx}=\frac{t_{zx}}{\mu_o},$$

$$\phi'=\frac{\omega_1^*\psi_o}{c_2^2}\phi,\ \in_1=\frac{\beta_o c_2^2}{K_o\omega_1^*},\ \tau_o'=\omega_1^*\tau_o,\ \tau_1'=\omega_1^*\tau_1,\ a'=\frac{\omega_1^* a}{c_2},\ T'=\frac{T}{T_o}, \qquad (16)$$

where

$$c_2=\left(\frac{\mu_o}{\rho}\right)^{\frac{1}{2}} \text{ and } \omega_1^*=\frac{c_2^2}{\kappa}.$$

Equations (10-(13), may be recast into the dimensionless form after suppressing the primes as:

$$\nabla^2 u + a_1\frac{\partial e}{\partial x}+a_2\frac{\partial \varphi}{\partial x}-a_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}=a_4\frac{\partial^2 u}{\partial t^2}, \qquad (17)$$

$$\nabla^2 w + a_1\frac{\partial e}{\partial z}+a_2\frac{\partial \varphi}{\partial z}-a_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}=a_4\frac{\partial^2 w}{\partial t^2}, \qquad (18)$$

$$\nabla^2\phi - a_5 e - a_6\phi - a_7\frac{\partial\varphi}{\partial t}+a_8 T=a_9\frac{\partial^2\phi}{\partial t^2}, \qquad (19)$$

$$\left(n^*+t_1\frac{\partial}{\partial t}\right)\nabla^2 T-\in_1\left(n_1\frac{\partial}{\partial t}+n_o\tau_o\frac{\partial^2}{\partial t^2}+t_2^2\frac{\partial^3}{\partial t^3}\right)e-a_{10}\frac{\partial\phi}{\partial t}$$

$$=\left(n_1\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}+t_2^2\frac{\partial^3}{\partial t^3}\right)T, \qquad (20)$$

where

$$a_1=\frac{\lambda_o+\mu_o}{\mu_o},\ a_2=\frac{b_o c_2^2}{\psi_o\,\mu_o\omega_1^{*2}}, a_3=\frac{\beta_o T_o}{\mu_o}, a_4=\frac{\rho c_2^2}{(1-\alpha^* T_o)\mu_o}, a_5=\frac{b_o\psi_o}{\alpha_o},$$

$$a_6 = \frac{\xi_{1o} c_2{}^2}{\omega_1^{*2} \alpha_o},\ a_7 = \frac{\omega_{1o} c_2{}^2}{\omega_1^* \alpha_o}, a_8 = \frac{m_o T_o \psi_o}{\alpha_o}, a_9 = \frac{\rho \psi_o c_2{}^2}{\alpha_o}, \in_1 = \frac{\beta_o c_2{}^2}{\omega_1^* K_o},$$

$$a_{10} = \frac{m_o c_2{}^4}{K_o \psi_o \omega_1^{*3}}.$$

Using the expression relating displacement components u(x,z,t) and w(x,z,t) to the scalar potential functions ψ_1(x,z,t) and $\psi_2(x,z,t)$ in dimensionless form

$$u = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, w = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x} \tag{21}$$

in equations (17)-(20), we obtain

$$(\nabla^2 (1 + a_1) - a_4 \frac{\partial^2}{\partial t^2})\psi_1 + a_2 \phi - a_3 (1 + \tau_1 \frac{\partial}{\partial t})T = 0, \tag{22}$$

$$a_5 \nabla^2 \psi_1 - (\nabla^2 - a_6 - a_7 \frac{\partial}{\partial t} - a_9 \frac{\partial}{\partial t^2})\phi - a_8 T = 0, \tag{23}$$

$$\in_1 (n_1 \frac{\partial}{\partial t} + n_o \tau_o \frac{\partial^2}{\partial t^2} + t_2{}^2 \frac{\partial^3}{\partial t^3})\nabla^2 \psi_1 + a_{10} \frac{\partial \phi}{\partial t} - \left[(n^* + t_1 \frac{\partial}{\partial t})\nabla^2 - (n_1 \frac{\partial}{\partial t} + \tau_o \frac{\partial}{\partial t^2} + t_2{}^2 \frac{\partial^3}{\partial t^3}) \right] T = 0, \tag{24}$$

$$(\nabla^2 - a_4 \frac{\partial^2}{\partial t^2})\psi_2 = 0 \quad . \tag{25}$$

Applying the Laplace and Fourier transforms

$$\hat{f}(x,z,s) = \int_0^\infty f(x,z,t) e^{-st} dt,$$

$$\tilde{f}(x,z,s) = \int_0^\infty \hat{f}(x,z,s) e^{i\xi x} dx, \tag{26}$$

on equations (22)-(25) and eliminating $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}\ and\ \tilde{\psi}_2$ from the resulting expressions, we obtain

$$\left(\frac{d^6}{dz^6}+Q\frac{d^4}{dz^4}+N\frac{d^2}{dz^2}+I\right)(\tilde{\psi}_1,\tilde{\phi},\tilde{T})=0\ , \tag{27}$$

$$\left(\frac{d^2}{dz^2}-\lambda_4{}^2\right)\tilde{\psi}_2=0\ , \tag{28}$$

where

$$Q=\frac{1}{b_1 f_1}\left[b_1(-3\xi^2 f_1-f_5 f_1-f_2)-a_4 s^2 f_1+a_2 a_5 f_1-a_3\in_1 f_3 f_4\right]\ ,$$

$$N=\frac{1}{b_1 f_1}\begin{bmatrix} b_1(3\xi^4 f_1+f_5 f_1 2\xi^2+2\xi^2 f_2+f_5 f_2+a_8 a_{10}s)+s^2(2a_4\xi^2 f_1+a_4 f_5 f_1 \\ +a_4 f_2)-a_2(a_5 f_1 2\xi^2+a_5 f_2+a_8\in_1 f_4)-a_3 a_5 a_{10} s f_3 \\ +2\xi^2 a_3\in_1 f_3 f_4+a_3\in_1 f_3 f_4 f_5 \end{bmatrix},$$

$$I=\frac{1}{b_1 f_1}\begin{bmatrix} -b_1 f_1\xi^6+\xi^4(-b_1 f_1 f_5-b_1 f_2-a_4 s^2 f_1+a_2 a_5 f_1-a_3\in_1 f_3 f_4) \\ +\xi^2(-b_1 f_5 f_2-a_8 a_{10}\, s b_1-a_4 s^2 f_5 f_1-a_4 s^2 f_2+a_2 a_5 f_2+a_2 a_8\in_1 f_4 \\ +a_3 a_5 a_{10} s f_3-a_3\in_1 f_3 f_4 f_5)+s^2(-a_4 f_2 f_5-a_4 a_8 a_{10} s) \end{bmatrix},$$

and

$$\mathrm{b}_1=1+a_1,\ f_1=\mathrm{n}^*+\mathrm{t}_1\mathrm{s},\ f_2=\mathrm{n}_1\mathrm{s}+\tau_o s^2+t_2{}^2 s^3,\ f_3=1+\tau_1 s,$$

$$f_4=n_1 s+n_o\tau_o s^2+t_2{}^2 s^3,\ f_5=a_5+a_7 s+a_9 s^2,\ \lambda_4{}^2=\xi^2+a_4 s^2$$

$$b_2=\frac{\lambda_o(1-\alpha^* T_o)}{\mu_o},\ b_3=2(1-\alpha^* T_o),\ b_4=\frac{b_o c_2{}^2(1-\alpha^* T_o)}{\omega_1{}^{*2}\psi_o\mu_o}$$

The roots of the equations (27) and (28) are $\pm\lambda_\ell$ (ℓ =1,2,3,4).

Assuming the regularity conditions, the solutions of equations (27)-(28) may be written as

$$\tilde{\psi}_1=A_1 e^{-\lambda_1 z}+A_2 e^{-\lambda_2 z}+A_3 e^{-\lambda_3 z}\ , \tag{29}$$

$$\tilde{\phi}=d_1 A_1 e^{-\lambda_1 z}+d_2 A_2 e^{-\lambda_2 z}+d_3 A_3 e^{-\lambda_3 z}, \tag{30}$$

$$\tilde{T}=e_1 A_1 e^{-\lambda_1 z}+e_2 A_2 e^{-\lambda_2 z}+e_3 A_3 e^{-\lambda_3 z}, \tag{31}$$

$$\tilde{\psi}_2 = A_4 e^{-\lambda_4 z}, \qquad (32)$$

where

$$d_\ell = \frac{a_{13}c_{\ell 1} - a_{\ell 1}c_{\ell 3}}{a_{12}c_{\ell 1} - a_{13}c_{12}}, e_\ell = \frac{a_{\ell 1}c_{12} - a_{12}c_{\ell 1}}{a_{12}c_{\ell 3} - a_{13}c_{12}} \quad (\ell = 1,2,3)$$

and

$$a_{\ell 1} = b_1\lambda_\ell^2 - \lambda_4^2, a_{12} = a_2, a_{13} = -a_3 f_3, c_{\ell 1} = \in_1 f_4(\lambda_\ell^2 - \xi^2),$$
$$c_{12} = \in_1 s, c_{\ell 3} = -f_1(\lambda_\ell^2 - \xi^2) + f_2.$$

Boundary Conditions

1. **Thermoelastic interaction due to thermal source**

The boundary conditions in this case are

$$t_{zz} = 0, \quad t_{zx} = 0, \quad \frac{\partial \phi}{\partial z} = 0, \quad T = T_o\delta(x)\delta(t) \qquad at \ z = 0. \qquad (33)$$

where T_o is the constant temperature applied on the boundary. $\delta(x)$, $\delta(t)$ are the Dirac delta functions.

Applying the Laplace and Fourier transforms defined by (26), to the boundary condition (33) we get

$$\tilde{t}_{zz} = 0, \quad \tilde{t}_{zx} = 0, \quad \frac{d\tilde{\phi}}{dz} = 0, \quad \tilde{T} = 1 \qquad at\, z = 0. \qquad (34)$$

Making use of equations (4), (16) and (21) and applying the Laplace and Fourier transforms defined by (26) and substituting the value of $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}\ and\ \tilde{\psi}_2$ from equations (29)-(32) in the boundary condition (34) we obtain the components of displacement, stress, change in volume fraction field and temperature distribution as

$$\tilde{u} = P[(-i\xi)(\Delta_1'' E_1 + \Delta_2'' E_2 + \Delta_3'' E_3) + \lambda_4 \Delta_4'' E_4],$$

$$\tilde{w} = P[(-\lambda_1\Delta_1^{"}E_1 - \lambda_2\Delta_2^{"}E_2 - \lambda_3\Delta_3^{"}E_3) + (-i\xi)\Delta_4^{"}E_4] ,$$

$$\tilde{t}_{zz} = P[(R_1\Delta_1^{"}E_1 + R_2\Delta_2^{"}E_2 + R_3\Delta_3^{"}E_3) + R_4\Delta_4^{"}E_4] ,$$

$$\tilde{t}_{zx} = P[(q_1\Delta_1^{"}E_1 + q_2\Delta_2^{"}E_2 + q_3\Delta_3^{"}E_3) + q_4\Delta_4^{"}E_4] ,$$

$$\tilde{\phi} = P[d_1\Delta_1^{"}E_1 + d_2\Delta_2^{"}E_2 + d_3\Delta_3^{"}E_3] ,$$

$$\tilde{T} = P[e_1\Delta_1^{"}E_1 + e_2\Delta_2^{"}E_2 + e_3\Delta_3^{"}E_3] \qquad (35)$$

where

$$P = \frac{1}{\Delta},$$

$$\Delta = (\lambda_2 d_2 e_3 - \lambda_3 d_3 e_2)(R_1 q_4 - R_4 q_1) + (\lambda_3 d_3 e_1 - \lambda_1 d_1 e_3)(R_2 q_4 - R_4 q_2) + (\lambda_1 d_1 e_2 - \lambda_2 d_2 e_1)(R_3 q_4 - R_4 q_3),$$

$$\Delta_1^{"} = \lambda_3 d_3 (R_2 q_4 - R_4 q_2) + \lambda_2 d_2 (R_4 q_3 - R_3 q_4),$$

$$\Delta_2^{"} = \lambda_3 d_3 (R_4 q_1 - R_1 q_4) + \lambda_1 d_1 (R_3 q_4 - R_4 q_3),$$

$$\Delta_3^{"} = \lambda_2 d_2 (R_1 q_4 - R_4 q_1) + \lambda_1 d_1 (R_4 q_2 - R_2 q_4),$$

$$\Delta_4^{"} = \lambda_3 d_3 (R_1 q_2 - R_2 q_1) + \lambda_2 d_2 (R_3 q_1 - R_1 q_3) + \lambda_1 d_1 (R_2 q_3 - R_3 q_2),$$

$$E_\ell = e^{-\lambda_\ell x} \quad (\ell = 1,2,3,4);$$

$$R_\ell = (b_2 + b_3)\lambda_\ell^{\,2} - b_2\xi^2 + b_4 d_\ell - (1 + \tau_1 s)e_\ell , \;\; q_\ell = 2(i\xi)\lambda_\ell \qquad (\ell = 1,2,3);$$

$$R_4 = i\xi\lambda_4 , \;\; q_4 = -(\xi^2 + \lambda_4^{\,2}).$$

Particular Cases

(i) In case of Independent of modulus of elasticity we take $\alpha^* = 0$ in the resulting expressions given by equations (35) with the changed values of a_4, b_2, b_3, b_4 as

$$a_4 = \frac{\rho c_2^{\,2}}{\mu}, \; b_2 = \frac{\lambda}{\mu}, \; b_3 = 2, \quad b_4 = \frac{b_o c_2^{\,2}}{\omega_1^{*2}\psi\,\mu}$$

and

$$\lambda = \lambda_o,\ \mu = \mu_o,\ \beta = \beta_o,\ \xi_1 = \xi_{10},\ m = m_o,\ b = b_o,$$

$$\alpha = \alpha_o,\ \psi = \psi_o,\ K = K_o,\ \omega_o = \omega_{1o}.$$

(ii) If we neglect the voids effect ($\alpha = b = \xi_1 = m = \psi = \omega_o = 0$) the expressions for components of displacement ,stress and temperature distribution are obtained in thermoelastic half-space under the dependence of modulus of elasticity by replacing Δ with Δ^* and $\Delta_\ell^{"}$ with Δ_ℓ^{***} $(\ell = 1,2,4)$ and $e_\ell, E_\ell, R_\ell, q_\ell, \lambda_\ell$ with $e_\ell^*, E_\ell^*, R_\ell^*, q_\ell^*, \lambda_\ell^*$ $(\ell = 1,2.)$ respectively and $d_\ell = \lambda_3 = R_3 = q_3 = \Delta_3^{"} = 0\,(\ell = 1,2,3.)$, in the equation (35) respectively with

$$\Delta^* = q_4(-R_1^* e_2^* + R_2^* e_1^*) + R_4(q_1^* e_2^* - q_2^* e_1^*),$$

$$\Delta_1^{***} = (R_2^* q_4 - R_4 q_2^*),$$

$$\Delta_2^{***} = (-R_1^* q_4 + R_4 q_1^*),$$

$$\Delta_4^{***} = (R_1^* q_2^* - R_2^* q_1^*),$$

$$e_\ell^* = -\frac{c_{\ell 1}}{c_{\ell 3}},\ E_\ell^* = e^{-\lambda_\ell^* z},\ R_\ell^* = (b_2 + b_3)\lambda_\ell^{*2} - b_2\xi^2 - (1+\tau_1 s)e_\ell^*,\ q_\ell^* = 2i\xi\lambda_\ell^*,$$

$$\lambda_\ell^{*2} = \frac{-A + (-1)^{\ell+1}\sqrt{A^2 - 4B}}{2},$$

with

$$A = \frac{(-1)}{b_1 f_1}\left[b_1(2\xi^2 f_1 + f_2) + a_4 s^2 f_1 + \in_1 a_3 f_3 f_4\right],$$

$$B = \frac{1}{b_1 f_1}\left[(b_1\xi^2 + a_4 s^2)(f_1\xi^2 + f_2) + \xi^2 a_3 \in_1 f_3 f_4\right],$$

Special Cases

We obtain the corresponding expressions for components of displacement, stress, change in volume fraction field and temperature distribution as given in equations (35) in all the theories of thermoelasticity with changed values of Q,N,I, by considering the following

Sub-Cases

I- Coupled thermoelasticity with voids

$$n^* = n_1 = 1, \quad t_1 = t_2 = \tau_o = \tau_1 = 0, \quad n_o \tau_o = 0$$

with

$$f_1 = 1, \ f_2 = s, \ \ f_3 = 1, \ \ f_4 = s.$$

II-L.S. Theory (with one relaxation time)

$$n^* = n_1 = 1 = n_o, \quad \tau_o > 0, \quad t_1 = t_2 = \tau_1 = 0$$

with

$$f_1 = 1, \ \ f_2 = s + \tau_o s^2, \ f_3 = 1, \ f_4 = s + \tau_o s^2.$$

III-G. L. Theory (with two relaxation times)

$$n^* = n_1 = 1, \ n_o = t_1 = t_2 = 0, \quad \tau_1 \geq \tau_o$$

with

$$f_1 = 1, \ f_2 = s + \tau_o s^2, \ \ f_3 = 1 + \tau_1 s, \ f_4 = s.$$

IV-G. N. Theory (thermoelasticity without energy dissipation)

$$n^* > 0, \quad n_1 = 0, \ n_o = 1, \ t_1 = 0, \quad t_2 = \tau_1 = 0, \quad \tau_o = 1$$

with $n^* =$const.has dimension(1/sec),and $(n^* K = K^*)$ is a charateristic const.

of this theory and $K^* = \dfrac{(\lambda + 2\mu) C_e}{4}$

with

$$f_1 = n^*, f_2 = s^2, f_3 = 1, f_4 = s^2.$$

V-G. N. Theory of type – III

$$n^* > 0, \quad n_1 = 0, \; n_o = 1, \; t_1 = 1, \quad t_2 = \tau_1 = 0, \quad \tau_o = 1$$

with

$$f_1 = n^* + s, \; f_2 = s^2, \; f_3 = 1, \; f_4 = s^2.$$

VI-Dual Phase-Lag (C.T. Theory)

$$n^* = n_1 = n_o = 1, \; t_1 = \tau_\theta > 0, \; \tau_o = \tau_q > 0, \; t_2^2 = \frac{1}{2}\tau_q^2, \; \tau_1 = 0, \quad \tau_q \ge \tau_\theta > 0$$ with

$$f_1 = 1 + \tau_\theta s, \; f_2 = s + \tau_q s^2 + \frac{1}{2}\tau_q{}^2 s^3, \; f_3 = 1, \; f_4 = s + \tau_q s^2 + \frac{1}{2}\tau_q{}^2 s^3$$

Conclusion

The transformed components of displacement, stress, change in volume fraction field and temperature distribution due to thermal source in different theories of thermoelasticity under the dependence of modulus of elasticity have been obtained. Various special cases of interest have been deduced from the present investigation for different theories of thermoelasticity.

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Applications of Various Mathematical Tools In Describing Different Concepts of Chemistry

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Abstract

Modern Chemistry is incomplete without Mathematics as all branches of chemistry make use of mathematics in one or the other way. So, recently, mathematical chemistry is of great interest for researchers to find novel applications of mathematics to chemistry. Mathematical chemistry is chiefly related to mathematical modeling of chemical phenomena. The main areas of research being chemical graph theory which use topology to study isomerism mathematically, and topological descriptors or indices development to find use in QSAR (Quantitative Structure-Activity Relationships); and chemical characteristics of group theory used in stereochemistry and quantum chemistry. Quantum chemistry involves various other mathematical tools like single-variable calculus, multi-variable calculus, differential equations, complex functions, group theory, probability and statistics and linear algebra etc. Similarly, many mathematical tools are extensively applied in Spectroscopy. Applications of Fourier transforms in chemistry including electron diffraction, X-ray diffraction, Microwave spectra, Infrared and Raman spectra and Nuclear Magnetic Resonance Spectra. Infrared spectroscopy is based upon harmonic oscillator and depends upon Group theory. Further, differentiation and integration are integral parts different chemistry topics like chemical kinetics, thermodynamics and statistical thermodynamics, chemical equilibrium, electrochemistry etc. In addition, mathematics helps in understanding other chemical phenomenon such as dielectric and magnetic properties of chemical matter. The present article provides an overview on how different mathematical tools assist in understanding various concepts and phenomena of chemistry.

Keywords: Chemistry, Mathematics, Operators, Quantum, Spectroscopy, QSAR

Introduction

Mathematics is an important constituent of all scientific branches [1], as well as everyday life. Both theoretical and applied math are equally important because both have been linked together and go side by side to each other. One of the areas where mathematics is increasingly used is chemistry. Math is very essential to elucidate chemistry concepts in more sophisticated manners. It is very difficult to make calculations in chemistry without knowing basic maths [1].

There are numerous applications of math in chemistry like using ratios and measurements for making solutions and dilutions; for balancing chemical equations, using vectors to understand the structures of crystals *etc* [2]. Mathematics is used in almost all branches of chemistry, for example organic chemistry, inorganic chemistry, physical chemistry, analytical chemistry, biochemistry, and environmental chemistry, *etc* [1].

The present paper summarizes how various mathematical tools are applied in explaining different concepts of chemistry.

Mathematical Chemistry

There are various applications of mathematics in the field of chemistry. A brief account of how different mathematical tools are extensively used in describing diverse concepts of chemistry.

The Fundamental Mathematics Tools

The concepts of addition, subtraction, multiplication, and division are used widely in chemistry, for example, calculating the molecular mass of components, volume, making solutions and dilutions, system of linear equations is used for balancing chemical equations *etc* [1].

Basic Math Topics Used in General Chemistry

Understanding of elementary math topics like significant figures, unit conversions, proportions, exponents and logarithms, probability and statistics, and summation notations, *etc* is necessary to explain different concepts of chemistry [2, 3].

Determining temperature, pressure, volume, density; taking measurements; performing dimensional analysis *etc* are impossible without having knowledge of math. Conversion of one unit to another unit involves the use of conversion factors, which are significant for building equality and relating to different units. The commonly measured property of a substance is density which is necessary in the majority of chemistry calculations, and one of the main fundamental quantities in science is temperature which is used widely ranging from weather forecasts to sophisticated medical services. Logarithm is used in calculations related to pH, pK_a, pk_b of acids and bases [4]. Scientists use different equipments to measure matter, and these measurements are very important since small errrors in these measurements can have a noteworthy impact on the research they are conducting.

Applied Mathematics

Numerous mathematical tools and techniques used by chemists to understand various chemistry concepts are basic trigonometry, algebra, graphing, calculus, and geometry, *etc* [2]. These tools improve quantitative skills as well as assists in the conceptual understanding of derivatives and integrals, which are employed in diverse chemistry concepts. Knowledge of three-dimensional geometry helps in visualizing three-dimensional graphs, structures and stereochemistry of molecules. Applied mathematics is very important in organic, inorganic and physical chemistry, and it expands afar from general chemistry.

Operators

A variety of operators, for example the Delta, Sigma, and Pi operators, are generally employed in chemistry for performing different calculations. For instance, to determine the optical absorbance of a reaction, the Delta operators must be used [1].

Topology and Graph Theory

Molecules exist in a spectacular variety of shapes expressed by their three-dimensional geometrical structure. An easy but very effectual approach to describe molecular structure is based on graph theory. Using a two-dimensional representation, though, molecular graphs keep the most crucial structural information of molecules *i.e*. the manner of connection among the atoms. This topological structure correlates incredibly well with the physicochemical properties (QSPR) and biological activities (QSAR) of chemical compounds. Numerous graph-invariants (also called topological descriptors or topological indices) have been recognized and employed to the structural analysis of chemical characteristics [5].

Different Concepts of Chemistry based on Mathematical Tools

Periodic Table

Dmitri Mendeleev performed a multivariate analysis of chemical information on the set of chemical elements and reduced such a huge multidimensional information to a simple law *i.e.* the periodic law. He processed information concerning stoichiometries of chemical compounds, reactivities, and physical properties which is a multidimensional problem. The trends in periodic table followed a topological structure. It has been stated by some other authors that various mathematical theories like group theory, number theory, order theory, information theory and topology *etc* are suitable to employ and repeat to some level the constitution of trends found by Mendeleev in 1869 [6, 7].

Quantum Chemistry

Quantum chemistry [8] has established to be remarkably triumphant in understanding the physical world. Chemistry as well as physics has benefited from insights from quantum mechanics, and mathematics plays an important role in quantum mechanics which involve use of differential equation in Schrodinger Wave Equation, Operators like Lapalacian operators, Hermitian operator, Hamiltonian operator *etc*. Shapes of different orbitals are found using radial wave functions and angular wave functions which involve exponential form of complex numbers, Sine, Cosine, Tangent functions *etc*. The computation of the wave function describes the electronic structure of atoms and molecules.

Group Theory

In chemistry, group theory [9] makes use of symmetry elements, matrices and determinant of matrix, orthogonality theorem etc. It leads to understand point groups and stereochemistry of the compounds. Group theory relates symmetry of a molecule to its physical properties, therby, leads to a quick easy way to determine the significant physical information of the molecule. The symmetry of a molecule give information about energy levels of the orbitals, symmetry of orbitals, probability of transitions between energy levels and even bond order also without regorous calculations.

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Spectroscopy

Many important applications of mathematics to chemistry involve spectra. The thought was that when a molecule shifts from one energy level to another, some "characteristic" of the molecule can be found out, i.e. different molecules can be acknowledged since the measurements in energy changes are characteristic of specific molecules. Spectroscopy [10] is used to determine the molecular formula, molecular mass and structural information of the compounds using different electromagnetic radiations. This work involves the idea of eigen values of matrices and other topics in linear algebra and operator theory. Fourier Transformation (such as FT-IR, FT-NMR *etc*) is used in instruments to understand the outcomes of spectroscopy.

Thermodynamics

Partial differential equations, differentiation, integration, delta operators, exponentials, summation *etc*. are some mathematical tools which are used in explaining different concepts of classical and statistical thermodynamics [8, 11] In classical thermodynamics, substances are assumed to be continuum, so, thermodynamic state functions can be employed. Quantities which are independent of the path of the process are known as 'state functions' and their characteristics can be described and expressed in mathematical language. Some basic state functions include pressure, volume, temperature, internal energy and entropy *etc*. The change in these functions is continuous in certain range. In accord with this postulation, calculus can be used into chemical thermodynamic functions. The total differential property of state functions is used in the derivation of Maxwell's relations and corresponding coefficient relations.

Chemical Kinetics

In chemical kinetics, the rate of a reaction can be measured by plotting graph between the concentration of any of the reactants as a function of time. Rate laws are expressed in differential form as well as integrated form. These rate laws are used to study the kinetics of the reaction [12].

Electrochemistry

Nernst equation, which, make use of logarithms is the core of electrochemistry. It is applied to calculate electrode potential of various single electrodes and equilibrium constant of a reversible cell. Further, EMF (electromotive force) of the cell is also based on logarithms [4].

QSAR

The quantitative structure-activity relationships (QSAR) became a basic tool in the area of drug discovery, a trend additionally reinforced with the development of the high-throughput methods for identifying lead compounds in drug design [13, 14] The software tools created for calculating topological descriptors of molecules have also contributed to the rapid development of this area of applied graph theory. QSAR have recently extensively developed with systematic methodology in drawing correlations (e.g., with consideration of a variety of multivariate regression techniques, sometimes developed in a purely chemical context). There is an enormous development of a range of different sorts of available quantities (such as quantum-chemically computed characteristics, other experimental properties, or simply molecular graph invariants) in order to make structure/property or structure/activity correlations, for instance, involved in toxicity evaluations and predominantly in drug design.

Crystallography

The mathematical crystallography was developed classically with the recognition of the Bravais lattices and crystal classes, followed by the influential detection of crystallographic space groups. Further, in crystallography [12], different crystal systems are explained on the basis of angles, elements of symmetry and point groups etc. The Bragg's equation is based upon the sine of the angle made by a beam of X-rays incident on the crystal. Using Bragg's equation we can determine the crystal structures of NaCl, KCl and CsCl.

Miscellaneous Concepts

The mathematical tool used in Refraction is trigonometry. According to the Snell's law, expression for refractive index of a medium is the ratio of the sine of the angle of incidence to the sine of angle of refraction when a ray of light travels from air or vacuum to a denser medium [12]. Further, different concepts like Conductance, Chemical equilibrium, Surface tension of a liquid, Viscosity of a liquid, van der Waals equation, colligative properties of dilute solutions, specific rotation & angle of rotation for a plane polarized light, dipole moment, the Clausius-Clapeyron equation in Phase equilibrium *etc*. are based upon various mathematical tools [4, 12]

These are only a small number of instances of how mathematics is applied in chemistry and what kind of math is used to explore differed concepts of chemistry. There are plenty of other applications of mathematics in chemistry. It can be clearly stated that math plays an essential role in fundamental sciences, predominantly in chemistry.

Conclusion

In this article, a brief account of the use of different mathematical tools in describing various concepts of chemistry is provided. So, it is quite apparent that with a basic knowledge of the mathematics, one can be easily well prepared to deal with the concepts and theories involved of chemistry.

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Cryptography

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Abstract

With the internet having reached a level that merges with our lives, growing explosively during the last several decades, data security has become a main concern for anyone connected to the web. Data security insures that our data is only accessible by the intended receiver and prevents any modification or alteration of data. In order to achieve this level of security various algorithms and methods have been developed. Cryptography can be defined as techniques that cipher data, depending on specific algorithms that make data unreadable to the human eye unless decrypted by algorithms that are predefined by the sender. Cryptography is a technique to achieve confidentiality of messages. The term has specific meaning in Greek: "secret writing". Now a days, however the privacy of individuals and organizations is provided through cryptography at a high level, making sure that information sent is secure in a way that the authorized receiver can access this information. With historical roots, cryptography can be considered an old technique that is still being developed. Examples reach 2000B.C.,when the ancient Egyptians used "secret hieroglyphics, as well as other evidence in the form of secret writings in ancient Greece or the famous Caesar Cipher of ancient Rome.

Key Words: Cryptography, Security, Algorithm, Cipher, Encryption, Decryption, Data Security

Introduction

Cryptography is a technique to achieve confidentiality of messages. The term has specific meaning in Greek: "secret writing". Now a days, however the privacy of individuals and organizations is provided through cryptography at a high level, making sure that information sent is secure in a way that the authorized receiver can access this information. With historical roots, cryptography can be considered an old technique that is still being developed. Examples reach 2000 B.C., when the ancient

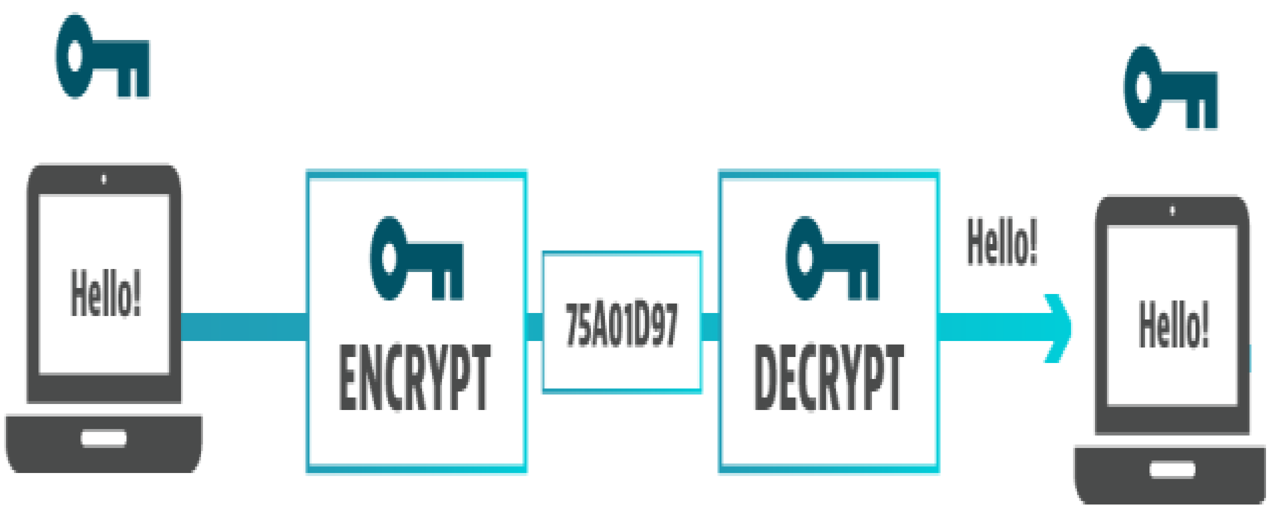
Egyptians used “secret hieroglyphics, as well as other evidence in the form of secret writings in ancient Greece or the famous Caesar Cipher of ancient Rome.

Billions of people around the globe use cryptography on daily basis to protect data and information, although most do not know that they are using it. In addition to being extremely useful, it is also considered highly brittle, as cryptographic systems can become compromised due to a single programming or specification error.

Cryptography Concept

The basic concept of a cryptographic system is to cipher information or data in order to achieve confidentiality of the information in a way that an unauthorized person would be unable to derive its meaning.

In cryptography the concealed information is usually termed **“plain text”**, and the process of disguising the plain text is defined as **“encryption”**; the encrypted text is known as **“cipher text”**. This process is achieved by a number of rules known as **“encryption algorithms”.** Usually, the encryption process relies on an **“encryption key”**, which is then give to the encryption algorithm as input along with the information. Using a **“decryption algorithm”**, the receiving side can retrieve the information using the appropriate **“decryption key”**.



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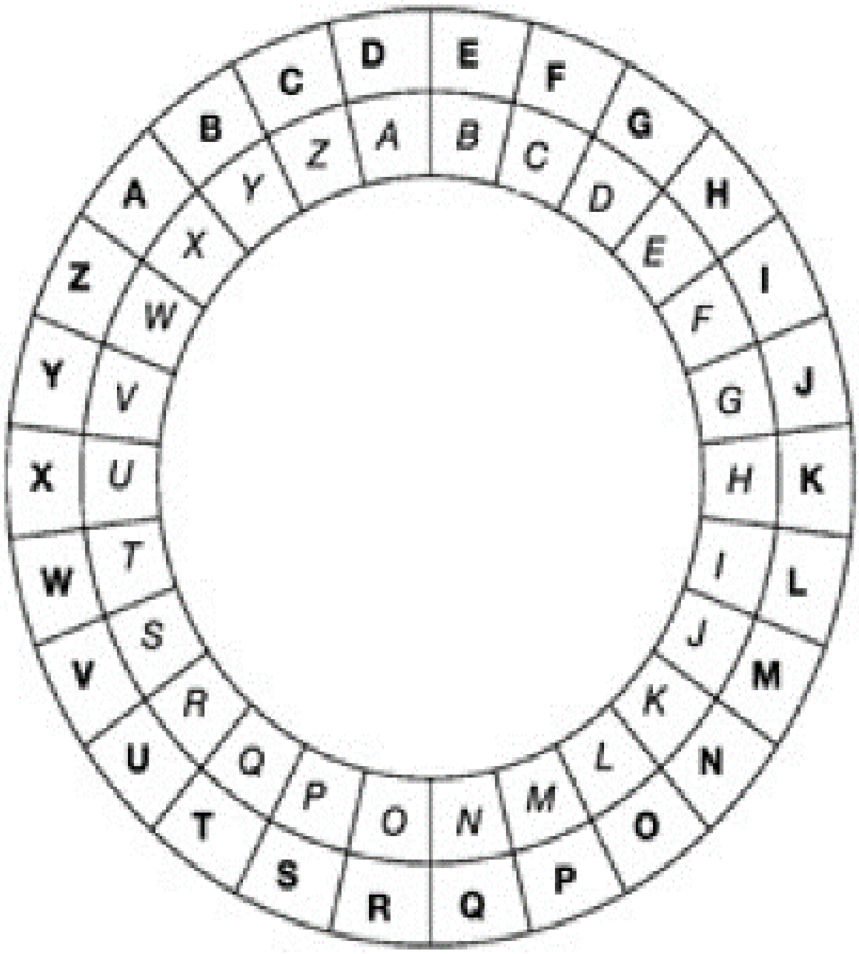
Caesar Cipher

This is one of the oldest and earliest examples of cryptography, invented by Julius Caesar, the emperor of Rome, during the Gallic Wars. In this type of algorithm the letters are encrypted by with the letters that come three places ahead of each letter in the alphabet. This means that a “shift” of 3 is used. As the Caesar cipher is one of the simplest examples of cryptography, it is simple to break. In order for the cipher text to be decrypted, the letters that were shifted get shifted three letters back to their previous positions.



Caesar Cipher encryption wheel

Simple Substitution Cipher

Take the Simple Substitutions Cipher, also known as Mono alphabetic Cipher, as an example. In a Simple Substitution Cipher, we take the alphabet letters and place them in random order under the alphabet written correctly, as seen here:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

E R J A U W P X H L C N G D I Q M T B Z S Y K V O F

In the encryption and decryption, the same key is used. The rule of encryption here is that "each letter gets replaced by the letter beneath it", and the rule of decryption would be the opposite. For instance, the corresponding cipher text for the plaintext MATHEMATICS is **NEBXWNEBHJT.**

Hill Cipher

Hill cipher is the multi letter cipher, developed by the mathematician Lester Hill in 1929.

THE HILL ALGORITHM

This encryption algorithm takes m successive plaintext letters and substitutes for them m cipher text letters. The substitution is determined by m linear equations in which each character is assigned a numerical value (a=0, b=1 ....z=25). For m=3, the system can be described as

$$c1 = (k11p1 + k12p2 + k13p3) \bmod 26$$

$$c2 = (k21p1 + k22p2 + k23p3) \bmod 26$$

$$c3 = (k31p1 + k32p2 + k33p3) \bmod 26$$

$$[\, c1 \quad c2 \quad c3] = [p1 \quad p2 \quad p3] \begin{bmatrix} k11 & k12 & k13 \\ k21 & k22 & k23 \\ k31 & k32 & k33 \end{bmatrix} \text{mod}26$$

$$\mathbf{C} = \mathbf{PK} \bmod 26.$$

This can be expressed in terms of row vectors and matrices where **C** and **P** are row vectors of length 3 representing the plaintext and cipher text, and **K** is a matrix representing the encryption key. Operations are performed mod 26.

Transposition Ciphers

All the techniques examined so far involve the substitution of a cipher text symbol for a plaintext symbol. A very different kind of mapping is achieved by performing some sort of permutation on the plaintext letters. This technique is referred to as a transposition cipher. The simplest such cipher is the **rail fence** technique, in which the plaintext is written down as a sequence of diagonals and then read off as a sequence of rows. For Example , to encipher the message “meet me after the toga party” with a rail fence of depth 2, we write the following:

m e m a t r h t g p r y

e t e f e t e o a a t

The encrypted message is

MEMATRHTGPRYETEFETEOAAT

This sort of thing would be trivial to cryptanalyze.A more complex scheme is to write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns. The order of the columns then becomes the key to the algorithm. For example,

Key: 4 3 1 2 5 6 7

Plaintext: a t t a c k p

o s t p o n e

d u n t i l t

w o a m x y z

Cipher text: TTNAAPTMTSUOAODWCOIXKNLYPETZ

Thus, in this example, the key is 4312567.To encrypt, start with the column that is labelled 1, in this case column 3.Write down all the letters in that column. Proceed to column 4, which is labelled 2, then column 2, then column 1, then columns 5, 6, and 7. A pure transposition cipher is easily recognized because it has the same letter frequencies as the original plaintext. For the type of columnar transposition just shown, cryptanalysis is fairly straightforward and involves laying out the cipher text in a matrix and playing around with column positions. Diagram and trigram frequency tables can be useful. The transposition cipher can be made significantly more secure by performing more than one stage of transposition. The result is a more complex permutation that is not easily reconstructed. Thus, if the foregoing message is re encrypted using the same algorithm,

Key: 4 3 1 2 5 6 7

Input: t t n a a p t

m t s u o a o

d w c o i x k

n l y p e t z

Output: NSCYAUOPTTWLTMDNAOIEPAXTTOKZ

To visualize the result of this double transposition, designate the letters in the original plaintext message by the numbers designating their position. Thus, with 28 letters in the message, the original sequence of letters is

01 02 03 04 05 06 07 08 09 10 11 12 13 14

15 16 17 18 19 20 21 22 23 24 25 26 27 28

After the first transposition, we have

03 10 17 24 04 11 18 25 02 09 16 23 01 08

15 22 05 12 19 26 06 13 20 27 07 14 21 28

Public key systems

The invention of public key encryption can be considered a cryptography revolution. It is obvious that even during the 70s and 80s, general cryptography and encryption were solely limited to the military and intelligence fields. It was only through public key systems and techniques that cryptography spread into other areas. Public key encryption gives us the ability to establish communication without depending on private channels, as the public key can be publicized without ever worrying about it. A summary of the public key and its features follows:

1) With the use of public key encryption, key distribution is allowed on public channels in which the system's initial deployment can be potentially simplified, easing the system's maintenance when parties join or leave.

2) Public key encryption limits the need to store many secret keys. Even in a case in which all parties want the ability to establish secure communication, each party can use a secure fashion to store their own private key. The public keys of other parties can be stored in a non-secure fashion or can be obtained when needed.

3) In the case of open environments, public key cryptography is more suitable, especially when parties that have never interacted previously want to communicate securely and interact. For example, a merchant may have the ability to reveal their public key online, and anyone who wants to purchase something can access the public key of the merchant as necessary when they want their credit card information encrypted.

Digital Signatures

Unlike cryptography, digital signatures did not exist before the invention of computers. As computer communications were introduced, the need arose for digital signatures to be discussed, especially in the business environments where multiple parties take place and each must commit to keeping their declarations and/or proposals. The topic of unforgeable signatures was first discussed centuries ago, except those were handwritten signatures. The idea behind digital signatures was first introduced in a paper by Diffie and Hellman titled “New Directions in Cryptography” [22].

Therefore, in a situation where the sender and receiver do not completely trust each other, authentication alone cannot fill the gap between them. Something more is required, i.e. the digital signature, in a way similar to the handwritten signature.

Digital Signature Requirements

The relationship that created the link between signature and encryption came into existence with the “digitalization” era that we are currently witnessing and living in. The requirements for an unforgettable signature schema would be:

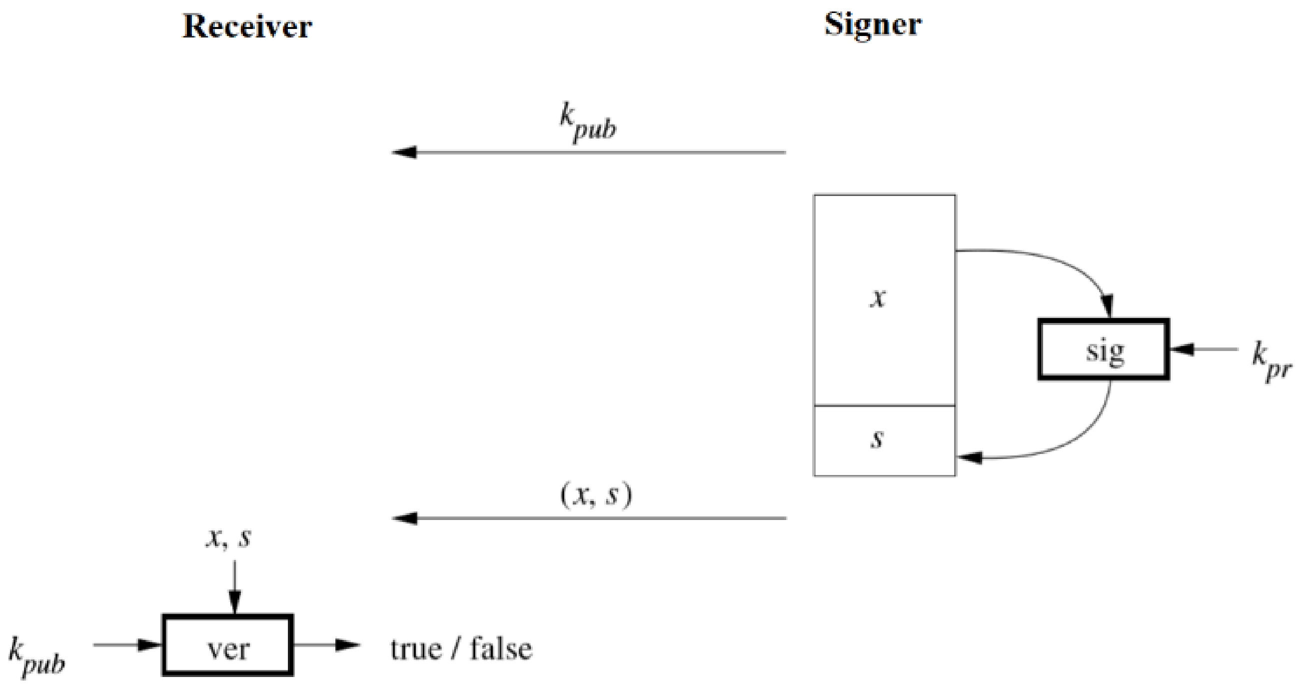
• Each user should have the ability to generate their own signature on any selected document they chose

• Each user should have the ability to efficiently verify whether or not a given string is the signature of another particular user.

• No one should have the ability to generate signatures on documents that the original owner did not sign.

Digital Signature Principles

Being able to prove that a user or individual generated a message is essential both inside and outside the digital domain. In today’s world, this is achieved through use of handwritten signatures. As for generating digital signatures, public-key cryptography is applied, in which the basic idea is that the individual who signs a document or message uses a private key (called private-key), while the individual receiving the message or document must use the matching public-key. The principle of the digital signature scheme is demonstrated in Figure



Digital signature principle

This process starts with the signer, who signs the message x. The algorithm used in the signing process is a function that belongs to the signer's private key (kpr), assuming that the signer will keep the private key secret. Thus, a relation can be created between the message x and the signature algorithm; the message x is also given to the signature algorithm as an input. After the message has been signed, the signature s is attached to the message x, and they are sent to the receiver in the pair of (x, s). It must also be noted that a digital signature is useless without being appended to a certain message, similar to putting a handwritten signature on a check or document.

The digital signature itself has an integer value that is quite large, e.g. a string with 2048 bits. In order for the signature to be verified, a verification function is needed in which both the message x and the signature s are given as inputs to the function. The function will require a public key in order to link the signature to the sender who signed it, and the output of the verification function would be either "true" or "false". The output would be true in a case in which the message x was signed through the private key that is linked with the other key, i.e. the public verification key. Otherwise, the output of the verification function would be false.

Difference between Digital Signature and Message Authentication

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When parties are communicating over an insecure channel, they may wish to add authentication to the messages that they send to the recipient so that the recipient can tell if the message is original or if it has been modified. In message authentication, an authentication tag is generated for a given message being sent; the recipients must verify it after receiving the message and ensure that no external adversary has the ability to generate authentication tags that are not being used by the communicating parties.

Message authentication can be said to be similar to digital signature, in a way, but the difference between them is that in message authentication, it is required that only the second party verify the message. No third party can be involved to verify the message's validity and whether it was generated by the real sender or not. In digital signature, however, third parties have the ability to check the signature's validity. Therefore, digital signatures have created a solution for message authentication.

Conclusion

Cryptography plays a vital and critical role in achieving the primary aims of security goals, such as authentication, integrity, confidentiality, and no-repudiation. Cryptographic algorithms are developed in order to achieve these goals. Cryptography has the important purpose of providing reliable, strong, and robust network and data security. In this paper, we demonstrated a review of some of the research that has been conducted in the field of cryptography as well as of how the various algorithms used in cryptography for different security purposes work. Cryptography will continue to emerge with IT and business plans in regard to protecting personal, financial, medical, and ecommerce data and providing a respectable level of privacy.

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Numerical Solutions of Nonlinear Schrodinger Equation via B-splines Based Differential Quadrature Method

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Abstract

PDEs are the best in simulating and mathematically modelling a variety of phenomena, including elastic behaviour, fluid flows, shallow water waves, gene mutation, and flow turbulence. It is not always feasible to solve these problems analytically when they are mathematically described. Therefore, it could be difficult for researchers to find analytical or exact solutions to these differential equations. Due to the complexity of these differential equations, there are several advanced numerical methods that may be used to solve them. Differential quadrature method (DQM) is a crucial advanced numerical technique that has shown to be the most effective in these conditions for obtaining numerical solution to these differential equations. In this study, Nonlinear Schrodinger Equation (NLSE) is numerically solved using B-spline differential quadrature method. The accuracy and efficiency of this method are shown by the findings, which are equivalent to those found in the literature and close to exact solution. The work, which is presented as figures and tables, is deemed positive.

Keywords: Differential quadrature method; B-spline; Nonlinear Schrodinger Equation; Partial differential equation.

Introduction

The nonlinear Schrodinger equation has applications in many fields, e.g., plasma physics [1], Bose-Einstein condensates [2], quantum mechanics and ocean dynamics [3], nonlinear optics [4], etc. A key component of optical fiber communication is the utilization of optical solitons, one of the most significant solutions to the nonlinear Schrodinger equation [4]–[9]. The one-dimensional nonlinear Schrodinger equation is given as follows [10]:

$$iu_t = \alpha u_{xx} + \beta |u|^2 u + g(x,t)u, x \in [a,b], t \geq 0, \quad (1)$$

with an initial: $u(x,0) = u_0(x)$,

with boundary conditions: $\lim_{|x\to\infty|} u(x,t) = 0$.

Where α, β are arbitrary real numbers, $g(x,t)$ denotes a bounded real-valued function, i is the imaginary unit and $u = c + id$ denotes the complex-valued wave function which shows the evolution of a slowly varying wave train in a stable dispersive physical system with no dissipation. The subscripts x and t represents partial derivatives for space and time, respectively, and u_t is the amplitude of the pulse envelope.

Many mathematicians and engineers have solved the nonlinear Schrodinger problem for a variety of reasons. By using the technique of unknown coefficients, Biswas et al. [11] investigated the optical characteristics of the cubic quartic NLSE. Kumar et al. [12] used the traditional Lie symmetry analysis to ensure the optical soliton solution of the NLSE with generalized anti-cubic nonlinearity. Remizov and Starodubtseva [13] used quasi-Feynman formulae to resolve the multidimensional Schrodinger problem. Remizov [14] solved the NLSE problem using a translation operator. By using the RB sub-ODE and He's semi-inverse methods, Abdel Wahed [1] has solved the (n+1) dimensional NLSE for an analytical answer. Malik et al. [7] used analytical methods to find novel optical solitons, including the Lie symmetry analysis and two iterations of the new extended generalized Kudryashov approach. Osman et al. [8] used two techniques for the perturbed NLSE in nonlinear optical fibers: the first is the extended modified auxiliary equation, and the second is a generalized Riccati equation approach. In addition, the extended trial equation method [15], spectral collocation method [16], split step finite difference method [17], trigonometric cubic B-spline with DQM [10], finite difference method, and the quartic B-spline based differential quadrature method [18], Crank-Nicolson based quintic B-spline differential quadrature method [19], modified cubic B-spine differential quadrature method [20], quintic B-spline Galerkin finite element method [21], exponential cubic B-spline collocation method [22], and exponential cubic B-spline based differential quadrature method [9], are few further effective numerical methods to the nonlinear Schrodinger equation.

Numerical approaches have been shown to be effective in the programming of the solution of differential equations. In order to help researchers locate the numerical solution to differential equations, many mathematical applications are available.

DQM is a well-known numerical technique, is employed to resolve partial differential equations (PDEs). Bellman and Casti [23], [24] presented this technique in the 1970s. This technique underwent revision in the 1980s [25] and was shown to be a useful numerical approach to issues in the physical and engineering sciences [26]. Differential equations in the biosciences, transport processes, fluid mechanics, static and dynamic structural mechanics, and lubrication mechanics may be solved numerically using DQM. Due to its properties of quick convergence, high accuracy, and computational power, it is currently a well-known numerical approach. Professor Chang Shu [27] has produced a book on DQM and its use in general engineering up to year 1999. The weighting coefficient formulation was enhanced by Quan and Chang [28], [29] which is a the most important component of DQM. It is effectively applied to estimate the weighting coefficients using a variety of test functions, including spline functions, Lagrange interpolation polynomials, sinc function, etc. There have been various PDEs that have been solved using DQM based on different basis functions, including quartic B-spline functions, modified cubic B-spline functions, and exponential cubic B-spline functions [30]–[34].

Expo-MCB basis functions also available in literature for solving various equations such as Burgers equation [35], multi-dimensional convection-diffusion equations [36], Sine-Gordon equation [37], Fisher's Reaction–Diffusion Equation [38], telegraph equation [39], etc.

The objective of this work is to find out the numerical solution of the NLS equation, which is a well-known nonlinear partial differential equation using DQM.

The structure of this paper is as follows: The numerical method used in this study the exponential cubic B-spline with the differential quadrature method is covered in Section 2. In section 3, the effectiveness and accuracy of method has been evaluated by using it to solve NLSE problems. Section 4 of this paper is a discussion of the findings.

Numerical Scheme

Exponential modified cubic B-spline differential quadrature method

DQM may be considered of as an approximation to a function's derivative by employing the linear summing of its values at certain discrete grid points over the problem domain. The appropriate weighting factors in the DQM approximation are calculated using “a set of basis functions” that cover the whole problem domain. It is possible to determine the weighting coefficients using a number of basis functions.

In this work, the “exponentially modified cubic B-spline basis functions” are used to obtain the weighting coefficients. DQM with Expo-MCB was adopted from the work of Arora et al. [9]. Now, using the specified MATLAB software for the recommended approach.

An ODE is produced by replacing the DQM-based approximations of the space derivatives with exponential B-spline basis functions, which may then be solved using any appropriate numerical method.

Numerical experiments and discussion

In this section, two numerical problems are considered to show the accuracy and efficiency of the proposed method. The error norm L_∞ is calculated by using the following definition:

$$L_\infty = \| u_{exact} - u_{computed} \|_\infty = \max_j |u_j^{exact} - u_j^{computed}|$$

Problem 1. Consider NLSE (1) with $\alpha = -0.5,\ \beta = 1,\ g(x,t) = cos^2(x),\ x \in [0,2\pi]$ and

$t > 0$. The exact solution of the problem is given by [10] $u(x,t) = sin(x)\, exp\,(^{-3it}/_2)$ with the initial condition as $u(x,0) = sin(x)$ and boundary conditions $u(0,t) = 0 = u(2\pi,t)$.

The comparison of numerical and exact solution is presented in figures 1 to 4. The accuracy of the method is presented with the help of L_∞ norm as shown in Table 1, using N=51 with

$\omega = 1$ and $\Delta t = 0.0001$. This approach yields superior results even when N is exactly half of the one compared in the literature.

Table 1. Comparison of error of solution of problem 1.

t	L_∞	
	Arora et al. [10] ($\mathrm{N} = 100$)	Present (N=51)
1	$1.69e-4$	$8.0553e-05$
5	$6.57e-4$	$3.8156e-04$
10	$2.17e-3$	$1.7590e-03$
20	$8.26e-3$	$7.5760e-03$

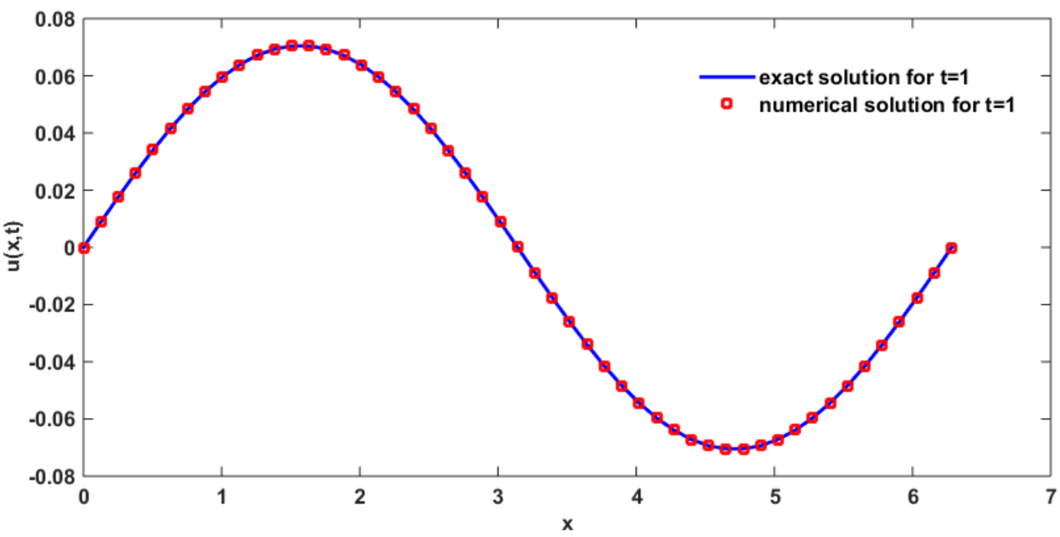

Fig. 1. The comparison of numerical and exact solution of problem 1 for N=51 at t=1.

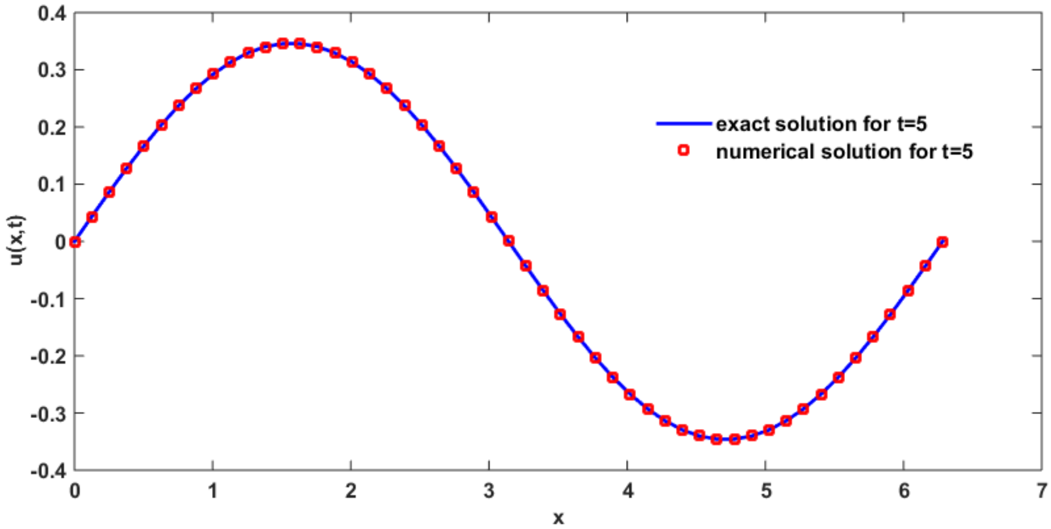

Fig. 2. The comparison of numerical and exact solution of problem 1 for N=51at t=5.

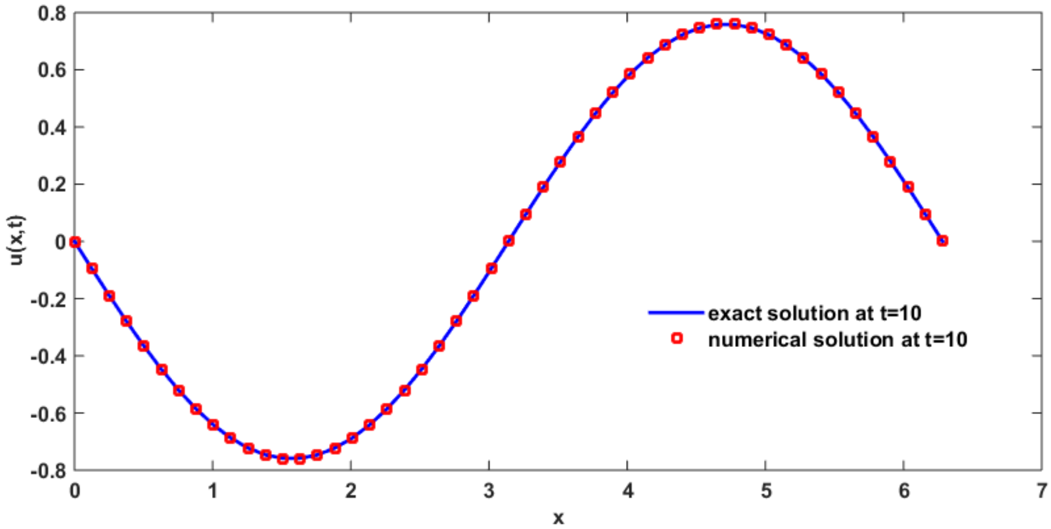

Fig. 3. The comparison of numerical and exact solution of problem 1 for N=51 at t=10.

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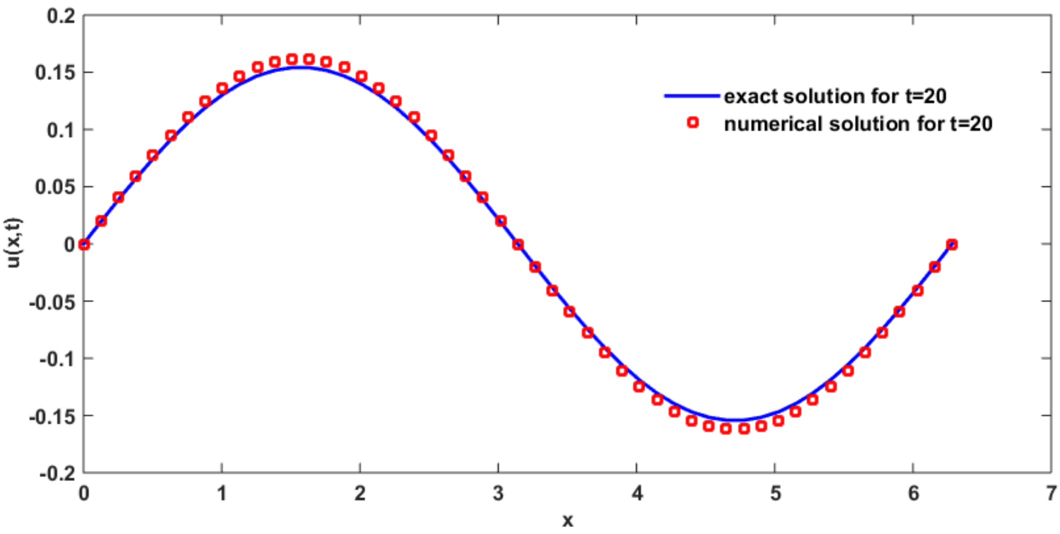

Fig. 4. The comparison of numerical and exact solution of problem 1 for N=51 at t=20.

Problem 2. Consider NLSE (1) with $\alpha = 1, \beta = 2$, $g(x,t) = 0$, $x \in [-15,15]$ and $t > 0$. The exact solution of the problem is given by [10] $u(x,t) = exp\,(-i(2x + 4 - 3t))\,sech(x + 2 - 4t)$ with the initial condition as: $u(x,0) = exp\,(-i(2x + 4))\,sech(x + 2)$ and boundary conditions: $u(-15,t) = 0 = u(15,t)$

The comparison of numerical and exact solution is presented in the figures 5 to 7. To show the accuracy of method L_∞ norm is shown in table 3 using N=301 with $\omega = 3$ and $\Delta t = 0.0001$. Results are better than the available literature results.

Table 2. Comparison of error of solution of problem 2.

t	L_∞	
	Arora et al. [10] ($\mathrm{N} = 200$)	Present (N=301)
0.5	$2.54e - 4$	$2.4820e - 04$
1.0	$1.97e - 4$	$1.8289e - 04$
1.5	$2.32e - 4$	$1.3055e - 04$
2.0	$3.40e - 4$	$1.2879e - 04$
2.5	$4.49e - 4$	$2.0733e - 04$
3.0	$6.68e - 4$	$3.1202e - 04$

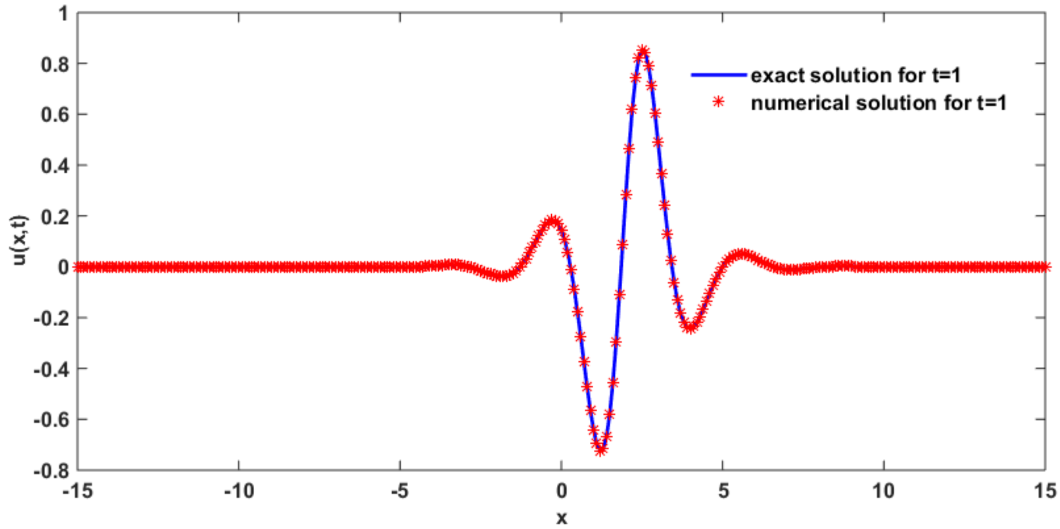

Fig. 5. The comparison of numerical and exact solution of problem 2 for N=301 at t=1.

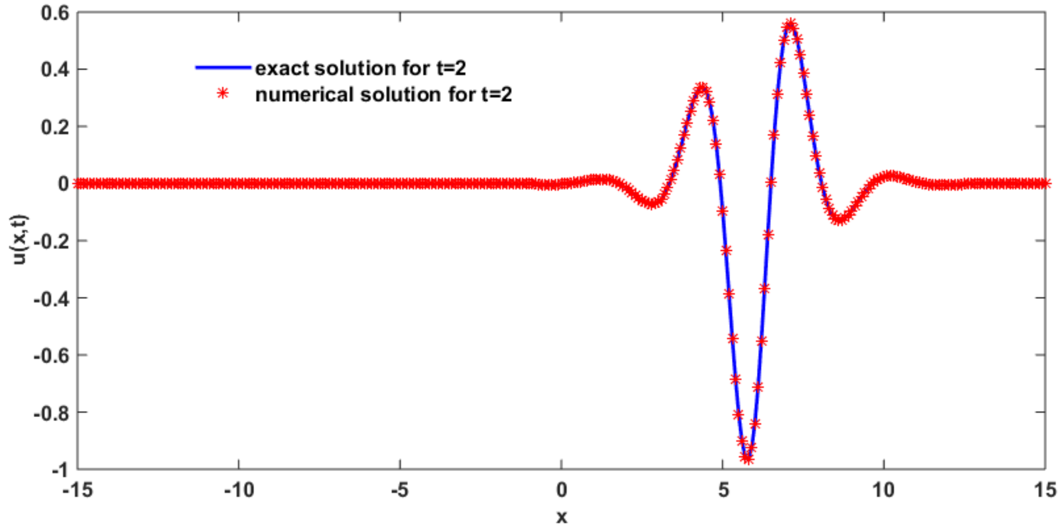

Fig. 6. The comparison of numerical and exact solution of problem 2 for N=301 at t=2.

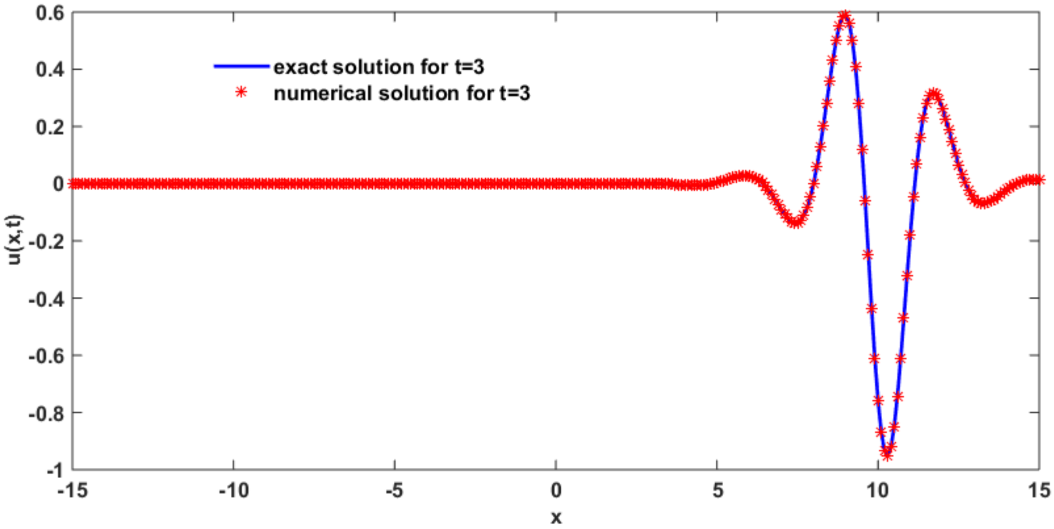

Fig. 7. The comparison of numerical and exact solution of problem 2 for N=301 at t=3.

Conclusion

In this study, differential quadrature method is utilized to determine the NLSE solutions numerically. In this method, the exponential cubic B-spline basis function is employed to determine the weighting coefficient. The accuracy and efficiency of the exponential cubic B-spline differential quadrature method have been validated by error norm L_∞, which demonstrates that the numerical solutions of this method are extremely close to the exact solutions, when compared to other numerical methods that have already been published.

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Singularity In Complex Plane

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Abstract

A singularity is the point on the complex plane at which a complex function either ceases to be defined or increases to an infinitely huge value. This point is referred to as being on the complex plane. To put it another way, this is the point in the function at which things begin to behave in an undesirable manner. Complex functions have the potential to exhibit singularities for a variety of reasons, including poles (locations where the function gets closer and closer to infinity), branch points (locations where the function has a branch with several values), and crucial singularities. Complex functions have the potential to exhibit singularities because of these various reasons (where the function becomes infinitely large but does not approach a specific value). Because they assist to shed light on how complicated functions behave, singularities are a notion that is particularly essential in complex analysis. By way of instance, the study of the singularities of a complex function enables us to determine the analytic properties of the function as well as acquire a better knowledge of how the function behaves in the complex plane. A singularity may also be used to establish the concept of a residue, which provides a measurement of the distinct behaviour of a function at a certain point in time. This can be done by using the singularity as a starting point. The idea of a residue may be formulated by using singularities in many ways. In the complex plane, singularities are sites at which complex functions act in unexpected or unusual ways. Singularities are also known as singularities. They are an essential component of complex analysis, and they provide light on the operation of complicated functions through providing this knowledge. To provide a brief explanation, singularities in the complex plane are sites at which complex functions act in unexpected or unusual ways. The study of singularities on the complex plane has attracted interest from a number of different subfields of mathematics, including complex analysis, topology, and differential equations, to name just a few. In the context of this discussion, the term "singularity" refers to a point on the complex plane at which a specific function either does not possess the capability to be well-defined or cannot be distinguished. Both the kind of singularity and its location inside *the complex plane have the potential to have a substantial impact on the way in which the function behaves. In this article, we examine the concept of singularities in the complex plane, as well as its use in a number of other fields of study. We discuss the many types of singularities, such as poles, essential singularities, and branch points, in addition to the features that are associated with each of these singularities individually. In addition to this, we explore the role of singularities within the theory of complex dynamics, as well as the classification of the many different types of singularities based on the way in which the function is behaving locally. This is done in conjunction with the previous point. The study of singularities in the complex plane is still a developing area of study, and new discoveries of technological breakthroughs and practical applications are discovered on a consistent basis. In conclusion, obtaining an understanding of the properties and behaviours of singularities in the complex plane is an essential step in developing a deeper familiarity with complex analysis and the numerous ways in which it can be applied in a wide variety of fields. This is because singularities play an important role in the complex plane.*

Key Words: Singularity, Complex Plane**,** function

Introduction

There are a number of nonlinear partial differential equations (PDE) with solutions that converge on singularities in a limited amount of time. The subject of whether or not a partial differential equation (PDE) has a singular behaviour is a very significant one, and it is of interest to mathematicians for their own reasons. This is because the presence of a singularity indicates the boundary of the PDE's applicability as a mathematical model and frequently has important implications for the physical world. The reason for this is due to the fact that the presence of a singularity indicates the boundary of the applicability of the PDE. As a consequence of this, there is a great amount of interest in the development of approaches that may offer signs as to whether or not a singularity is arising, where it is forming, and on the nature of the singularity itself. In this review article, we are going to focus on methods that are based on the analytic continuation in the complex domain of numerical solutions of PDEs that are derived via spectral discretization. This will be our primary area of investigation.

There are a few distinct nonlinear partial differential equations (PDE) that have solutions that converge on singularities in a constrained period of time. The question of whether or not a partial differential equation (PDE) has a singular behaviour is a highly important one, and mathematicians find it interesting for a variety of different reasons. This is due to the fact that the existence of a singularity denotes the limit of the PDE's applicability as a mathematical model, and it typically has significant repercussions for the physical world. The occurrence of a singularity denotes the limit of the PDE's application, which is why this is the case. The rationale for this can be found in the previous sentence. As a result of this, there is a great deal of interest in the development of methods that may offer signs as to whether or not a singularity is arising, where it is forming, and on the nature of the singularity itself. This interest is due to the fact that such methods may offer a glimpse into the future. In this article, we are going to concentrate on approaches that are based on the analytic continuation in the complex domain of numerical solutions of PDEs that are generated by spectral discretization. These numerical solutions are derived via the spectral discretization method. Our major focus throughout our research will be on this particular location. [1-5]

The motion of the analogous inviscid vortex sheet is predicted by the BR equations, but the development of the thin vortex layers is predicted by the NS equations. The movement of the large inviscid vortex sheet is predicted by the BR equations, but the growth of the thin vortex layers is predicted by the NS equations. The NS equations' task is to predict the development of the thin vortex layers, and the BR equations' task is to predict the motion of the corresponding inviscid vortex sheet. It is generally known that the Kelvin-Helmholtz instability may occur in the Birkhoff-Rott equation, which controls how an inviscid vortex sheet moves. This is because the motion of an inviscid vortex sheet is controlled by the Birkhoff-Rott equation. This is so because the Birkhoff-Rott equation is in charge of determining how an inviscid vortex sheet will move. This theory is based on the idea that even seemingly little disruptions have the potential to quickly snowball into much larger ones. This type of instability has several effects, but ill-posedness—which emerges as a blow up in the curvature—is the most noticeable. [6-8]

Vortex Layers

Making plans and getting started

Inside of the periodic domain that is two-dimensional $D^* = [-L_x/2, L_x/2] \times [-L_y/2, L_y/2]$ Let's say for the sake of argument that the flow in issue is both viscous and incompressible. The Navier-Stokes equations, which, according to the definition of the vorticity stream function, are: Our working hypothesis is that the Navier-Stokes equations are:

$$\frac{\partial \omega^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla_{\mathbf{x}^*} \omega^* = \nu \nabla_{\mathbf{x}^*}^2 \omega,$$
$$\nabla_{\mathbf{x}^*}^{\perp} \psi^* = \mathbf{u}^*,\ \nabla_{\mathbf{x}^*}^2 \psi^* = -\omega^*$$

Where

$\mathbf{x}^* = (x^*, y^*) \in D^*, \nabla_{\mathbf{x}^*} = (\partial_{x^*}, \partial_{y^*})$
$\nabla_{\mathbf{x}^*}^{\perp} = (\partial_{y^*}, -\partial_{x^*}), \mathbf{u}^* = (u^*, v^*)$ is the velocity

field ψ^* is the stream function, ω^* is the Both v and the vorticity of the fluid are valid methods for determining a fluid's kinematic viscosity.

When we make use of, we have the capability of transforming the equations into a form that does not depend on any dimensions. It has the qualities of length and width.

$\lambda = L_x/2\pi$ and the quantity $\Gamma = \int_{D^*} \omega_0^* dS^*$, where ω_0^* is the first piece of information. Therefore, nondimensional quantities are defined as the following:

$$(x, y) = \frac{(x^*, y^*)}{\lambda},\ t = t^* \frac{\Gamma}{\lambda^2},\ (u, v) = (u^*, v^*)\frac{\lambda}{\Gamma},\ \omega = \frac{\omega^* \lambda^2}{\Gamma},$$

The Reynolds number, despite the fact that it is,

$$Re = \frac{\Gamma}{\nu}$$

Fixed and Movable Singularities

As was said at the beginning of the article, there are two unique types of singularities: fixed singularities and movable singularities. It is necessary for us to differentiate between these two types of singularities. These ideas are now defined in a manner that is more specific than it was in the past. In the vast majority of instances, the collection of unchanging singularities $\Phi \subset \mathbf{C}$ is a group of points on the complex plane where a solution to an equation could behave in a way that is not typical of the behaviour of all solutions to the equation. These points are called singularities. On the other hand, it's feasible that a solution won't have a singularity at any point in the space it occupies. Φ [9-15]

First-Order Rational Equations

In his Stockholm lectures, P. Painlev'e presented a definition of fixed singularities for first-order rational equations; for discussions about this issue, please refer to the works authored by Ince and Hille. Imagine for a second that was a part of the equation.

$$y' = \frac{P(z,y)}{Q(z,y)}$$

Polynomials P and Q are examples of polynomials in y with coefficients that belong to a certain class of functions, such as the domain of algebraic functions. These coefficients belong to a certain category of functions. Assuming that the formula on the right-hand side of (1) is in reduced terms, or that the polynomials P and Q do not share a common factor, we proceed as follows. [16-18]

Definition 1. Let Φ_0 be the gathering of singular points corresponding to the coefficients of P and Q in such a manner that $D = \mathbf{C} \setminus \Phi_0$ is the largest domain on which analytic behaviour can be seen for all coefficients, and it includes all of the coefficients. It is possible to consider the union as defining the collection of constant singularities for (1).

$$\Phi = \Phi_0 \cup \Phi_1 \cup \Phi_2 \cup \Phi_3 \text{ where}$$
$$\Phi_1 = \{\zeta \in D: Q(\zeta, y) \equiv 0\},$$
$$\Phi_2 = \{\zeta \in D: P(\zeta,\eta) = Q(\zeta,\eta) = 0 \text{ for some } \eta \in \mathbf{C}\},$$
$$\Phi_3 = \{\zeta \in D: \grave{P}(\zeta,0) = \grave{Q}(\zeta,0) = 0\}.$$

Here $\grave{P}$ and $\grave{Q}$ are polynomials in $u = 1/y$ such that $u' = \frac{\tilde{P}(z,u)}{\tilde{Q}(z,u)}$ where $\tilde{P}$ and $\tilde{Q}$ are, once again, communicated via the use of shortened words.

Any solution singularity that is not one of the "fixed singularities" is referred to as a "movable singularity," and the word "movable singularity" is used to characterize this phenomena. Painlev'e demonstrated that all movable singularities are algebraic for any solution to (1), i.e., they are either poles or algebraic branch points. This discovery was important. Each conceivable remedy has this flaw. This demonstrates that the solution is embodied by an extension of a convergent series in a cut neighborhood centre on a mobile singularity z0. [19-22]

$$y(z) = \sum_{k=k_0}^{\infty} c_k (z - z_0)^{k/n}, k_0 \in \mathbf{Z}, n \in \mathbf{N}.$$

For evidence that supports the truth of this argument, have a look at the textbooks that were written by authors such as Hille or Ince Example, for instance. Take a look at the formula, would you?.

$$y' = \frac{1 + y^2}{z^2}$$

Which of these provide the most comprehensive response?

$$y(z) = \tan\left(c - \frac{1}{z}\right)$$

$c \in \mathbf{C}$ being the constant that is used in the process of integration. The spot where the singularity may be discovered, which can be located at

$z = 0$ does not depend on the in any manner at all preexisting circumstances and belongs to the set Φ The places where the remaining singularities may be found, which may be found here:

$z = \left(c - (2k + 1)\frac{\pi}{2}\right)^{-1}$ c determines how things change, and as a result, people are free to roam around.

Objectives of The Study

1. To study on Fixed and movable singularities
2. To study on Vortex layers

Research Method

The singularity of a solution that is not part of the set of fixed singularities is known as a moveable singularity.. This singularity is the cornerstone of research method since it is not part of the set of fixed singularities. Included in rational equations are a definition of fixed singularities and a basis on the natural extension of Wigner matrices. Both of these things have i.i.d. entries up to symmetry limitations, which is a requirement for the inclusion of these concepts.[23-25] Secondary sources include things like periodicals, journals, newspapers, web sites, and articles, all of which are used by our organization.

Result

In order to provide a space that can be measured $\mathfrak{A}$ in addition to a separate group $\mathbb{D} \subseteq \mathbb{C}$ The notation is used to talk about numbers that are complicated. $\mathscr{B}(\mathfrak{A}, \mathbb{D})$ the domain of measurable functions that have clearly defined bounds on $\mathfrak{A}$ Having guiding concepts in mind $\mathbb{D}$. Let $(\mathfrak{X}, \pi(\mathrm{d}x))$ be a place for measurement that has positive a measurement that is constrained but not equal to zero. π. be a place for measuring that has positive measure that is restricted but not zero and does not have zero measure. $a \in \mathcal{B}(\mathfrak{X}, \mathbb{R})$ in addition to having a favourable, symmetrical, and $s_{xy} = s_{yx}$function $s \in \mathcal{B}(\mathfrak{X}^2, \mathbb{R}_0^+)$ The next item that we are going to have a look at is the quadratic vector equation (QVE),

$$-\frac{1}{m(z)} = z + a + Sm(z),\ z \in \mathbb{H},$$

for a function $m: \mathbb{H} \to \mathcal{B}(\mathfrak{X}, \mathbb{H}), z \mapsto m(z)$, ' where $S: \mathcal{B}(\mathfrak{X}, \mathbb{C}) \to \mathcal{B}(\mathfrak{X}, \mathbb{C})$ is the integral operator with kernel s,

$$(Sw)_x := \int s_{xy} w_y \pi(\mathrm{d}y),\ x \in \mathfrak{X}, w \in \mathcal{B}(\mathfrak{X}, \mathbb{C}).$$

We equip the space $\mathcal{B}(\mathfrak{X}, \mathbb{C})$ with its natural norm,

$$\| w \| := \sup_{x \in \mathfrak{X}} |w_x|,\ w \in \mathcal{B}(\mathfrak{X}, \mathbb{C}).$$

With this norm $\mathcal{B}(\mathfrak{X},\mathbb{C})$ is a Banach space. For an operator T on $\mathcal{B}(\mathfrak{X},\mathbb{C})$ we write $\| T \|$ for the standard deviation of the induced operator norm

The following is considered to be folklore, in accordance with the standards of the canon of literary criticism: In the interest of full disclosure, we have given its evidence, which has been adapted to be consistent with our circumstances.[26-27]

Proposition 1: (Existence and uniqueness). The QVE provides a new approach to the problem. In every instance, $x \in \mathfrak{X}$ Estimating the likelihood of something happening is possible in only one approach. $v_x(\mathrm{d}\tau)$ on $\mathbb{R}$ such that

$$m_x(z) = \int_{\mathbb{R}} \frac{v_x(\mathrm{d}\tau)}{\tau - z}, \ z \in \mathbb{H}.$$

Each of these efforts receives support from the compact in its many forms. interval $[-\kappa, \kappa]$ with

$$\kappa := \| a \| + 2 \| S \|^{1/2}.$$

The family $(v_x)_{x\in\mathfrak{X}}$ ... constitutes a measurable function $v: \mathfrak{X} \to \mathcal{M}(\mathbb{R}), x \mapsto v_x$, where $\mathcal{M}(\mathbb{R})$ indicates the space of probability measures that are fitted with the weak topology and centred on R. [28-30]

Wigner Type Matrices

In the past, matrices of the Wigner type were simply referred to as matrices of the Wigner type. They are a natural extension of Wigner matrices, which include identically replicated elements up to the point when symmetry requirements are applied. These matrices H are both self-adjoint and centred, and there is not a diagonal in any of those properties. elements. $\mathbb{E}h_{ij} = 0$, may have their own separate entries provided that they do not break the rules. requirements for symmetry, which include ar., h_{ij} are independent for $i \leq j$. Furthermore, let us consider the following: $N|h_{ij}|^2$ possess the ability to be consistently incorporated. For the purpose of simplicity, let's assume that the variances of the components of H ultimately converge to a continuous, symmetric, and uniform piecewise 1/2-Holder distribution..

$q(x,y) = q(y,x)$, profile function $q: [0,1]^2 \to \mathbb{R}_0^+$with a non-vanishing diagonal, inf $|x - y| \leq \varepsilon q(x,y) > 0$ for some $\varepsilon > 0$, i.e.,

$$\mathbb{E}|h_{ij}|^2 = \frac{1}{N} q\left(\frac{i}{N}, \frac{j}{N}\right).$$

The next thing that will be demonstrated is the empirical spectrum measurements of the matrices. $H^{(N)}$ 'converge, as $N \to \infty$, to a non-random limit,

$\rho^{(N)}(\mathrm{d}\tau) \to \rho_S(\mathrm{d}\tau)$ probably not very strongly at all.

The next thing that will be demonstrated is the empirical spectrum measurements of the matrices.

ρ_S is determined by applying the solution m to the QVE in the configuration $(\mathfrak{X}, \pi(\mathrm{d}x)) = ([0,1], \mathrm{d}x)$ In the case when an equals zero, the integral kernel of S may be determined by using the asymptotic variance profile. $s_{xy} := q(x,y)$ .As an illustration of (with the Gaussian distribution serving as the base), (with the extra stipulation that the fourth moments must have a profile), and (with bounded higher moments). By making use of a cut-off argument in the standard style, bounded moment conditions may have their restrictions relaxed.[31-42]

We are able to provide a deeper level of comprehension on the limiting eigenvalue distribution S by making use of the theorem. Actually, $\rho_S(\mathrm{d}\tau) = \rho_S(\mathrm{d}\tau)\mathrm{d}\tau$ has a density that is Holder-continuous and contains singularities of degree that total no more than three, according to the definition of the term. In addition to this, the theorem states that the limiting spectral density, S, is established by determining whether or not q is piecewise. Both the Holder continuous and the 1/2-Holder continuous will be used. sustained for the course of a single period $[-\tau_0, \tau_0]$. and has square root singularities at the edges 0 and 0 of its domain.

In general, there is a possibility that cusps already exist in even the simplest circumstances that are not noteworthy. An instance of this issue is provided by the block profile of a 2×2 configuration.[43-45]

$$q := \alpha \mathbb{1}_{I \times I} + \beta(\mathbb{1}_{I \times I^c} + \mathbb{1}_{I^c \times I}) + \gamma \mathbb{1}_{I^c \times I^c},$$

where,, and are all constants that are in the positive, $I = [0,], andIc = [,1]$ for any value that is between $[0,1/2]$ and $[0,1/2]$. Consider, for example, the extraordinary situation: $\beta = 1$ and $\gamma = 1/\alpha$ the alternative $\delta = \delta_c(\alpha)$ with $\alpha > 2$ the alternative

$$\delta_c(\alpha) := \frac{(\alpha - 2)^3}{9(\alpha^3 - 2\alpha^2 + 2\alpha - 1)}$$

Our attention has been drawn to the fact that the solution m of the QVE, which corresponds to the 2 2-profile (2) through, S_{xy} = For some component functions H H, q(x, y) has the form m(z) = $(z)1I + v(z)1I$ c, which may alternatively be written as $m(z) = (z)1I + v(z)1I$ c. (z). By using this approach, the original QVE might be simplified into a two-dimensional system. The ansatz is made up of the following components: (,). Even in this relatively simple situation, you will still need to solve a quartic polynomial in order to get the QVE solution for general,, and. Despite this, the Theorem indicates that there will never be a quartic singularity of S. This is because S has an infinite number of dimensions. [46]

Translation Invariant Correlations

Let $\underline{\xi} = (\xi_{ij} : i, j \in \mathbb{Z})$ be a group of independent and uncorrelated random variables that are indexed by $\mathbb{Z}^2$. Let $\theta_{pq}, p, q \in \mathbb{Z}$, imply a change in position of $^{\xi}$, , in such a way that the component with the identifiers I j) of $\theta_{pq}\underline{\xi}$ is $\xi_{i+p,j+q}$ Given a measurable function $\Phi: \mathrm{RZ}^2 \to \mathrm{R}$, such that

$$\mathbb{E}\Phi(\underline{\xi}) = 0,\ \mathbb{E}\Phi(\underline{\xi})^2 < \infty, \text{ and } \sum_{p,q \in \mathbb{Z}} \left| \mathbb{E}\Phi\left(\theta_{pq}\underline{\xi}\right) \Phi(\underline{\xi}) \right| < \infty,$$

We build a series of translation-invariant random matrices starting with $\mathrm{H} = \mathrm{H(N)}$ and going all the way up to $\mathrm{H} = \mathrm{H(H)}$

$$h_{ij} := \frac{\Phi\left(\theta_{ij}\underline{\xi}\right) + \Phi\left(\theta_{ji}\underline{\xi}\right)}{\sqrt{N}}.$$

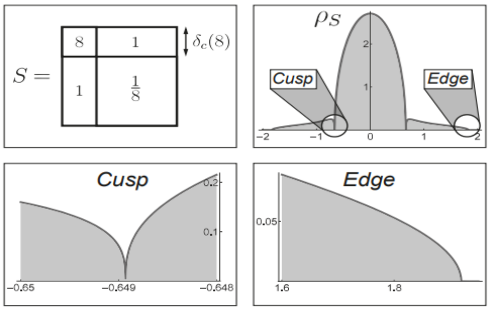

Figure 1: a 22 block profile with a cusp singularity in the density .

It has been brought to our attention that the QVE's solution m, which corresponds to the 2.2-profile (3.2) through s_{xy} = For some component functions H H, q(x, y) has the form m(z) = (z)1I + v(z)1I c, which may alternatively be written as m(z) = (z)1I + v(z)1I c. (z). This ansatz allows the original QVE to be simplified into a two-dimensional system. The following elements comprise the ansatz: (,). Despite the situation's relative simplicity, in order to obtain the QVE solution for general, you will still need to solve a quartic polynomial. Despite this, the Theorem indicates that a quartic singularity of S will never occur in the future.[47]

Applications

In the lines that follow, we are going to discuss three different applications of random matrix theory. By using the notation H = H (N), we identify a sequence of self-adjoint random matrices with entries hi j = h (N) I j on some probability space with expectation E. In this case, N may be either an even number or an odd number. The following is how the induced normalised empirical spectral measurements are defined:

$$\rho^{(N)}(A) := \frac{\left|\operatorname{Spec}\left(H^{(N)}\right) \cap A\right|}{N}$$

Borel's equations may be solved by setting A of R. From this point forward, the term 'S(d' will be used to refer to the average generating measure for the QVE when operator S is involved. That is to say that [48-50]

$$\rho_S(\mathrm{d}\tau) := \int_X v_x(\mathrm{d}\tau)\pi(\mathrm{d}x)$$

Conclusion

I matrices contain correlated entries with a correlation structure that is translation-invariant, and (ii) matrices have centred independent components with variances provided by S. Our analysis of the limiting eigenvalue density showed that it is composed entirely of square-root singularities and cubic-root cusps. Since no other singularities exist, this must be the case. We show that this solution's dependency on z may be represented by a variety of Stieltjes transformations of probability measures v on R. The symbol R stands for the entire class of probability measurements. It is possible to do this by performing a transformation on the z-values. With the exception of a finite number of algebraic singularities of degree no more than three, we prove that on S, v has a real analytic density under certain circumstances. In this article, we examine the singularities in the solution of as a function of z. Our results are tabulated for your convenience. With the exception of a few of places specific to the situation, we prove that m(z) is analytic in z all the way down to the real axis. Our fundamental theorem states that these singularities are algebraic and can only be of degree two or three given specific natural constraints on S. With this constraint, the pool of viable options for fixing the issue is narrowed down significantly. To date, this is our most important finding. The building blocks of m(z) are, in reality, Stieltjes transformations of a family of densities on R. Support for both densities is the same infinitely large set of small intervals.

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A Comparative study of Euler Method and Adomain Decomposition Method (ADM)

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Abstract

If a solution is represented by mean of infinite series, a finite portion of the series can be taken as approximate solution. There are many ways to solve the ordinary differentiable equation of first order but in following paper, we will discuss two methods to obtain the approximate solutions for the ordinary Differential Equation of first order. , the first one is "Adomain Decomposition Method (ADM)" and the second one is "Euler's Method". Moreover, we will make comparison between the solutions obtained by the two methods. The numerical results are presented through table.

Keywords: Differential transformation method, Adomain Decomposition Method, Ordinary Differential Equation, solution.

Introduction

Differential equations play an important role in the study of many natural sciences physical sciences and engineering. Some time due to some complexities it is difficult to solve the differential equation by direct method, so we adopt some approximation methods. By using these methods we find the solution in the series form. Many researchers have done a lot of work in this field. But Adomain decomposition method was firstly introduced by George Adomain (1986, 1988, and 1994). Nhawu *et al.*[1] investigated ADM method to solve some first-order differential equations. It is shown that the series solutions converge to the exact solution for each problem. It is observed that the method is particularly suited for initial value problems with oscillatory and exponential solutions. **Jafar et al.[2]** used Adomain decomposition method to solve for ordinary differential equation of any order by converting it into a differentiable equation of order one. Theoretical considerations are being discussed and converge of the method of these systems is addressed in this article. Later on Abdul at al.[4] investigated a new medication of Adomain decomposition method for linear and non linear operators. They conducted comparative study between the new modification and the modified main decomposition method. An algorithm for ADM

Method is given by Wenhai Chen at al.[3]. In the present paper we will compare Euler method and Adomain decomposition method.

Adomain Decomposition Method (ADM)

The Adomain Decomposition Method is a semi analytical method to solve the ordinary differential method. This method is very effective for solving wide class of non linear partial and ordinary differential equation.

The transformation of the kth derivative of a function in one variable is as follows:

$$L^n = \frac{d^n}{dx^n}$$

And the inverse transformation is defined as

$$L^{-n} = \int_0^x \int_0^x \ldots\ldots \int_0^x \quad \text{(n times)} \qquad (2)$$

Euler's Method

Euler's method is a numerical technique used to solve ordinary differential equation of the form

$$\frac{dy}{dx} = f(x,y)$$

$y(0)=y_0$

The Euler's method is

$y_{k+1} = y_k + h\, f(x_k, y_k)$

Numerical Example

Consider an ODE of first order for one parameter, say f(x,y).

Let the differential equation be, $\frac{dy}{dx}$ **= x+y** with initial condition, y(0)=1. Assume h=0.1. Now let us find the value of y(1).

From the given, y(0) = 1 →x0=0 ; y0 =1 and h=0.1

By Euler's method

y(0.1)=1+0.1(0+1)

y(0.1)=1.1

y(0.2)=1.1+0.1(0.1+1.1)

Y(0.2)=1.22

Y(0.3)=1.22+0.1(0.2+1.22)

Y(0.3)=1.36

Y(0.4)=1.36+0.1(0.3+1.36)

y(0.4)=1.53

Y(0.5)=1.53+0.1(0.4+1.53)

Y(0.5)=1.72

Y(0.6)=1.72+0.1(0.5+1.72)

Y(0.6)=1.94

Y(0.7)=1.94+0.1(0.6+1.94)

Y(0.7)=2.19

Y(0.8)=2.19+0.1(0.7+2.19)

Y(0.8)=2.48

Y(0.9)=2.48+0.1(0.8+2.48)

Y(0.9)=2.81

Y(1)=2.81+0.1(0.9+2.81)

Y(1)=3.81

By Adomain Decomposition Method and Inverse Adomain Decomposition Method

Operator L = $d/_{dx}$ and L^{-1}=$\int_0^x dx$

Y(x) =1 + x + $\frac{x^2}{2}$ + $\frac{x^3}{3}$ + $\frac{x^4}{12}$ + ….

Using intial condition y (0) = 1 and by substituting the successive values

From the above relation we obtain

y (0.1)= 1.1103

y (0.2)= 1.2428

y (0.3)= 1.3997

y (0.4)= 1.5835

y(0.5)=1.7969

y(0.6)=2.0428

y(0.7)=2.3243

y(0.8)=2.6448

y(0.9)=3.0077

y(1)=3.4167

Exact Solution

The exact solution of equation above is

Y=-x-1+2*exp(x)

This is obtained as

Equation is of the form $\frac{dy}{dx} + Py = Q$

Where P and Q are function of x .

Integrating factor (I.F)=$e^{\int Pdx}$

Solution is

y. (I.F) =$\int Q(I.F.)dx + c$

Where c is constant of integration, which is obtained from intial condition.

The values obtained from Euler's method and **ADM** method are tabulated below:

Table 1 - Comparison of Accuracy Between Euler method and ADM method

Method	**Euler's Method**	**ADM**	**Exact Solution**
Y(0)	**1**	**1**	**1**
Y(0.1)	**1.1**	**1.1103**	**1.1103**
Y(0.2)	**1.22**	**1.2428**	**1.2428**
Y(0.3)	**1.36**	**1.3997**	**1.3997**
Y(0.4)	**1.53**	**1.5835**	**1.5836**
Y(0.5)	**1.72**	**1.7969**	**1.7974**
Y(0.6)	**1.94**	**2.0428**	**2.0442**
Y(0.7)	**2.19**	**2.3243**	**2.3275**
Y(0.8)	**2.48**	**2.6448**	**2.6511**
Y(0.9)	**2.81**	**3.0077**	**3.0192**
Y(1)	**3.18**	**3.4167**	**3.4366**

Conclusion

In this paper we have discussed ADM Method and Euler method separately. Then we did proper comparison of these two methods with examples. From above table we can see that approximate error of solution that we are getting by Euler method is much more then ADM method. So we can easily conclude that ADM Method which is rarely known is far better and easy to apply then Euler method.

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Two-dimensional deformations in a nonlocal transversely isotropic functionally graded thermoelastic medium with rotation due to thermal load

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Abstract

The present research work deals with the analysis of two-dimensional deformations in a rotating, nonlocal, functionally graded (non-homogeneous), transversely isotropic thermoelastic half-space under the application of a thermal load in the context of Lord and Shulman theory. Material properties are assumed to be graded in z*-direction and the normal mode analysis is used to obtain the exact expressions for the normal displacement, stress components and temperature field. The profiles of field variables are computed numerically and limned graphically to observe the disturbances induced in the medium. Certain particular cases of interest have also been deduced from the current investigation.*

Keywords: Nonlocal; Functionally graded materials; Thermoelasticity; Green-Lindsay theory; Normal mode Analysis.

Introduction

The theory of thermoelasticity is concerned with the relationship between elastic properties of a material and its temperature. Biot [1] gave a new impulse to do research works in the area of thermoelasticity by presenting equations of thermoelasticity with coupling of temperature and deformation fields. A generalization of the theory of thermoelasticity was put forward by Lord and Shulman [2], which involves single relaxation time in equation for heat conduction. They developed the theory by including a heat flux rate term in the classical Fourier's equation of thermal conduction. As a result, it ensures the finite speed of heat propagation because the obtained heat equation is hyperbolic.

Functionally graded materials (FGMs) are defined as the advanced materials whose thermal and elastic characteristics vary continuously corresponding to change in spatial coordinates. These types of specifications develop spatial heterogeneity in the materials. FGMs are designed to work in high temperature field and as a result, these materials are highly helpful in nuclear reactors, space technology and aviation applications. Lagrangian finite element formulations was proposed by Reddy and Chin [3] to analyze the pseudodynamic thermal vibrations induced in functionally graded elastic cylinders. A problem on one-dimensional transient thermal stresses in nonhomogeneous plates, spheres and cylinders was solved by Wang and Mai [4] by adopting finite element technique. Abbas and Zenkour [5] examined the electro-magnetic responses of a nonhomogeneous thermoelastic cylinder in the purview of LS theory, with the help of finite element technique. Thi [6] analyzed the thermal vibration responses of functionally graded porous plates with varying thickness resting on two-parameter based elastic foundations adopting finite element method.

Eringen [7] established the theory of nonlocal elasticity by applying the idea of nonlocality to the field of elasticity. According to the nonlocal theory, the stress field at a particular point of an elastic continuum body not only depends on the strain field at that point, but also on the strains at other points of the continuous body. The concept of nonlocality has been extended to elasticity and various other fields by Eringen and his co-author [8-10]. A nonlocal generalization of Fourier's law of heat conduction in two-dimensional thermal lattices is presented by Challamel *et al*. [11]. Saeed and Abbas [12] examined the non-local thermoelastic interactions in a nanoscale material in the context of Green and Naghdi theory (without energy dissipation). Recently, Kumar *et al*. [13] discussed the propagation of photo-thermal waves in a nonlocal semiconductor in the context of two phase lag theory of thermoelasticity.

The objective of the present research work is to analyze the deformations in a rotating nonlocal functionally graded, transversely isotropic thermoelastic medium due to a thermal load in the context of Lord-Shulman theory. The normal mode analysis is used to obtain exact expressions of the field variables.

Basic Equations and Mathematical Formulation

Following Eringen [9], Challamel *et al*. [11] and Lord and Shulman [2], the constitutive relations and governing field equations for a rotating nonlocal transversely isotropic functionally graded thermoelastic medium are as:

The Constitutive Relations

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{ij}=\sigma_{ij}^L=C_{ijkl}e_{kl}-\beta_{ij}\theta\delta_{ij} \tag{1}$$

$$\left(1-\varepsilon^2\nabla^2\right)\rho T_0\eta=\left(\rho T_0\eta\right)^L=\rho C_E\theta+T_0\beta_{ij}e_{ij}\,, \tag{2}$$

$$e_{ij}=\frac{1}{2}\left(u_{k,j}+u_{j,i}\right), \tag{3}$$

The Equation of Motion

$$\sigma_{ji,j}=\rho\left[\ddot{\boldsymbol{u}}+\left(\boldsymbol{\Omega}\times\left(\boldsymbol{\Omega}\times\boldsymbol{u}\right)\right)+\left(2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}}\right)\right]_i, \tag{4}$$

Nonlocal Generalization of FOURIER'S Law

$$\left(1-\varepsilon^2\nabla^2\right)\left(1+\tau_0\frac{\partial}{\partial t}\right)q_i=-K_{ij}\theta_{,j}\,, \tag{5}$$

Energy equation:

$$\rho T_0\dot{\eta}=-q_{i,i}\,, \tag{6}$$

Heat conduction equation:

$$\left(K_{ij}\theta_{,j}\right)_{,i}=\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left[\rho C_E\theta+T_0\beta_{ij}e_{ij}\right], \tag{7}$$

where $i,j,k,l=1,2,3,\sigma_{ij}$ are the components of stress tensor, $\varepsilon=e_0 s$ is nonlocal parameter, e_0 is the material constant corresponding to the internal characteristic length *s*, C_{ijkl} are the elastic coefficients, e_{ij} are the components of strain tensor, β_{ij} are the components of thermal elastic coupling tensor, σ is the specific entropy, $\boldsymbol{\Omega}$ is the rotation vector, C_E is the specific heat at constant strain, $e_{kk}=e$ is the cubical dilatation, δ_{ij} is Kronecker delta, u_i are the components of displacement vector $\boldsymbol{u},\theta=T-T_0,T$ is absolute temperature, T_0 is temperature of the material in its natural state. The quantities σ_{ij}^L and $\left(\rho T_0\eta\right)^L$ correspond to local thermoelastic solid. Here, a dot indicates partial temporal derivative, comma denotes material derivative and the summation convention is used.

For a functionally graded medium, the parameters $C_{ijkl}, \beta_{ij}, K_{ij}$ and ρ are no longer constant but become space-dependent. Hence we consider

$$\left[C_{ijkl}, \beta_{ij}, K_{ij}, \rho\right] = f(\boldsymbol{x})\left[C'_{ijkl}, \beta'_{ij}, K'_{ij}, \rho'\right],$$

where $C'_{ijkl}, \beta'_{ij}, K'_{ij}$ and ρ' are constants and $f(\boldsymbol{x})$ is given non-dimensional function of the space variable $\boldsymbol{x} = (x, y, z)$. Using these values of parameters, equations (1)-(3) take the following forms

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{ij} = \sigma_{ij}^L = f(\boldsymbol{x})\left[C'_{ijkl}e_{kl} - \beta'_{ij}\theta\delta_{ij}\right], \quad (8)$$

$$\boldsymbol{\sigma}_{ij,j} = f(\boldsymbol{x})\rho'\left[\ddot{\boldsymbol{u}} + \left(\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\right) + \left(2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}}\right)\right]_i, \quad (9)$$

$$\left[f(\boldsymbol{x})\left(K'_{ij}\theta_{,j}\right)\right]_{,i} = f(\boldsymbol{x})\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\rho' c_E\theta + \beta'_{ij}T_0 u_{i,j}\right]. \quad (10)$$

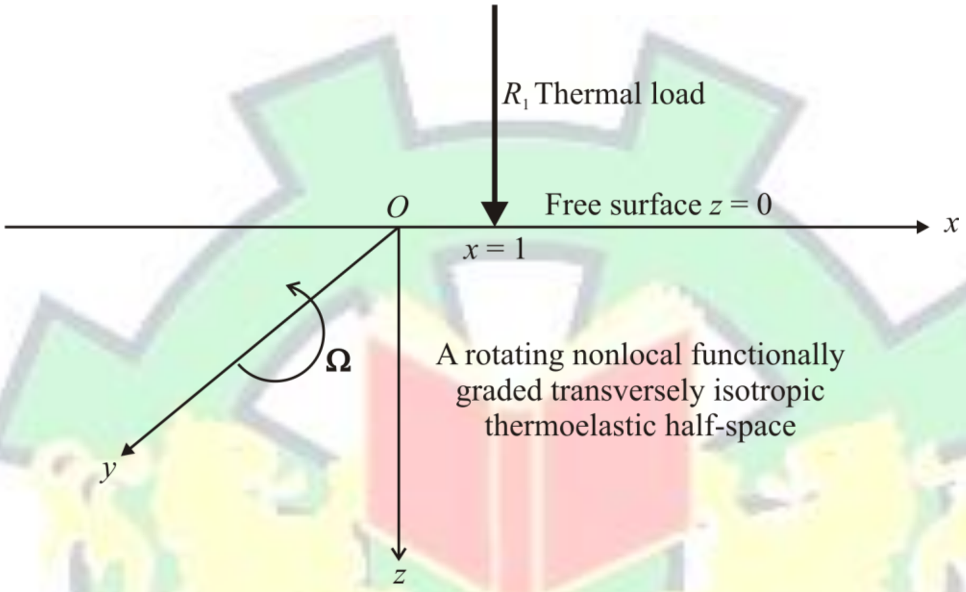
A model made up of a rotating nonlocal functionally graded transversely isotropic thermoelastic half-space $(-\infty \le x \le \infty, z \ge 0)$ subjected to a mechanical load under the purview of Lord-Shulman theory is considered, as shown in Figure 1. The half-space is assumed to be transversely isotropic as its elastic and thermal properties are symmetric about the perpendicular to the plane of isotropy. The model formulation is restricted to *zx*-plane and therefore all the field quantities are independent of the spatial variable *y*. Hence, the displacement vector $\boldsymbol{u}$ and rotation vector $\boldsymbol{\Omega}$ are as

$\boldsymbol{u} = (u, 0, w)$ such that $u = u(x,z,t)$, $w = w(x,z,t)$ and $\boldsymbol{\Omega} = (0, \Omega, 0)$. (11)

The material properties of the model are assumed to be graded in *z*-direction only, so we take $f(\boldsymbol{x})$ as $f(z)$. Along with these assumptions, the stresses obtained from equation (8) in *xz*-plane, can be expressed as

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{xx} = \sigma_{xx}^{L} = f\left(z\right)\left(C'_{11}\frac{\partial u}{\partial x} + C'_{13}\frac{\partial w}{\partial z} - \beta'_{11}\theta\right), \tag{12}$$

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{zz} = \sigma_{zz}^{L} = f\left(z\right)\left(C'_{13}\frac{\partial u}{\partial x} + C'_{33}\frac{\partial w}{\partial z} - \beta'_{33}\theta\right), \tag{13}$$



$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{zx} = \sigma_{zx}^{L} = f\left(z\right)C'_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right). \tag{14}$$

Figure 1: Geometry of the problem

In view of restrictions (11), inserting the stress components defined in equations (12)-(14) into the equation of motion (9), one can obtain

$$\left(1-\varepsilon^2\nabla^2\right) f\left(z\right)\rho'\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right] = \frac{\partial f\left(z\right)}{\partial z}\left[C'_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$+ f\left(z\right)\left[C'_{11}\frac{\partial^2 u}{\partial x^2} + \left(C'_{13} + C'_{44}\right)\frac{\partial^2 w}{\partial x\partial z} + C'_{44}\frac{\partial^2 u}{\partial z^2} - \beta'_{11}\frac{\partial \theta}{\partial x}\right], \tag{15}$$

$$\left(1-\varepsilon^2\nabla^2\right) f\left(z\right)\rho'\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w + 2\Omega\frac{\partial u}{\partial t}\right] = \frac{\partial f\left(z\right)}{\partial z}\left[C'_{13}\frac{\partial u}{\partial x} + C'_{33}\frac{\partial w}{\partial z} - \beta'_{33}\theta\right]$$

$$+ f\left(z\right)\left[C'_{44}\frac{\partial^2 w}{\partial x^2} + \left(C'_{13} + C'_{44}\right)\frac{\partial^2 u}{\partial x\partial z} + C'_{33}\frac{\partial^2 w}{\partial z^2} - \beta'_{33}\frac{\partial \theta}{\partial z}\right]. \tag{16}$$

By using summation convention, the heat conduction equation (10) in *xz*-plane, takes the form

$$\left(\frac{\partial f(z)}{\partial z}\right)K'_{33}\frac{\partial\theta}{\partial z}+f(z)\left[K'_{11}\frac{\partial^2\theta}{\partial x^2}+K'_{33}\frac{\partial^2\theta}{\partial z^2}\right]=f(z)\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial}{\partial t}\right)$$

$$\left[\rho' C_E\theta+T_0\beta'_{11}\frac{\partial u}{\partial x}+T_0\beta'_{33}\frac{\partial w}{\partial z}\right]. \qquad (17)$$

For convenience, the governing field equations can be normalized by introducing the following set of non-dimensional quantities:

$$\left(\hat{x},\hat{z},\hat{u},\hat{w},\hat{\varepsilon}\right)=\frac{\omega^*}{c_0}\left(x,z,u,w,\varepsilon\right),\ \hat{\Omega}=\frac{\Omega}{\omega^*},\ \left(\hat{t},\hat{\tau}_0\right)=\omega^*\left(t,\tau_0\right),$$

$$\left(\hat{\sigma}_{ij},\hat{\sigma}^L_{ij}\right)=\frac{1}{\rho' c_0^2}\left(\sigma_{ij},\sigma^L_{ij}\right),\ \hat{\theta}=\frac{\theta}{T_0}, \qquad (18)$$

where

$$\omega^*=\frac{C'_{11}C_E}{K'_{33}},\ c_0^2=\frac{C'_{11}}{\rho'}.$$

Exponential Variation of Non-Homogeneity

The thermo-mechanical properties of the model are considered non-homogeneous along *z*-direction. In order to incorporate the non-homogeneity of the model, let us consider $f(z)=e^{-nz}$, where *n* is non-homogeneity parameter and this implies that the material properties of the considered model are varying exponentially along the *z*-direction.

With the help of dimensionless quantities defined in (18) and expression of function $f(z)$ as $f(z)=e^{-nz}$, the governing equations (12)-(17) transform to the following non-dimensional forms (while dropping the hats)

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{xx}=\sigma^L_{xx}=e^{-nz}\left[\frac{\partial u}{\partial x}+I_1\frac{\partial w}{\partial z}-I_2\theta\right], \qquad (19)$$

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{zz}=\sigma_{zz}^L=e^{-nz}\left[I_1\frac{\partial u}{\partial x}+I_3\frac{\partial w}{\partial z}-I_4\theta\right], \tag{20}$$

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{zx}=\sigma_{zx}^L=e^{-nz}\left[I_5\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right], \tag{21}$$

$$\left(1-\varepsilon^2\nabla^2\right)\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}\right]=-n\left[I_5\frac{\partial u}{\partial z}+I_5\frac{\partial w}{\partial x}\right]$$

$$+\left[\frac{\partial^2 u}{\partial x^2}+I_6\frac{\partial^2 w}{\partial x\partial z}+I_5\frac{\partial^2 u}{\partial z^2}-I_2\frac{\partial\theta}{\partial x}\right], \tag{22}$$

$$\left(1-\varepsilon^2\nabla^2\right)\left[\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\right]=-n\left[I_1\frac{\partial u}{\partial x}+I_3\frac{\partial w}{\partial z}-I_4\theta\right]$$

$$+\left[I_5\frac{\partial^2 w}{\partial x^2}+I_6\frac{\partial^2 u}{\partial x\partial z}+I_3\frac{\partial^2 w}{\partial z^2}-I_4\frac{\partial\theta}{\partial z}\right], \tag{23}$$

$$\left(I_0\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}\right)-n\frac{\partial\theta}{\partial z}=\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left[\theta+I_7\frac{\partial u}{\partial x}+I_8\frac{\partial w}{\partial z}\right], \tag{24}$$

where

$$I_0=\frac{K'_{11}}{K'_{33}},\left[I_1,I_2,I_3,I_4,I_5\right]=\frac{1}{C'_{11}}\left[C'_{13},\left(\beta'_{11}T_0\right),C'_{33},\left(\beta'_{33}T_0\right),C'_{44}\right],$$

$$I_6=\left(I_1+I_5\right),\ I_7=\frac{\beta'_{11}}{\rho' c_E},\ I_8=\frac{\beta'_{33}}{\rho' c_E}.$$

Solution Methodology

In this section, normal mode analysis is used to get the exact solutions without any presumed restrictions on the physical quantities. So, the physical quantities under consideration and the stresses can be decomposed in terms of normal modes in the following form

$$\left[u, w, \sigma_{ij}, \theta\right](x, z, t) = \left[u^*, w^*, \sigma_{ij}^*, \theta^*\right](z)\exp\left(\omega t + \imath m x\right), \tag{25}$$

where m is the wave number in x-direction, ω is the frequency, $\imath$ is the imaginary unit, and u^*, w^*, θ^* and σ_{ij}^* are the amplitudes of the functions u, w, θ and σ_{ij} respectively.

Introducing expression (25) to equations (22)-(24), one can get

$$\left(J_{11}D^2 - J_{12}D - J_{13}\right)u^*(z) + \left(J_{14}D^2 + J_{15}D - J_{16}\right)w^*(z) - J_{17}\theta^*(z) = 0, \tag{26}$$

$$\left(J_{14}D^2 - J_{15}D + J_{21}\right)u^*(z) + \left(J_{22}D^2 + J_{23}D + J_{24}\right)w^*(z) + \left(I_4 D - J_{25}\right)\theta^*(z) = 0, \tag{27}$$

$$J_{31}u^*(z) + J_{32}Dw^*(z) + \left(D^2 - nD - J_{33}\right)\theta^*(z) = 0, \tag{28}$$

where

$$D = \frac{d}{dz},\ J_{11} = I_5 + \varepsilon^2\left(\omega^2 - \Omega^2\right),\ J_{12} = nI_5,\ J_{13} = m^2 + \varepsilon^2\left(\omega^2 - \Omega^2\right),$$

$$J_{14} = 2\Omega\omega\varepsilon^2,\ J_{15} = \imath m I_6,\ J_{16} = \imath n m I_5 + 2\Omega\omega\left(1 + \varepsilon^2 m^2\right),\ J_{17} = \imath m I_2,$$

$$J_{21} = \imath n m I_1 = 2\Omega\omega\left(1 + \varepsilon^2 m^2\right),\ J_{22} = -I_3 - \varepsilon^2\left(\omega^2 - \Omega^2\right),\ J_{23} = nI_3,$$

$$J_{24} = I_5 m^2 + \left(1 + \varepsilon^2 m^2\right)\left(\omega^2 - \Omega^2\right),\ J_{25} = nI_4,\ J_{26} = \left(1 + \tau_0\omega\right)\omega,$$

$$J_{31} = -\imath m I_7 J_{26},\ J_{32} = -I_8 J_{26},\ J_{33} = I_0 m^2 + J_{26}.$$

Equations (26)-(28) form a system of three linear differential equations in physical quantities $u^*(z), w^*(z)$ and $\theta^*(z)$. By adopting elimination procedure, the following differential equation of order six is obtained

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$$\left[D^6 + A_1 D^5 + A_2 D^4 + A_3 D^3 + A_4 D^2 + A_5 D + A_6\right]\left(u^*, w^*, \theta^*\right)(z) = 0, \qquad (29)$$

where

$$A_1 = \frac{L_{12}L_{27} - L_{11}L_{28} + L_{13}L_{26} - L_{15}L_{23} + L_{16}L_{22} - L_{17}L_{21}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$A_2 = \frac{L_{11}L_{29} - L_{12}L_{28} + L_{13}L_{27} + L_{14}L_{26} - L_{15}L_{24} - L_{16}L_{23} + L_{17}L_{22} - L_{18}L_{21}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$A_3 = \frac{L_{11}L_{30} + L_{12}L_{29} - L_{13}L_{28} + L_{14}L_{27} - L_{15}L_{25} - L_{16}L_{24} - L_{17}L_{23} + L_{18}L_{22}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$A_4 = \frac{L_{12}L_{30} + L_{13}L_{29} - L_{14}L_{28} - L_{16}L_{25} - L_{17}L_{24} - L_{18}L_{23}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$A_5 = \frac{L_{13}L_{30} + L_{14}L_{29} - L_{17}L_{25} - L_{18}L_{24}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$A_6 = \frac{L_{14}L_{30} - L_{18}L_{25}}{L_{11}L_{27} + L_{12}L_{26} + L_{15}L_{22} - L_{16}L_{21}},$$

$$L_{11} = I_4 J_{11},\ L_{12} = J_{14}J_{17} - I_4 J_{12} - J_{11}J_{25},\ L_{13} = J_{12}J_{25} - I_4 J_{13} - J_{15}J_{17},$$

$$L_{14} = J_{13}J_{25} + J_{17}J_{21},\ L_{15} = I_4 J_{14},\ L_{16} = J_{17}J_{22} + I_4 J_{15} - J_{14}J_{25},$$

$$L_{17} = J_{17}J_{23} - I_4 J_{16} - J_{15}J_{25},\ L_{18} = J_{16}J_{25} + J_{17}J_{24},\ L_{21} = J_{11},$$

$$L_{22} = J_{12} + nJ_{11},\ L_{23} = nJ_{12} - J_{13} - J_{11}J_{33},\ L_{24} = nJ_{13} + J_{12}J_{33},$$

$$L_{25} = J_{13}J_3 + J_{17}J_{31},\ L_{26} = J_{14},\ L_{27} = J_{15} - nJ_{14},\ L_{28} = J_{16} + nJ_{15} + J_{14}J_{33},$$

$$L_{29} = nJ_{16} - J_{15}J_{33} + J_{17}J_{32},\ L_{30} = J_{16}J_{33}.$$

The solution of equation (29), which is bounded as $z \to \infty$, is given by

$$\left(u^*, w^*, \theta^*\right)(z) = \sum_{j=1}^{3}\left(H_j, H_j', H_j''\right)(m,\omega)e^{-\lambda_j z}, \text{ for } \mathrm{Re}\left(\lambda_j\right) > 0, \qquad (30)$$

where H_j, H'_j and H''_j are expressions which depend upon ω and *m*. Using the solutions (30) in the system of equations (26)-(28), one can get the following expressions

$$\left[u^*, w^*, \theta^*\right](z) = \sum_{j=1}^{3}\left[1, N_{1j}, N_{2j}\right] H_j\left(m, \omega\right) e^{-\lambda_j z}\text{ , for } \mathrm{Re}\left(\lambda_j\right) > 0\text{ ,} \qquad (31)$$

where

$$N_{1j} = \frac{L_{11}\lambda_j^3 - L_{12}\lambda_j^2 + L_{13}\lambda_j - L_{14}}{\left(-L_{15}\lambda_j^3 + L_{16}\lambda_j^2 - L_{17}\lambda_j + L_{18}\right)},$$

$$N_{2j} = \frac{\left(J_{11}\lambda_j^2 + J_{12}\lambda_j - J_{13}\right) + \left(J_{14}\lambda_j^2 - J_{15}\lambda_j - J_{16}\right)N_{1j}}{J_{17}}.$$

In view of solution equation (31), normal stress (20) and shear stress (21) take the form

$$\left[\sigma_{zz}^*, \sigma_{zx}^*\right](z) = \sum_{j=1}^{3}\left[N_{3j}, N_{4j}\right] H_j\left(m, \omega\right) e^{-\lambda_j z - nz}\text{ , for } \mathrm{Re}\left(\lambda_j\right) > 0\text{ ,} \qquad (32)$$

where

$$N_{3j} = \frac{\iota I_1 m - I_3\lambda_j N_{1j} - I_4 N_2}{\left[1 - \varepsilon^2\left(\lambda_j^2 - m^2\right)\right]},\; N_{4j} = \frac{I_5\left(-\lambda_j + \iota m N_{1j}\right)}{\left[1 - \varepsilon^2\left(\lambda_j^2 - m^2\right)\right]}.$$

Application: Thermal load is subjected to the boundary of the half-space

The surface of the rotating nonlocal functionally graded transversely isotropic thermoelastic half space i.e. the plane $z = 0$, is subjected to a thermal load R_1 as shown in Figure 1. Therefore, the boundary conditions can be written as

$$\theta\left(x, 0, t\right) = R_1, \qquad (33)$$

$$\sigma_{zz}\left(x, 0, t\right) = 0, \qquad (34)$$

$$\sigma_{zx}\left(x, 0, t\right) = 0, \quad \text{at } z = 0. \qquad (35)$$

Using expressions of non-dimensional quantities in (18) and normal mode technique defined in (25), the boundary conditions (33)-(35) transform to

$$\theta^* = R_1^*, \tag{36}$$

$$\sigma_{zz}^* = 0, \tag{37}$$

$$\sigma_{zx}^* = 0, \quad \text{at } z = 0. \tag{38}$$

where R_1^* is defined by the expression $R_1 = R_1^* \exp(\omega t + imx)$.

Using expressions (31) and (32), the boundary conditions (36)-(38) yield a non-homogeneous system of three linear equations, which can be written in matrix form as

$$\begin{bmatrix} N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \\ N_{41} & N_{42} & N_{43} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} R_1^* \\ 0 \\ 0 \end{bmatrix}. \tag{39}$$

The expressions for $H_j, (j = 1, 2, 3)$ obtained by solving the system (39) are as

$$H_1 = \frac{\Delta_1}{\Delta}, \; H_2 = \frac{\Delta_2}{\Delta}, \; H_3 = \frac{\Delta_3}{\Delta}, \tag{40}$$

where

$$\Delta_1 = R_1^* \left(N_{32} N_{43} - N_{33} N_{42} \right),$$

$$\Delta_2 = R_1^* \left(N_{33} N_{41} - N_{31} N_{43} \right),$$

$$\Delta_3 = R_1^* \left(N_{31} N_{42} - N_{32} N_{41} \right),$$

$$\Delta = N_{21} \left(N_{32} N_{43} - N_{33} N_{42} \right) - N_{22} \left(N_{31} N_{43} - N_{33} N_{41} \right) + N_{23} \left(N_{31} N_{42} - N_{32} N_{41} \right).$$

Substitution of (40) in (31) and (32) provides us the following expressions of physical fields

$$u^*(z) = \frac{1}{\Delta} \left[\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z} \right], \tag{41}$$

$$w^*(z) = \frac{1}{\Delta}\left[N_{11}\Delta_1 e^{-\lambda_1 z} + N_{12}\Delta_2 e^{-\lambda_2 z} + N_{13}\Delta_3 e^{-\lambda_3 z}\right], \quad (42)$$

$$\theta^*(z) = \frac{1}{\Delta}\left[N_{21}\Delta_1 e^{-\lambda_1 z} + N_{22}\Delta_2 e^{-\lambda_2 z} + N_{23}\Delta_3 e^{-\lambda_3 z}\right], \quad (43)$$

$$\sigma_{zz}^*(z) = \frac{1}{\Delta}\left[N_{31}\Delta_1 e^{-\lambda_1 z - nz} + N_{32}\Delta_2 e^{-\lambda_2 z - nz} + N_{33}\Delta_3 e^{-\lambda_3 z - nz}\right], \quad (44)$$

$$\sigma_{zx}^*(z) = \frac{1}{\Delta}\left[N_{41}\Delta_1 e^{-\lambda_1 z - nz} + N_{42}\Delta_2 e^{-\lambda_2 z - nz} + N_{43}\Delta_3 e^{-\lambda_3 z - nz}\right]. \quad (45)$$

Particular Cases

Neglecting Non-Homogeneity Effect

By setting $n = 0$ i.e. non-homogeneity function $f(x) = 1$ (in equations (8)-(10), then one can investigate the disturbances in a rotating homogeneous transversely isotropic thermoelastic half-space.

Ignoring Nonlocal Effect

By setting nonlocality parameter as $\varepsilon = 0$ in the basic field equations of this paper, we shall investigate the disturbances in a rotating functionally graded transversely isotropic thermoelastic half-space.

Without Rotation

To discuss the problem in a nonlocal functionally graded transversely isotropic thermoelastic medium due to a mechanical load in the context of Lord-Shulman theory, it is sufficient to set the value of rotation parameter (angular velocity) as zero i.e. $\Omega = 0$.

Numerical Results and Discussion

To illustrate the analytical procedure presented earlier, we now present some numerical results by depicting the variations of normal displacement, normal stress, shear stress and temperature distribution with the help of computer programming using the MATLAB software. For the purpose of illustration, we have chosen a magnesium crystal-like material. The material constants are taken from Eringen [9] and Chadwick and Seet [14]:

$$C'_{11} = 5.974\times10^{10}\, N\ m^{-2},\ C'_{13} = 2.17\times10^{10}\ N\ m^{-2},\ C'_{33} = 6.17\times10^{10}\, N\ m^{-2},$$

$$C'_{44} = 1.639\times10^{10}\, N\ m^{-2},\ \rho' = 1.74\times10^{3}\ kg\ m^{-3},\ T_0 = 298\ K,\ \varepsilon = 0.039\ m,$$

$$\tau_0 = 0.2\ s,\ \beta'_{11} = 2.68\times10^{6}\ N\ m^{-2}\ \deg^{-1},\ \beta'_{33} = 2.68\times10^{6}\ N\ m^{-2}\ \deg^{-1},$$

$$K'_{11} = 1.7\times10^{2}\ W\ m^{-1}\ \deg^{-1},\ K'_{33} = 1.7\times10^{2}\ W\ m^{-1}\ \deg^{-1},\ n = 1.0,$$

$$c_E = 1.04\times10^{3}\ J\ kg^{-1}\ \deg^{-1},\ \omega = 1.0,\ R_1^{*} = 1,\ \Omega = 0.2,\ m = 1.1.$$

Utilizing the above numerical values of the parameters, the values of the non-dimensional field variables have been evaluated and results are presented in the form of the graphs at different positions of z at $t = 0.01s$ and $x = 1.0$.

Figures 2-4 illustrate the effect of rotation parameter on normal displacement, normal stress and temperature field respectively for three different values of rotation parameter (0.2, 0.1 and 0.0).

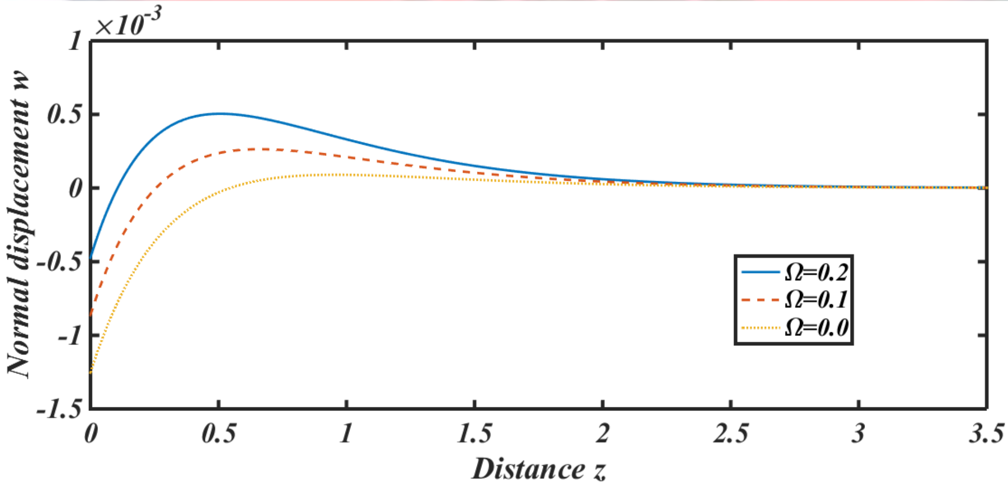

Figure 2: Effect of rotation on the profile of normal displacement.

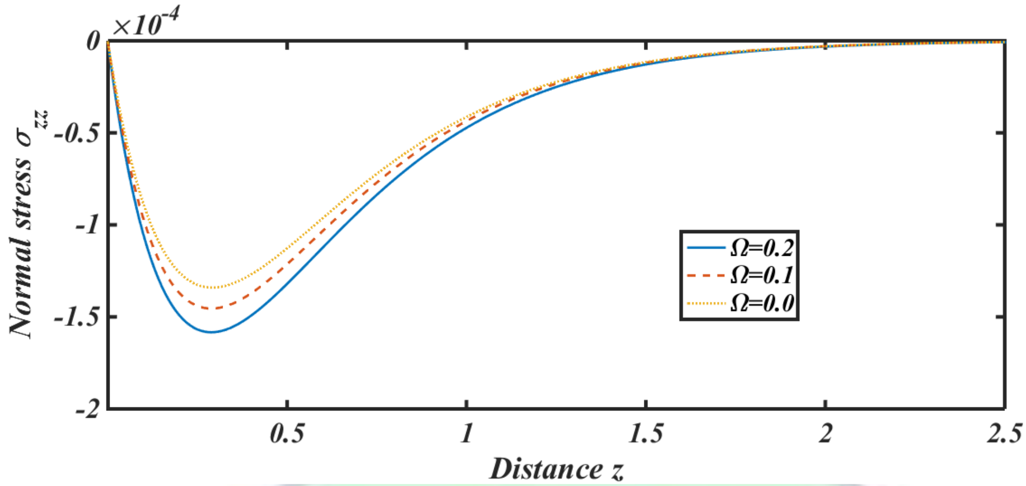

Figure 3: Effect of rotation on the profile of normal stress.

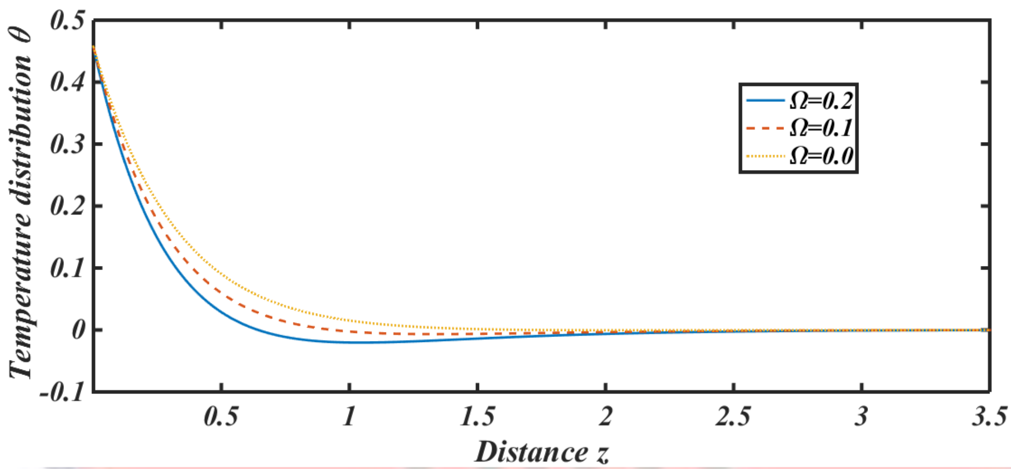

Figure 4: Effect of rotation on the profile of temperature distribution field.

It can be seen from these figures that rotation parameter has a significant effect on the profiles of all the physical fields i.e. normal displacement, normal stress and temperature distribution field.

Conclusion

The current investigation provides a mathematical model to study the two-dimensional deformations in a rotating nonlocal functionally graded, transversely isotropic thermoelastic medium under a thermal load within the framework of Lord-Shulman theory, by using normal mode technique. The following conclusions can be drawn according to the analysis of this study:

1. The rotation parameter has a mixed effect on the profiles of normal

displacement and temperature distribution, while it has an increasing effect on the profile of normal stress throughout the domain.

2. Time t has a significant increasing effect on the profiles of all physical fields.
3. From all the figures, it has been observed that all the physical fields have non-zero values only in the bounded region of space, which is in accordance with the notion of generalized thermoelasticity theory and supports the physical facts.

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Role of Mathematics in Business

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Abstract

Mathematics is important part of our life. We cannot think about our life without mathematics. Mathematics is applied in every work .It is a natural way to organize and to think. Mathematics is even used in business world. It is used everywhere in economics, science, engineering, accounting, finance etc. Mathematics used in business as a critical tool for students of business, practitioners of business anyone else who need to use mathematical concepts in business world. Business mathematics help companies to make better decisions understand problems more throughly and see their long -term impact.

This paper focuses on role of mathematics in Business and various areas where mathematics is applied. Uses of various mathematical concept such as Arithmetic, Algebra, Calculus, Probability, Statistics which are used to make decision about pricing, production, and investment in business and help in achieving business objectives viz. maximizing profits, minimizing costs and ensuring optimal utilization of resources.

Keywords: Mathematics, Business, Business Decision, Commerce.

Introduction

Business mathematics is mathematics used by commercial enterprises to record and manage business operations. Commercial organizations use mathematics in accounting, inventory management, marketing, sales forecasting, and financial analysis. Mathematics typically used in commerce includes elementary arithmetic, elementary algebra, statistics and probability. Business management can be done in a more effective way by use of more advanced mathematics such as calculus, matrix algebra and linear programming. Elementary algebra is often included as well, in the context of solving practical business problems. The practical applications typically include checking accounts; price discounts payroll calculations, simple and compound interest, consumer and business credit, and mortgages and [revenues]. In today's era of globalization, mathematics is one of the primary elements and proved very useful in the field of business. Business organizations are using mathematics in the field of accounting (viz. financial accounting, cost accounting, corporate accounting, management accounting) inventory management, sales forecasting, marketing, financial analysis (Karatzas & Shreve, 1998) [1]. Without mathematics one feels helpless in every aspect of business and commerce.

Objectives of Study

1. To study areas in which mathematics is applied in business.
2. To know the application of mathematics in business in form of Statistics, Algebra, Probability etc.

Methodology

The study is descriptive in nature. Data is collected through secondary sources like websites, research paper, books, Journals.

Areas OF Mathematics in Business

A business can be described as an organization or enterprising entity that engages in professional, commercial or industrial activities. There can be different types of businesses depending on various factors. Some are for-profit, while some are non-profit. Similarly, their ownership also makes them different from each other. For instance, there are sole proprietorships, partnerships, corporations, and more. Business is also the efforts and activities of a person who is producing goods or offering services with the intent to sell them for profit. Some Areas of business where mathematics is included:

Finance: Financial math is study of mathematics that is used in financial decision-making. It includes concepts such as the time value of money, interest rates, annuities, present value, amortization, future value, and risk management. These topics are important in understanding how to make financial decision for businesses and individuals.

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Accounting: Accounting math centers around the accurate recording of financial transactions. This includes topics such as debits and credits, assets and liabilities, double-entry bookkeeping, journal entries, and various financial statements. Accounting math is an extremely critical part of record keeping for businesses, and it is important for accounting professionals to understand how to properly record transactions.

Economics: Economics is the study of how people use resources to produce goods and services. It includes microeconomics, which focuses on individual economic decision-making, and macroeconomics, which looks at the economy as a whole. There are several important topics contained within economics such as supply and demand, market equilibrium, production and costs, perfect competition, monopoly, and externalities. All of these concepts are important in understanding how businesses operate and make decisions within their greater business environments.

Marketing: Marketing is the process of exploring, creating, and delivering value to meet the needs of a target market in terms of goods and services; Marketing is currently defined by the American Marketing Association (AMA) as "the activity, set of institutions, and processes for creating, communicating, delivering, and exchanging offerings that have value for customers, clients, partners, and society at large". Mathematics help in Analysis of marketing research information various Statistical records are maintained for building and maintaining an extensive market various tools and technique are used for Sales forecasting.

Personnel: Good personnel management is responsible for creating and maintaining a harmonious working environment. This includes ensuring that the compensation and benefits strategy for the business encourages success, employee disciplinary and grievance procedures, effective communication, and solid health and safety policies. Mathematics is used to determined Labour turnover rate, Employment trends, Performance appraisal, Wage rates and incentive plans.

Research and Development: Mathematics is even used for Development of new product lines, optimal use of resources, Evaluation of existing products.

Application of Mathematics in Business

Mathematics is used in almost every field of daily life. Business involves the buying and selling of goods in order to earn profit, it uses mathematics to record, classify, summarize and analyze the business transactions. So mathematics is used by commercial enterprises to record and manage the business operations such as, elementary arithmetic involving fractions, decimals, percentage, elementary algebra, statistics and probability. Now a day's business management is using advanced mathematics such as calculus, matrix algebra and liner programming. Practical applications include checking accounts, forecasting the sales, price discounts, mark-ups, mark-downs, payroll calculations, simple and compound interest, reducing wastage of resources (Veer & Shukla, 2009) [2]. Some applications of mathematics in business and commerce are listed below:

Arithmetic: This is one of the most basic types of math, and it is used for a variety of purposes. Arithmetic is used to keep track of financial transactions, calculate prices, and make basic predictions. Linear algebra serves a purpose of powerful tool for its application in business. As total cost, revenue, supply, demand and population are all related with a system of linear equations. Leontief (1987) [5] derived a production equation in input-output analysis and got Noble prize for his contribution. The model given by him was X=CX+d, where x is the production factor, c is consumption matrix and d is demand vector.

Algebra: Algebra is the branch of mathematics that deals with the manipulation of equations and variables. Algebra is used in business to solve problems involving equations. . For example, businesses use algebra to calculate the break-even point, which is the point at which Total revenue equals Total expenses. The basic idea in accounting is that total wealth of business is called Assets. There are two possible claims on assets (A) called liabilities (L) and capital(C). By using mathematical relation A=L+C, accountants use mathematics in order to arrive the total cost and taking decision regarding manufacturing or buying the product. The total cost formula for business is T= a+bx; where „T" is total cost, „a" is fixed cost, „b" is cost per unit produced and „x" is no. of units produced.

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Geometry: Geometry is a branch of mathematics that is focused on the properties of spaces and shapes. Geometry is used in business for a variety of purposes, such as understanding spatial relationships, designing buildings and products, and measuring area and volume.

Calculus: Calculus is another branch of mathematics made up of two fields- differential calculus and integral calculus. Differential calculus plays valuable role in management and business for decision making in production (e.g. supply of raw material, wage rates and taxes). As an advanced branch of mathematics, calculus focuses heavily on functions and derivatives. Functions examine the relationship between two or more variables, or entities that take on different values. Mathematicians and economists often use letters, such as X and Y, to symbolize particular variables. If the value of Y changes as the value of X changes, then the two variables have a functional relationship. Derivatives, meanwhile, consider the rate of change in one variable relative to the change in another. Calculus, by determining marginal revenues and costs, can help business managers maximize their profits and measure the rate of increase in profit that results from each increase in production. As long as marginal revenue exceeds marginal cost, the firm increases its profits.

Probability: Probability is the branch of mathematics that deals with the study of random events. Probability is used in business to make decisions about risk. For example, businesses use probability to calculate the likelihood of an event occurring, such as the likelihood of a product being defective. Probability theory serves as a useful tool for decision making, estimating number of defective units, sales expected and also in business policies. Through the use of statistical (regression) techniques Levine and Zervos (1998) [3] attempted to find the empirical relationship between various measures of stock market development, banking development, and long-run economic growth**Statistics:** Statistics is the study of data collection, analysis, interpretation, presentation, and organization. It is a critical tool in business for understanding trends, making predictions, and drawing conclusions from data. Some important topics in statistics include data collection methods, probability, random variables, distributions, hypothesis testing, and regression analysis. These concepts are important for understanding and using data to make decisions in business.

Conclusion

Thus, it is evident that mathematical methods and tools become crucial part of the business organization. Application of mathematics becomes necessary from the beginning i.e. from buying or estimating the cost of product to the end sales and earning profits. Mathematical formulae help business to do financial analysis using ratios, percentages, equations. The objective of minimizing cost & maximizing profit is achieved through linear programming and calculus. The estimation of future returns & profitability is done through probability distributions. It also helps in sale forecasting & risk evaluation. Matrices play important role in variety of solutions for consumer relationships and logistics management. Statistics helps in collection, presentation and analysis of data to arrive at conclusions (Chiang & Storey, 2012) [8]. Statisticians have developed many tools for application and which can be utilized for business improvement. Statistical Thinking and Methods need to become part of the knowledge base of an organization (Abraham, B., 2007)

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Financial Mathematics in Share Market Challenges and Future Ahead

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Abstract

Financial mathematics in share market is the merchandise of applying mathematics to portfolio choice theory and option pricing theory. With the rapid development of the profitable situation, the products and derivatives of the financial industry are continuously optimized and innovative, and new financial goods and services are gradually increasing. The operation of financial markets, the blueprint and pricing of financial derivatives, and the analysis and supervision of risk become very imperative, and the research and development of financial mathematics is fetching more and more important. Therefore, it is of realistic significance to analyze the specific application of mathematics in the monetary field.

Financial mathematics, also called investigative finance, mathematical economics and mathematical finance, is an interdisciplinary focus of mathematics and finance that arose in the late 1980s and early 90s. Financial mathematics in share markets chiefly uses the modern mathematical theory and method (such as stochastic analysis, stochastic most advantageous control, portfolio analysis, nonlinear analysis, multivariate arithmetical analysis, mathematical programming, up to date computational methods etc.) of financial (including banking, speculation, bonds, funds, stocks, futures, options and other financial instruments and markets) analysis the number of theory and put into practice. The core problem is the selection theory of the optimal outlay strategy and the asset pricing theory under the doubtful condition. Financial mathematics not only have a direct effect on the novelty of financial instruments and financial markets in the share markets, drive efficiently, but also for the company's investment decision-making and assessment of project research and development (such as real options) and menace management in financial institutions has been extensively used.

Keywords: Financial, Mathematics, Share, Market, Challenges, Model

Introduction

Applying arithmetics to the financial field is based on some financial or economic assumptions, and uses abstract mathematical methods to build mathematical models of how the financial mechanism works. Financial mathematics chiefly includes the basic concepts and methods of mathematics, the associated natural science methods and so on Financial mathematics

They are useful in various forms of entry theory. The use of mathematics is to convey, reason, and prove the fundamental principles of finance. From the nature of financial mathematics, financial mathematics is an important branch of finance in share market. Therefore, financial mathematics in share markets is completely based on the surroundings and foundation of financial theory. The people who slot in in financial mathematics through formal financial academic instruction will have more advantages in this context. Finance is used as a sub discipline of economics of identity development, though it has an attribute enough from the economic independence, but it still requires economic standard and economic technology related as locale. At the same time, financial mathematics also needs fiscal knowledge, tax theory and secretarial principles as the background of knowledge.

Financial News in Stock Market

In the modern financial theory, mathematics in the turf of finance is another important relevance is analyzed in option pricing and investment decision using discrepancy game method, [5,6] and the application of this aspect has made outstanding achievements.

Because the whole law of financial market does not harmonise with the hypothesis of steady state, the abnormal variation of securities will lead to abnormal alteration in the process of abnormal fluctuation, and this kind of variation will not obey the Brown motion. At this position, we need to use stochastic dynamic model to study and analyze the complete decision-making of securities investment.

This method is not only in hypothesis or in practice, but also has a enlarged deviation. The financial problems in share markets and interferences by using the differential method to non geometry in the financial field of the Brown allocation has important use, not only can effectively relax this postulation can also be uncertain disturbances become unreceptive to the illusion of hand. The stability (robustness) of the powerful portfolio strategy can be obtained through the engrossing analysis of the whole uncertain problem.

Observation

The research of these scholars has unswervingly led to the emergence of the capital asset pricing (CAPM) model. As one of the prognostic model for risk assets based on expected profit equilibrium on the basis of CAPM, explains the configuration of market equilibrium in investors by Markowitz's theory of investment management under the circumstances of the theory of the relationship between the predictable return and expected risk in a simple linear relationship between the turn of phrase of it, that there is a positive link the relationship between a beta scale an asset to the expected rate of return and measure the risk value of assets.

It should be said that, as a variety of objective of risk asset equilibrium price verdict theory, single index model, and based on CAPM not only simplifies the computation process of portfolio selection, the Markowitz portfolio assortment theory in the real world a big step forward, but also makes the securities theory from the previous qualitative examination to quantitative analysis, empirical turn from the normative, then the securities speculation theory and practical operation, which has an enormous influence even to the development of pecuniary theory and practice, has become the theoretical basis of recent finance

Discussion

Stochastic optimal control is sophisticated in the development of the control theory gradually developed, through the relevance of Behrman principle in amalgamation optimization, measure theory and functional analysis method of stochastic quandary analysis. This method was formed in the late 60s of the last century, and became established gradually in the early 70s. From the application of stochastic optimal control theory, the rejoinder of financial experts in this field is very quick.

Stochastic Optimal Control Model

At the beginning of 70s, the finance research field which appeared a few articles related to economics papers, including Merton (Merton) are discussed using the method of unremitting time consumption and portfolio, the portfolio analysis between them is additional consistent with the actual situation; and Brock (Brock) and Millman (Mirman) in random changes, using disconnected time method of optimal economic growth are discussed. Subsequently, the stochastic optimal control method has been applied in most financial fields in share markets.

Result

From the construction of differential game application, selection pricing and investment judgment in the capital asset pricing model and stochastic optimization theory to survey three aspects of the important application of mathematics in the ground of finance in share markets, reflects the significant role of mathematics in modern financial analysis. There are a huge number of things on which a bank may have risk, so they are grouped (categorised) into "asset classes".

Share Market

The most common 5 types of asset class are:

- **Interest rate**
- **Foreign exchange**
- **Credit**
- **Equities**

Interest rate

These are the tariff at which individuals, banks, corporates and sovereigns (i.e. governments) borrow and lend money. Rates are usually quoted to the nearest 0.01% and range in term from overnight to 30 years or beyond.

Foreign exchange

The rate at which one currency can be sold to buy another. If I have 1,000 pounds and an overdraft of 1,400 dollars, then the net value is zero if the exchange rate is 1.4. If the rate goes to 1.39, then I have a net overdraft.

Credit:

If I loan money to Argentina, then I should demand a much higher rate of interest than if I mortgage money to the UK, because there is a greater likelihood that Argentina will not repay its debt and I won't get my money back.

Equities: This is just a different name for shares, such as Marks & Spencer or Microsoft.

Conclusion

As trades come closer to selling (the simplest deal can take 10 seconds from start to finish, more complex ones can effortlessly take 6 months) the trader will become increasingly involved. He is the human being who commits the bank to taking on risk, and will have jeopardy limits within which he operates. Ultimate responsibility for a transaction pricing falsehood with the trader. When a deal is done, the sales/structuring effort moves on to something else, and the seller is responsible for ensuring that the deal is managed throughout its life so as not to lose money.

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Recent Development In Fixed Point Theory

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Abstract

This article basically demonstrated the recent developments in the fixed point theory. Various discussions and are presented on the relation of fixed point theorems to applications and areas are delineated in the future research directions as well.

Introduction

Fixed point theory is an important, attractive and vastly functional branch of mathematics. It is wholly grown up but still continues to be a vibrant area of research. Study of fixed point theorems is being pursued within several mathematical domains such as: Classical Analysis, Functional Analysis, Topology, Operator Theory and Algebraic Topology. This theory plays an important role in proving the existence and sometimes uniqueness of the differential equations, integral equations, partial differential equations etc. Presently lots of research work is going on in this field.

Let $\mathfrak{I}$ be a self map of a nonempty set $\mathcal{A}$. A point $a \in \mathcal{A}$ such that $\mathfrak{I}a = a,$ is called a fixed point of the map $\mathfrak{I}$. The significance of the fixed point theory occur primarily in the fact that a large number of the equations appearing in the different physical formulations may be changed to fixed point equations.

Historically, in 1912, L.E.J. Brouwer[1] introduced the idea of the fixed point and stated that "a continuous map on a closed unit ball in $\mathcal{R}_n$ has a fixed point". Schauder[2] extended Brouwer's outcome to compact convex set in Banach space in 1930.

The first important result of this theory is the revolutionary effort of Stefan Banach[3], available in 1922 is widely famous as the "Banach Contraction Principle". This principle stated that

if $\mathfrak{I}$ is self mapping of a complete metric space $(\mathcal{A}, d)$ fulfilling

$$d(\mathfrak{I}a, \mathfrak{I}b) \leq k d(a, b),$$

for every a, b in $\mathcal{A}$ and $0 < k < 1$, then there always be a unique fixed point for mapping $\mathfrak{I}$ in $\mathcal{A}$.

The Banach Contraction Principle is the foundation stone over which the whole bulk of results on fixed point rest. Subsequently, many extensions of this theorem enriched the theory of fixed points by a number of authors Tychonoff[4], Lefschetz[5], Kakutani[6], Tarski[7], Edelstein[8], Kannan[9], Chatterjea[10], Zamfirescu[11], Ćiric[12], Jungck[13] and references thereof.

For an extensive study of fixed point theory there are some special books by Aksoy and Khamsi[14], Dugundji and Granaj[15], Goebel and Kirk[16], Istratescu[17], S.P. Singh[18], Hasser[19] and Smart[20].

The essential condition for Banach fixed point theorem is that the mapping $\Im$ must be continuous in the space. But in 1968, Kannan[9] developed a fixed point result for the mapping that is not necessarily continuous. He gave his result as if $\Im$ is self mapping of a complete metric space $(\mathcal{A}, d)$ fulfilling the property

$$d(\Im a, \Im \mathscr{b}) \leq \mathscr{k}[d(\Im a, a) + d(\Im \mathscr{b}, \mathscr{b})],$$

for all $a, \mathscr{b}$ in $\mathcal{A}$ and $0 < \mathscr{k} < \frac{1}{2}$, then a unique fixed point exists for mapping $\Im$ in $\mathcal{A}$.

In 1969, S.P. Singh[18] demonstrated that there exists a positive integer ℓ and a number $0 < \mathscr{k} < \frac{1}{2}$, for all $a, \mathscr{b}$ in $\mathcal{A}$ such that

$$d(\Im^{\ell} a, \Im^{\ell} \mathscr{b}) \leq \mathscr{k}[d(a, \Im^{\ell} a) + d(\mathscr{b}, \Im^{\ell} \mathscr{b})].$$

S. Reich[21] in 1971, developed a result that there exist non negative numbers λ, μ, σ fulfilling $\lambda + \mu + \sigma < 1$, for all $a, \mathscr{b}$ in $\mathcal{A}$ such that

$$d(\Im a, \Im \mathscr{b}) \leq \lambda d(a, \Im a) + \mu d(\mathscr{b}, \Im \mathscr{b}) + \sigma d(a, \mathscr{b}).$$

In 1972, Zamfirescu[11] acquired a very exciting fixed point theorem by merging the contractive conditions of Banach, Kannan and Chatterjea. It stated that there are $\lambda, \mu, \sigma \in \Re$ with $0 < \lambda < 1, 0 < \mu, \sigma < \frac{1}{2}$, such that for all $a, \mathscr{b}$ in $\mathcal{A}$ at least any one of following is true

1. $d(\Im a, \Im \mathscr{b}) \leq \lambda d(a, \mathscr{b})$,
2. $d(\Im a, \Im \mathscr{b}) \leq \mu[d(a, \Im a) + d(\mathscr{b}, \Im \mathscr{b})]$,
3. $d(\Im a, \Im \mathscr{b}) \leq \mu[d(a, \Im \mathscr{b}) + d(\mathscr{b}, \Im a)]$.

In the same year i.e. in 1972, Bianchini[22] developed a result in which he stated that there always be a number α, $0 < \alpha < 1$, such that for all $a, \mathscr{b}$ in $\mathcal{A}$

$$d(\Im a, \Im \mathscr{b}) < \alpha \max\{d(a, \Im a), d(\mathscr{b}, \Im \mathscr{b})\}.$$

Yen[23] in 1972, demonstrated that there exist positive integers λ, μ and a number α, $0 < \alpha < 1$, such that for every $a, \mathscr{b}$ in $\mathcal{A}$

$$d\left(\Im^{\lambda} a, \Im^{\mu} b\right) \leq \alpha d(a, b).$$

In 1973, Hardy and Rogers[24] proved that there exist non negative constants satisfying $\sum_{j=1}^{5} \beta_j < 1$ such that for all a, b in $\mathcal{A}$

$$d(\Im a, \Im b) \leq \beta_1 d(a, b) + \beta_2 d(a, \Im a) + \beta_3 d(b, \Im b) + \beta_4 d(a, \Im b) + \beta_5 d(b, \Im a).$$

Ciric[12] in 1974, initiated a fresh thought via contractive definition as there exists a number $0 \leq \alpha < 1$ such that for each a, b in $\mathcal{A}$

$$d(\Im a, \Im b) \leq \alpha \rho(a, b),$$

where $\rho(a, b) = max\{d(a, b), d(a, \Im a), d(b, \Im b), d(a, \Im b), d(b, \Im a)\}$.

In 1976, Fisher[25] proved the result with each a, b in $\mathcal{A}$

$$d(\Im a, \Im b) \leq \alpha\{d(\Im b, a) + d(\Im a, b)\}.$$

Jaggi[26] initialized the new idea of rational expression in 1977. He stated that there exist numbers γ, δ such that $0 \leq \gamma + \delta < 1$ and for each a, b in $\mathcal{A}$ with $a \neq b$

$$d(\Im a, \Im b) \leq \gamma d(a, b) + \delta d(a, \Im a) \frac{d(b, \Im b)}{d(a, b)}.$$

In 1977, Rhoades's condition for contractive mapping[27] stated that

$$d(\Im a, \Im b) < max\left\{d(a, b), d(a, \Im a), d(b, \Im b), \frac{d(a, \Im b) + d(b, \Im a)}{2}\right\}.$$

In 1980, Jaggi and Das[28]obtained various fixed point theorem with function satisfies the condition

$$d(\Im a, \Im b) \leq \gamma d(a, b) + \delta d(a, \Im a) \frac{d(b, \Im b) d(b, \Im a)}{d(a, b) + d(a, \Im a) + d(b, \Im a)}.$$

In this way, a lot of work has been reported in the literature on this line by using different classes of contraction type conditions. For an excellent comparisons of various contraction conditions, one may refer Rhoades[29-30], Collaco and Silva[31], Murthy[32], Singh and Tomar[33].

Preliminaries and Basic Definitions

Definition 1.2.1 Let X and Y be two non-empty sets such that $X \cap Y \neq \infty$. A point $\tau \in X$ is fixed point of mapping $\Im: X \rightarrow Y$ if and only if $\Im(\tau) = \tau$, i.e. τ is invariant under any type of transformation.

Remark 1.2.2 It is not always possible that every mapping have fixed points.

Example 1.2.3 Mapping $\Im: \Re \rightarrow \Re$ defined as $\Im(a) = a^2 + 6a + 6$. Clearly, $-2, -3$ are fixed points of $\Im$.

Example 1.2.4 Mapping $\Im: \Re \rightarrow \Re$ defined as $\Im(a) = a^2 - 9a + 14$. Then $\Im(2) = 2$.

Example 1.2.5 Mapping $\mathfrak{I}: \mathfrak{R} \rightarrow \mathfrak{R}$ defined as $\mathfrak{I}(a) = a + 5$. Then no fixed point exists for $\mathfrak{I}$.

Example 1.2.6 The projection $(a, b) \rightarrow b$ of $\mathfrak{R}^2$ onto the Y-axis has all points of Y-axis as are fixed points.

The concept of metric space is initialized by Frechet[34] in 1906. He explored the study of such spaces and their applications to various areas of mathematics.

Definition 1.2.7[34] A mapping $d: \mathcal{A} \times \mathcal{A} \rightarrow \mathfrak{R}$ in a non-empty set $\mathcal{A}$ satisfies the axioms for every $a, b \in \mathcal{A}$:

(M-I) $d(a, b) > 0$,

(M-2) $d(a, b) = 0$ if $a = b$,

(M-3) $d(a, b) = d(b, a)$,

(M-4) $d(a, b) \leq d(a, c) + d(c, b)$.

d is metric on $\mathcal{A}$ or also called distance function and the pair $(\mathcal{A}, d)$ is known to be a metric space.

Example 1.2.8[34**]** Consider $\mathfrak{R}$ be the set of real numbers and $\mathcal{A}$ be non empty set. A mapping defined by $d: \mathcal{A} \times \mathcal{A} \rightarrow \mathfrak{R}$ for every $a, b \in \mathcal{A}$

$$d(a, b) = \begin{cases} 0 & if \quad a = b \\ 1 & if \quad a \neq b \end{cases}$$

then d is metric on $\mathcal{A}$ and it is known as discrete metric space.

Definition 1.2.11[34**]** A sequence $\{a_n\}$ in $\mathcal{A}$ is called convergent if there always be a unique element $a_0 \in \mathcal{A}$ such that $\lim_{n \to \infty} d(a_n, a_0) = 0$.

Definition 1.2.12[34**]** A sequence $\{a_n\}$ in $\mathcal{A}$ is known to be Cauchy sequence if for every $\varepsilon > 0$ there always be a positive integer n_0 such that $m, n > n_0$, $d(a_n, a_m) < \varepsilon$.

Definition 1.2.13[34**]** Metric space $(\mathcal{A}, d)$ is known to be complete if every Cauchy sequence is convergent in it.

Example 1.2.14[34**]** Usual metric space $(\mathfrak{R}, d)$ is complete.

Definition 1.2.15[34**]** Consider $(\mathcal{A}_1, d_1)$ and $(\mathcal{A}_2, d_2)$ be metric spaces, then mapping $\mathcal{F}: (\mathcal{A}_1, d_1) \rightarrow (\mathcal{A}_2, d_2)$ is continuous at the point $a_0 \in \mathcal{A}$ if for every $\varepsilon > 0$ there always be $\delta >$ 0 such that $d_2(\mathcal{F}(a), \mathcal{F}(a_0)) < \varepsilon$ whenever $d_1(a, a_0) < \delta$.

Rudolph Lipschitz[35] invented the idea of Lipchitz condition. In 1965, idea was developed in metric space.

Definition 1.2.16[35] An into mapping $\mathcal{F}: \mathcal{A} \to \mathcal{A}$ of a metric space $(\mathcal{A}, d)$ satisfies the Lipchitz condition if there always be a real number k such that

$$d\big(\mathcal{F}(a), \mathcal{F}(b)\big) \leq k\, d(a, b), \text{ for every } a, b \in \mathcal{A}.$$

Definition 1.2.17[35] An into mapping $\mathcal{F}: \mathcal{A} \to \mathcal{A}$ of a metric space $(\mathcal{A}, d)$ is known to be contraction if

$$d\big(\mathcal{F}(a), \mathcal{F}(b)\big) \leq k\, d(a, b), \text{ for every } a, b \in \mathcal{A} \text{ and } 0 \leq k < 1.$$

Definition 1.2.18[35] An into mapping $\mathcal{F}: \mathcal{A} \to \mathcal{A}$ in a metric space $(\mathcal{A}, d)$ is known to be a non expensive if

$$d\big(\mathcal{F}(a), \mathcal{F}(b)\big) \leq d(a, b), \text{ for every } a, b \in \mathcal{A}.$$

In 1976, Jungck[36], gave an extension to Banach's contraction theorem by applying the notion of commuting mappings.

Definition 1.2.19[36] Mappings $\mathcal{F}_1, \mathcal{F}_2: \mathcal{A} \to \mathcal{A}$ in a metric space $(\mathcal{A}, d)$ are said to be commuting mappings iff $\mathcal{F}_1\mathcal{F}_2 = \mathcal{F}_2\mathcal{F}_1$.

Definition 1.2.20[36] Let $\mathcal{F}_1, \mathcal{F}_2: \mathcal{A} \to \mathcal{A}$ be mappings in a metric space $(\mathcal{A}, d)$, then $\{\mathcal{F}_1, \mathcal{F}_2\}$ is said to weakly commuting pair on $\mathcal{A}$ if

$$d(\mathcal{F}_1\mathcal{F}_2 a, \mathcal{F}_2\mathcal{F}_1 a) \leq d(\mathcal{F}_1 a, \mathcal{F}_2 a), \text{ for every } a \in \mathcal{A}.$$

Definition 1.2.21[36] Let $\mathcal{F}_1, \mathcal{F}_2: \mathcal{A} \to \mathcal{A}$ be mappings in a metric space $(\mathcal{A}, d)$. A point $a \in \mathcal{A}$ is a coincidence point of $\mathcal{F}_1$ and $\mathcal{F}_2$ if $\mathcal{F}_1 a = \mathcal{F}_2 a$.

Jungck[37] also gave extension to commuting mappings and weakly commuting mappings by introducing compatible mappings.

Definition 1.2.22[37] Two self mappings $\mathcal{F}_1$ and $\mathcal{F}_2$ of a metric space $(\mathcal{A}, d)$ are said to be compatible if

$$\lim_{n\to\infty} d(\mathcal{F}_1\, \mathcal{F}_2 a_n, \mathcal{F}_2\mathcal{F}_1 a_n) = 0,$$

whenever $\{a_n\}$ is a sequence in $\mathcal{A}$ such that $\lim_{n\to\infty} \mathcal{F}_1 a_n = \lim_{n\to\infty} \mathcal{F}_2 a_n = t$, for some $t \in \mathcal{A}$.

Definition 1.2.23[37] Two self mappings $\mathcal{F}_1$ and $\mathcal{F}_2$ of a metric space $(\mathcal{A}, d)$ are said to be compatible of Type A if

$$\lim_{n\to\infty} d(\mathcal{F}_2\mathcal{F}_1 a_n, \mathcal{F}_1\mathcal{F}_1 a_n) = 0 \text{ and } \lim_{n\to\infty} d(\mathcal{F}_1\mathcal{F}_2 a_n, \mathcal{F}_2\mathcal{F}_2 a_n) = 0,$$

whenever $\{a_n\}$ is a sequence in $\mathcal{A}$ such that $\lim_{n\to\infty} \mathcal{F}_1 a_n = \lim_{n\to\infty} \mathcal{F}_2 a_n = t$, for some

Expansive Mapping is familiarized by Wang et al.[38] in 1984 and they showed some fixed point theorems in a complete metric spaces.

Definition 1.2.24[38] Let $(\mathcal{A}, d)$ be a complete metric space and let an onto self map $\mathcal{F}$ be on $\mathcal{A}$ such that

$$d(\mathcal{F}a, \mathcal{F}b) \geq \rho\, d(a, b), \text{ for all } a, b \in \mathcal{A} \text{ with } \rho > 1.$$

Then $\mathcal{F}$ has a unique fixed point in $\mathcal{A}$.

In 1984, Khan et al.[39] introduced altering distance function.

Definition 1.2.25[39] A function $\emptyset$ from $[0, \infty)$ to $[0, \infty)$ is an altering distance mapping if subsequent assumptions are fulfilled:

1. $\emptyset$ is non-decreasing and continuous,
2. $\emptyset(t) = 0$ implies t =0.

In 1998, with the help of control mapping, Alber and Guerre-Delabriere[40], introduced weakly contractive mappings.

Definition 1.2.26[40] Mapping $\mathcal{F}: \mathcal{A} \to \mathcal{A}$ is known to be weakly contractive if there always be a function $\emptyset$ from $[0,\infty)$ to $[0,\infty)$ such that $\emptyset$ is non-decreasing, continuous and $\emptyset(t) = 0$ iff t =0 and

$$d(\mathcal{F}a, \mathcal{F}b) \leq d(a, b) - \emptyset\big(d(a, b)\big), \text{ for all } a, b \in \mathcal{A}.$$

In 2002, the concept of E.A. property in metric spaces has been introduced by Aamri and El Moutawakil[41].

Definition 1.2.27[41] Two self mappings $\mathcal{F}_1, \mathcal{F}_2$ in a metric space $(\mathcal{A}, d)$ satisfy E.A. property if there exists a sequence $\{a_n\}$ in $\mathcal{A}$ such that

$$\lim_{n\to\infty} \mathcal{F}_1 a_n = \lim_{n\to\infty} \mathcal{F}_2 a_n = t \text{ for some } t \in \mathcal{A}.$$

Later on in 2006, Bhaskar and Lakshmikantham[42] initiated the idea of coupled fixed points. Then Lakshmikantham[43] developed these consequences to establish coupled coincidence theorems and common coupled fixed point.

Definition 1.2.28[42] A point $(a, b) \in \mathcal{A} \times \mathcal{A}$ is known to be a coupled fixed point for a mapping $\mathcal{F}: \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ if

$$\mathcal{F}(a, b) = a \text{ and } \mathcal{F}(b, a) = b.$$

Definition 1.2.29[43] A point $(a, b) \in \mathcal{A} \times \mathcal{A}$ is known to be a coupled coincidence point for the functions $\mathcal{F}_1, \mathcal{F}_2 : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ if

$$\mathcal{F}_1(a, b) = \mathcal{F}_2(a, b) \text{ and } \mathcal{F}_1(b, a) = \mathcal{F}_2(b, a).$$

Definition 1.2.30[43] A point $(a, b) \in \mathcal{A} \times \mathcal{A}$ is known to be a common coupled fixed point of the mappings $\mathcal{F}_1, \mathcal{F}_2 : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ if

$$a = \mathcal{F}_1(a, b) = \mathcal{F}_2(a, b) \text{ and } b = \mathcal{F}_1(b, a) = \mathcal{F}_2(b, a).$$

Various Gernalizations of Metric Space

In recent period, new spaces originated by many researchers. The key in setting up these spaces is to weaken axioms of metric spaces. Some of the gernalizations of metric spaces are given below.

$\mathcal{D}$-metric space

$\mathcal{D}$-Metric Space is introduced by Dhage[48] in 1984. Many authors has developed the concept and proved many results. Here we discuss basic definitions and theorems of $\mathcal{D}$-metric space.

Definition 1.3.1[44] In a non empty set Π, a function $\mathcal{D}: \Upsilon \times \Upsilon \times \Upsilon \to \mathfrak{R}$ satisfies the conditions for every $p, q, r, \alpha \in \Upsilon$:

(D1) $\mathcal{D}(p, q, r) \geq 0$,

(D2) $\mathcal{D}(p, q, r) = 0$ if and only if $p = q = r$,

(D3) $\mathcal{D}(p, q, r) = \mathcal{D}\{\pi(p, r, q)\}$, where π is a permutation,

(D4) $\mathcal{D}(p, q, r) \leq \mathcal{D}(p, q, \alpha) + \mathcal{D}(p, \alpha, r) + \mathcal{D}(\alpha, q, r)$.

The pair $(\Upsilon, \mathcal{D})$ is known as $\mathcal{D}$-metric space.

b-metric Space

Inspired from the research works of Bakhtin[45], Czerwik[46] described an axiom that was lighter than the triangular inequality and he developed idea of b-metric space.

Definition 1.3.2[46] A real-valued mapping $d: \Pi \times \Pi \to [0, \infty)$ on a non empty set Π is known to be b-metric on Π with constant $s \geq 1$, if it satisfies for every $p, q, \alpha \in \Pi$:

(b1) $d(p, q) = 0$ if $p = q$,

(b2) $d(p, q) = d(q, p)$,

(b3) $d(p, q) \leq s\{(p, \alpha,) + (\alpha, q)\}$.

The pair (Π, d) is known to be b -metric space.

Clearly, when $s = 1$, concept of b-metric space is coincide with metric space.

$\mathcal{G}$-metric Space

In 2005, Mustafa and Sims[47] initiated an innovative description of the generalized metric by introducing $\mathcal{G}$-metric space. Literature on $\mathcal{G}$-metric space has developed a lot in recent time and many fixed points results on $\mathcal{G}$-metric space have appeared.

Definition 1.3.24[47] A mapping $\mathcal{G}: \Pi \times \Pi \times \Pi \to [0, \infty)$ on a non empty set Π satisfies the properties for every $p, q, r, \alpha \in \Pi$:

(G-1) $\mathcal{G}(p, q, r) = 0$ if and only if $p = q = r$,

(G-2) $0 < \mathcal{G}(p, p, q)$, with $p \neq q$,

(G-3) $\mathcal{G}(p, p, q) \leq \mathcal{G}(p, q, r)$, with $q \neq r$,

(G-4) $\mathcal{G}(p, q, r) = \mathcal{G}\{\pi(p, r, q)\}$, where π is a permutation,

(G-5) $\mathcal{G}(p, q, r) \leq \mathcal{G}(p, \alpha, \alpha) + \mathcal{G}(\alpha, q, r)$.

The pair $(\Pi, \mathcal{G})$ is known to be $\mathcal{G}$-metric space.

$\mathcal{G}_b$-metric space

In 2014, by merging the notion of b-metric space and $\mathcal{G}$-metric space, Aghajani et al.[48] obtained the perception of $\mathcal{G}_b$-metric space.

Definition 1.3.90[48**]** Assume that a mapping $\mathcal{G}_b: \Pi \times \Pi \times \Pi \to [0, \infty)$ on a non empty set Π with constant $s \geq 1$, satisfies the following properties for every $p, q, r, \alpha \in \Pi$:

(Gb-1) $\mathcal{G}_b(p, q, r) = 0$ if and only if $p = q = r$,

(Gb-2) $0 < \mathcal{G}_b(p, p, q)$, with $p \neq q$,

(Gb-3) $\mathcal{G}_b(p, p, q) \leq \mathcal{G}_b(p, q, r)$, with $q \neq r$,

(Gb-4) $\mathcal{G}_b(p, q, r) = \mathcal{G}_b\{\pi(p, r, q)\}$, where π is a permutation,

(Gb-5) $\mathcal{G}_b(p, q, r) \leq s\{\mathcal{G}_b(p, \alpha, \alpha) + \mathcal{G}_b(\alpha, q, r)\}$.

The pair $(\Pi, \mathcal{G}_b)$ is known to be $\mathcal{G}_b$-metric space.

Clearly every $\mathcal{G}_b$-metric space is a $\mathcal{G}$-metric space but converse is not true.

Open Problems Researchers face many problems that make the fixed point and its study in constant development, whether in terms of restricting contraction contractive and non-expansive conditions or using auxiliary functions in different spaces. Among these open problems we mention:

(i) Structure of the fixed points sets.
(ii) Approximation of fixed points.
(iii) Abstract metric theory

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Role of Mathematics in Economics: An Analytical Study

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Abstract

Mathematics is a science of numbers and shapes. It studies the subject matter of quantity, structure, space and change. Mathematics is the art of understanding, enumerating and functioning with error and uncertainty. It is not only concerned with everyday problems but also with using imagination, intuition and reasoning to find new ideas and to solve puzzling problems. At the present time the world is at ease recognizing mathematics as a grouping of all those descriptions.

Economics is a social science concerned mainly with explanation and study of the production, consumption, exchange and distribution of goods and services. Economics is "an inquiry into the nature and causes of the wealth of nations" according to Adam Smith the father of economics. It is the knowledge and principles of household management. The economic philosophy of Hebrews years back to about did not think any economic problem without linking the points with the obtainable philosophical, ethical and political structure at the time. Consequently, as in the case of mathematics, the world is now contented in admitting economics as a grouping of all these explanations.

The role of mathematics in economics is an important source for mathematics to improve undergraduate mathematics lessons and better house the requirements of students of economics. Use of mathematics in economics represents impression of the techniques and viewpoints of economists. Its goals are not proposed to teach economics, but rather to give mathematicians an intelligence of what mathematics is exercised at the undergraduate level in different field of economics, and to give students with the prospects to concern their mathematics in pertinent economics perspectives. Mathematics allows economists to do quantifiable analysis. There are reasons for the application of mathematics in economics, mathematical techniques required to make and realize economic ideas and to make contented discussion about economics. Application of mathematics assists in efficient understanding of the correlation and in origin of definite consequences which would either be unfeasible during verbal argument. Mathematics in the present time a very vital instrument exercised in economic study.

Objectives and Methodology of the Study

The main focus of study is on analyzing the relationship between mathematics and economics and the use of different mathematical tools in the concepts of economics. The Main objectives are:-

- 1. To know about the relationship between mathematics and economics.
- To learn the application of mathematics in the theoretical concepts of economics.
- To learn about the application of various mathematical techniques in the study of economics.

This study is descriptive in nature. The study is based on the reference of various books, journals, research articles and websites.

Application of Mathematics in Economics

The use of quantitative technique in economics is the application of mathematics and statistics in economic analysis The key principle of using quantitative tools in economics is to present assiduousness in presuming economic theories and measure economic parameters so that they can be compared with related other values. Subjects like mathematics and statistics provide many tools to analyze various aspects of economic theories and principles considering the data related to individuals, societies, and nations. Economics is concerned with consumption, production, exchange, distribution of wealth, saving, investment, and income, etc. So, it is said that the study of modern economics is incomplete lacking of the knowledge of quantitative techniques.

A mathematical function explains the links among two or more variables. That is, a function states reliance of one variable on one or more variables. Therefore, if the value of a variable Y depends on variable X, mathematically we may explain it Y=f (X). The term entails that each value of the variable Y is decided by an exclusive value of the variable X and Y is known as the dependent variable and X is the independent variable. In economics Demand is a function of price and production is a function of factors of production. Normally we state that demand (D) depends on the price. The major use of mathematics in economics can be doted as to study cause and outcome relationship, analyze three or more values, convert a sentence into symbols, express economic phenomena algebraically find the slope of curves, study trigonometric function, marginal and totalconcepts and study linear and non-linear programming problems. In the study of economics, we can show a cause and effect relationship between different inter-related variables. By applying

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mathematical notations, we can explain more accurately. Such as the quantity demanded of a gooddepends on its price. Then we write quantity demanded of good X as the dependent variable and its price as an independent variable as $D_X = f \quad P_X$.

The marginal analysis of economic conclusions is based on mathematical ideas of derivatives. The marginal idea in economics is the most significant. The marginal utility is the first older derivative DU/DY of the total utility function. all marginal concepts for example marginal rate of substitution, marginal rate of technical substitution, marginal productivity, marginal production, marginal cost, marginal revenue, marginal propensity to consume marginal rate of invest, marginal propensity to save are the first order derivatives of the related functions. Total utility function is U= f (Q). In the differentiation concept is useful to obtain the marginal functions from the total functions.

Mathematical abilities and methods assist to resolve various problems simply and rationally. There are different variables in economics in which it is required to set up a relationship between them by numerical calculation or graphical presentation. In both cases, mathematical tools are very significant. Cost affects profit, price level affects demand, investment is affected by market demand and interest rate, etc. are the different dependent and independent variables whose relation can be verified through mathematics. The demand curve, supply curve, profit maximization, cost minimization, and other algebraic types of problems can be solved by using mathematics.Graphically the value of DU/DY is the slope of a curve. This method is used to find out the rate of change/slope in economics of the curves like demand curves, revenue curves, cost curves, indifference curves. Quadratic Function or second Degree function is yet another mathematical concept and used in cost function in economics.

Derivative, integration, transformation, coordinate geometry, linear programming, etc. are very much useful and essential mathematical tools for successful economic transactions. Statistics is one of the most powerful subjects for explaining eventsand it deals with statistical data and statistical methods, which havea remarkable role in a proper understanding of the economic problems and formulation of economic policies and strategies. Thus, statistics are an essential part of economics. All situations like profit and loss, demand and supply, investment, production, and so many other data can be analyzed by statistical tools and methods. In the modern technological and competitive world, the economic activities at large volumes cannot run without comparing and analyzing other similar types of businesses around the globe or regions.

The economist forever estimates some indicators of the economic environment more time and relate the information to expect the fact of the economy shortly. Mathematics has prevailing logic and tools to set up a relation among the variables which are broadly used in economics. Long and difficult theoretical expressions of economic theories can be stated in the short and understanding form only with the help of mathematical tools and techniques. We can locate the exact shares and coefficients of various economic relations with the help of math. Economic theories obtain further tangible and accurate form because of mathematics and they are then broadly applied and identified. Logarithm, equations, functions, linear programming, integral calculus, game theory, geometry, coordinates geometry, etc., and applied mathematics is significant parts that have a very pivotal role in the field of economics.

The use of diagrams charts are popular in mathematical study in the case of two variables. If the variables are three or more, we cannot use graphs and the verbal description will also beconfused. In this situation, we can use mathematics instead. This makes study simplest and exact. Occasionally mathematical signs are needed instead of the use of economic expressions and equations are used instead of theoretical verdicts. Special findings from mathematical action can be best taken by applying signs and that assists in concluding the effects faster than the conclusion drawn from the graphical technique andothers. Algebra is used in economics for the solution of simple, simultaneous, and quadratic equations. Matrix algebra is used to resolve concurrent equations examined in multi-market or general equilibrium models. Coordinate geometry is exercised to explain the connection among diagrams and equations. It is also used to discover the slope of supply curve and demand curve. For the study of trigonometric functions counting complex origins of quadratic or higher degree equations, trigonometry is applied. The discussion of marginal concepts like marginal utility, marginal production, marginal cost, marginal revenue, marginal physical production, marginal profit, and total utility, total production total cost, total revenue, total physical production and total profit differential calculus is applied. Mathematical ideas are also used in solving linear and non-linear equations and problems. Mechanisms similar to the game theory can also be applied in study and resolving the difficulties related with duopoly and oligopoly markets.

Conclusion

Mathematics can surely assist the household management by helping to examine the production, distribution, consumption, exchange and distribution of goods and services with quantifying the quantifiable. The different economic concepts properly can be understood with the application of different tools of mathematics in economics. The analysis of various economic ideas remains incomplete without the application of mathematics in economics. The exercise of mathematical methods in economic concepts makes easier and interesting to the subject of economics. We should remember the fact that conceptual mathematical thoughts have not forever specified increase to instant real outcomes.

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Cyclotomic Numbers in the Ring $R_{4p^n} = GF(l)[x]/(x^{4p^n} - 1)$

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Abstract

Explicit expressions for all the 4(nd+1) Cyclotomic Numbers in the ring $R_{4p^n} = GF(l)[x]/(x^{4p^n} - 1)$ *, where p and l are distinct odd primes*

$o(l)_{4p^n} = \phi(p^n)$ */d, (n ≥ 1) an intger, are obtained **Theorem2.1.***

Keywords: Primitive root, Cyclotomic Cosets, Cyclotomic Number,

2000 AMS Mathematical Subject Classification: 20C05, 94B05, 16S34.

Introduction

Let GF(l) be a field of odd prime order l .Let $\eta \ge 1$ be an integer with gcd(*l*, η)=1.Let $R_\eta = GF(l)[x]/(x^\eta - 1)$ .The Cyclotomic Numbers in R_η have been obtained by Arora and Pruthi [1,2] .When $m = p^n q$ where p, q are distinct odd primes and *l* is a primitive root mod p^n and q both with gcd ($\phi(p^n)$ /2, $\phi(q)$ /2) = 1, the Cyclotomic Numbers in R_η have been obtained by, G.K.Bakshi and Madhu Raka [4]. In this paper, we consider the case when $\eta = 4p^n$ $o(l)_{p^n} = \phi(p^n)$ /d, (n ≥ 1) where p and l are distinct odd primes . Explicit expressions for all the 4(nd+1) Cyclotomic Numbers in the ring $R_{4p^n} = GF(l)[x]/(x^{4p^n} - 1)$, where p and *l* are distinct odd primes $o(l)_{4p^n}$ = $\phi(p^n)$ /d, (n ≥ 1) an intger, are obtained.

Cyclotomic Numbers in $R_{4p^n} = GF(l)[x]/(x^{4p^n} - 1)$

2.1. For $0 \le s \le \eta - 1$, let $C_s = \{s, sl, sl^2, ..., sl^{t_s - 1}\}$, where t_s is the least positive integer such that $sl^{t_s} \equiv s \pmod{\eta}$ be the cyclotomic coset containing s. corresponding to the cyclotomic coset C_s containing s and its elements are called Cyclotomic Numbers.

Lemma2.1. Let p, l be distinct odd primes and n ≥ 1, an integer, $o(l)_{4p^n} = \frac{\phi(4p^n)}{d}$.Then $o(l)_{4p^{n-j}} = \frac{\phi(p^{n-j})}{d}$, for all $0 \le j \le n-1$.

Proof . Trivial.

Lemma2.2. For given distinct odd primes p, l there always exists fixed integer g satisfying $(g, 4pl) = 1$, 1 < g < 4p, $o(g)_{4p} = \phi(4p)$

Where g ≢ l, l, l^2, l^3 …, $l^{\frac{\phi(4p)}{d}-1}$

Lemma2.3 There always exists fixed integer g satisfying $(g, 4pl) = 1$, 1 < g < 4p, $o(g)_{4p} = \phi(4p)$

And $g^j \not\equiv$,l^k mod(4p) …, $l^{\frac{\phi(4p)}{d}-1}$ for any j and k For 0 ≤ j ≤d –1 and For 0 ≤ k ≤ $\frac{\phi(4p)}{d}$ –1further for any j , 1 ≤ j ≤ n the set { l, l, l^2 ,l^3 …, $l^{\frac{\phi(4p^{n-i})}{d}-1}$ g,g l,g l^2 , gl^3 …, $l^{\frac{\phi(4p^{n-i})}{d}-1}$ g^2, $g^2 l$, $g^2 l^2$, $g^2 l^3$ …, $g^2 l^{\frac{\phi(4p^{n-i})}{d}-1}$ g^{d-1}, $g^{d-1} l$, $g^{d-1} l^2$, $g^{d-1} l^3$ …,

$g^{d-1} l^{\frac{\phi(4p^{n-i})}{d}-1}$ } forms a reduced residue system modulo $4p^{n-j}$

Proof . Lemma4 [1]

Theorem2.1. If η = $4p^n$ (n ≥ 1), then the 4(nd+1) cyclotomic cosets modulo $4p^n$ are given by (i) C_0 = {0}, C_{p^n} = {p^n} C_{2p^n} = { $2p^n$ }, C_{3p^n} = { $3p^n$}

For 0 ≤ k ≤d –1 and For 0 ≤ i ≤n –1

(i) $C_{g^k p^i}$ = {$g^k p^i$, $g^k p^i l$, …, $g^k p^i l^{\frac{\phi(4p^{n-i})}{d}-1}$},

(ii) $C_{2g^k p^i}$ = {$2g^k p^i$, $2g^k p^i l$, …, $2g^k p^i l^{\frac{\phi(4p^{n-i})}{d}-1}$},

(iii) $C_{3g^k p^i}$ = {$3g^k p^i$, $3g^k p^i l$, …, $3g^k p^i l^{\frac{\phi(4p^{n-i})}{d}-1}$},

(iv) $C_{4g^k p^i}$ = {$4g^k p^i$, $4g^k p^i l$, …, $4g^k p^i l^{\frac{\phi(4p^{n-i})}{d}-1}$},

where g is defined as in lemma2.2.

Proof. C_0 = {0}, C_{p^n} = {p^n} C_{2p^n} = { $2p^n$ }, C_{3p^n} = { $3p^n$ }are trivial.

For $0 \le k \le d-1$ and For $0 \le i \le n-1$

Since by our choice $o(l)_{4p^{n-i}} = \frac{\phi(4p^{n-i})}{d}$ Hence $C_{g^k p^i}$ is the cyclotomic coset containing $g^k p^i$. Similarly $C_{2g^k p^i}$ is the cyclotomic coset containing $2g^k p^i$ .On same lines we can say that $C_{3g^k p^i}$ and $C_{4g^k p^i}$ are the cyclotomic coset containing $3g^k p^i$ and $4g^k p^i$.

We now claim that the cyclotomic cosets obtained in (i)-(iv) above are the only cyclotomic cosets modulo $4p^n$.

By constructions of cyclotomic cosets in (i)-(iv) it then follows easily that :

$|C_0|=1, |C_{p^n}|=1, |C_{2p^n}|=1, |C_{3p^n}|=1$

For $0 \le k \le d-1$ and For $0 \le i \le n-1$

$|C_{g^k p^i}|, |C_{2g^k p^i}|, |C_{3g^k p^i}|$, and $|C_{4g^k p^i}| = \frac{\phi(4p^{n-i})}{d}$

Then, by order considerations, it follows that the sum :

$$|C_0| + |C_{p^n}| + |C_{2p^n}| + |C_{3p^n}| + |\sum_{k=0,i=0}^{d-1,n-1} C_{G^K P^I}| + |\sum_{k=0,i=0}^{d-1,n-1} C_{2G^K P^I}|$$

$$+\left|\sum_{k=0,i=0}^{d-1,n-1} C_{3G^{K}P^{I}}\right|+\left|\sum_{k=0,i=0}^{d-1,n-1} C_{4G^{K}P^{I}}\right|$$

$$= 4 + 4d\sum_{,i=0}^{,n-1} \frac{\phi(4p^{n-i})}{d}$$

$=4+4\ (p^n -1)= 4p^n$

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Eigen Value Approach To Elastodynamic Deformation In Rotating Orthotropic Micropolar Viscoelastic Medium

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Abstract

The present study deals with the elastodynamic response in homogeneous orthotropic micropolar viscoelastic medium under the effect of rotation due to the time harmonic source. An eigen value approach after using Fourier transform has been employed to solve the problem. The components of displacements, microrotation and stresses are obtained in the transformed domain. A numerical inversion technique has been applied to obtain the components of resulting quantities in the physical domain. The resulting quantities are computed numerically and depicted graphically to describe the effect of viscosity and rotation for a specific model. Some special cases are deduced.

Keywords: Micropolar, orthotropic, viscoelastic, eigen value approach and Fourier transform

Introduction

The study of viscoelastic behaviour is very important since many engineering phenomena, the materials may describe viscoelastic behaviour as an unintentional side effect. Viscoelasticity is of interest in many branches of material science, solid state physics, metallurgy, steel, aluminum, copper, synthetic polymer. In reality all materials deviate from Hooke's law in various ways by exhibiting viscous-like as well as characteristics. The Kelvin-Voigt model is one of the macroscopic mechanical models used to represent the viscoelastic behavior of the material. This model described the elastic response of stress when the deformation is time dependent but recoverable. Eringen , (1967).investigated the linear theory of viscoelasticity for micropolar materials. The constitutive equations of stress and couple stress rate dependent materials, strain and microrotation rate dependent materials and continous memory dependent micropolar elastic medium are studied. The growth equations and propagation conditions, which govern the propagation of waves in micropolar viscoelasticity are discussed by Mc Carthy and Eringen (69). They studied the coupling between the discontinuities in the macroscopic and microscopic fields. De Cicco and Nappa (1999) studied the problem of Saint Venant's principle in the dynamic theory of micropolar viscoelastic solids. 19

The formulation and solution of anisotropic problems is more difficult than their isotropic counterparts. In the last years, many problems related to elastodynamic response of an anisotropic continuum have been studied by some researchers. Iesan (1974) investigated the static theory of anisotropic micropolar elastic media and also proved the positive definiteness of the operator for the boundary value problem. Passarella, Tibullo and Zampoli (2011) presented the plane strain problem in orthotropic micropolar elastic media and studied the existence of progressive plane waves under the strong ellipticity conditions.

Most of the large bodies like the moon, the earth and other planets have an angular velocity and the earth behavies like a huge magnet. So, it is become important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. The problems relating to rotating media have been discussed by many researchers. Othman, Atwa, Jahangir and Khan (2013) worked on the effect of magnetic field and rotation on generalized thermo-microstretch for a homogeneous isotropic elastic solid whose surface is subjected to a mode-I crack under the Green-Nagadhi theory. El-Karamany [22] derived Maysel's formula in the generalized linear micropolar thermoviscoelasticity. Magana and Quintanilla (2014) represented the uniqueness and analyticity of solutions in isotropic micropolar thermoviscoelastic materials. Khan et al (2020) discussed thermally developed unsteady viscoelastic micropolar nanofluid with modified heat/ mass fluxes. Abouelregal et al (2023) worked on thermo magneto interaction in a viscoelastic micropolar medium by considering a higher order two phase delay thermoeastic model.

In the present paper, we have obtained the displacements, microrotation and stresses in an orthotropic micropolar viscoelastic solid under the effect of rotation subjected to the time harmonic source by applying Fourier transform. The solutions are determined by using eigen value approach and applying numerical inversion transform technique. The effects of viscosity and rotation are illustrated numerically and represented graphically. Some particular cases are also deduced from the present investigation.

Basic Equations

The governing equations for micropolar elastic solids with rotation in the absence of body forces and body couples are given by Eringen (1967), Iesan (1974) and Schoenberg and Censor (1973) as

$$t_{ji,j} = \rho[\ddot{\boldsymbol{u}}_i + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u}_i) + 2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}_i], \qquad (1)$$

$$\epsilon_{ijr} t_{jr} + m_{ji,j} = \rho j \ddot{\boldsymbol{\phi}}_i. \qquad (2)$$

In these equations, $\boldsymbol{u}_i$ are the components of displacement vector, $\boldsymbol{\phi}_i$ are the components of microrotation vector, t_{ij} are the components of force stress tensor, m_{ij} are the components of couple stress tensor, ϵ_{ijk} is permutation symbol, ρ is the density of the medium, j is the microinertia. The medium is rotating uniformly with respect to an inertial frame and the rotation vector in $(x,\ y,\ z)$ rectangular Cartesian frame is $\mathbf{\Omega} = \Omega\hat{\mathrm{n}}$, where $\hat{\mathrm{n}}$ is the unit vector denoting the direction of axis of rotation. The displacement equation of motion has two terms: centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$, due to the time varying motion only and $2\mathbf{\Omega} \times \dot{\mathbf{u}}$ is the Coriolis acceleration.

Formulation of the Problem

A homogeneous and isotropic medium of an infinite extent is considered and rectangular Cartesian coordinate system (x, y, z) is chosen. We consider xy- plane with the displacement vector $\mathbf{u} = (u_1,\ u_2,\ 0)$, microrotation vector $\boldsymbol{\phi} = (0, 0, \phi_3)$ and angular velocity $\mathbf{\Omega} = (0, 0, \Omega)$. With these consideration, the governing equations take the form

$$A_{11} \frac{\partial^2 u_1}{\partial x^2} + (A_{12} + A_{78}) \frac{\partial^2 u_2}{\partial x \partial y} + A_{88} \frac{\partial^2 u_1}{\partial y^2} - K_1 \frac{\partial \phi_3}{\partial y} = \rho \left[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega \frac{\partial u_2}{\partial t} \right], \quad (3)$$

$$(A_{12} + A_{78}) \frac{\partial^2 u_1}{\partial x \partial y} + A_{77} \frac{\partial^2 u_2}{\partial x^2} + A_{22} \frac{\partial^2 u_2}{\partial y^2} - K_2 \frac{\partial \phi_3}{\partial x} = \rho \left[\frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 + 2\Omega \frac{\partial u_1}{\partial t} \right], \quad (4)$$

$$B_{66} \frac{\partial^2 \phi_3}{\partial x^2} + B_{44} \frac{\partial^2 \phi_3}{\partial y^2} - \chi \phi_3 + K_1 \frac{\partial u_1}{\partial y} + K_2 \frac{\partial u_2}{\partial x} = \rho j \frac{\partial^2 \phi_3}{\partial t^2}, \qquad (5)$$

where

$$K_1 = A_{78} - A_{88},\ K_2 = A_{77} - A_{78},\ \chi = K_2 - K_1.$$

The constitutive relations for the orthotropic micropolar elastic solid are given by Iesan (1974) as

$$t_{11} = A_{11}\epsilon_{11} + A_{12}\epsilon_{22}, \quad t_{12} = A_{77}\epsilon_{12} + A_{78}\epsilon_{21}, \quad t_{21} = A_{78}\epsilon_{12} + A_{88}\epsilon_{21},$$

$$t_{22} = A_{12}\epsilon_{11} + A_{22}\epsilon_{22}, \quad m_{13} = B_{66}\frac{\partial \phi_3}{\partial x}, \quad m_{23} = B_{44}\frac{\partial \phi_3}{\partial y}, \tag{6}$$

where

$$\epsilon_{ij} = u_{j,i} + \epsilon_{ji3}\phi_3, \tag{7}$$

and ϵ_{ij} are the components of micropolar strain tensor.

We assume the viscoelastic nature of the material which is described by the Voigt (1887) model of linear elasticity. So, we replace the micropolar elastic constants $\bar{A}_{11}, \bar{A}_{22}, \bar{A}_{77}, \bar{A}_{88}, \bar{A}_{12}, \bar{A}_{78}, \bar{B}_{44}, \bar{B}_{66}$ by their complex modulii as

$$\bar{A}_{11} = A_{11} + A_{11}^{\nu}\frac{\partial}{\partial t}, \quad \bar{A}_{22} = A_{22} + A_{22}^{\nu}\frac{\partial}{\partial t}, \quad \bar{A}_{77} = A_{77} + A_{77}^{\nu}\frac{\partial}{\partial t}, \bar{A}_{88} = A_{88} + A_{88}^{\nu}\frac{\partial}{\partial t},$$

$$\bar{A}_{12} = A_{12} + A_{12}^{\nu}\frac{\partial}{\partial t}, \quad \bar{A}_{78} = A_{78} + A_{78}^{\nu}\frac{\partial}{\partial t}, \bar{B}_{44} = B_{44} + B_{44}^{\nu}\frac{\partial}{\partial t}, \quad \bar{B}_{66} = B_{66} + B_{66}^{\nu}\frac{\partial}{\partial t}, \tag{8}$$

We assume the non-dimensional variables as

$$x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad u_i' = \frac{u_i}{h}, \quad \phi_3' = \frac{\bar{A}_{11}}{K_1}\phi_3, \quad t_{ij}' = \frac{t_{ij}}{\bar{A}_{11}},$$

$$t' = \sqrt{\frac{\bar{A}_{11}}{\rho h^2}}\,t, \quad m_{ij}' = \frac{h}{\bar{B}_{66}}m_{ij}, \quad {\omega'}^2 = \frac{\rho h^2}{\bar{A}_{11}}\omega^2, \quad \Omega' = \frac{\Omega}{\omega}, \tag{9}$$

where h is the parameter of dimension of length.

Assuming the time harmonic solution as

$$(u_1,\ u_2,\ \phi_3) = \big(u_1(x,y),\ u_2(x,y),\ \phi_3(x,y)\big)e^{\iota\omega t}, \tag{10}$$

where ω is the angular frequency.

Making use of dimensionless quantities defined by (9) in (3)-(5), after suppressing the primes and with the aid of (10), we obtain

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{\bar{A}_{11}}{\bar{A}_{88}}\frac{\partial^2 u_1}{\partial x^2}+\frac{(\bar{A}_{12}+\bar{A}_{78})}{\bar{A}_{88}}\frac{\partial^2 u_2}{\partial x\partial y}-\frac{K_1^2}{\bar{A}_{11}\bar{A}_{88}}\frac{\partial\phi_3}{\partial y}$$

$$=-\left[\left(\frac{\bar{A}_{11}\omega^2}{\bar{A}_{88}}\right)u_1+\left(\frac{\rho \mathrm{h}^2\Omega^2\omega^2}{\bar{A}_{88}}\right)u_1+\left(\frac{2\iota h\Omega\omega^2\,\sqrt{\bar{A}_{11}\rho}}{\bar{A}_{88}}\right)u_2\right], \tag{11}$$

$$\frac{\partial^2 u_2}{\partial y^2}+\frac{(\bar{A}_{12}+\bar{A}_{78})}{\bar{A}_{22}}\frac{\partial^2 u_1}{\partial x\partial y}+\frac{\bar{A}_{77}}{\bar{A}_{22}}\frac{\partial^2 u_2}{\partial x^2}-\frac{K_1K_2}{\bar{A}_{22}}\frac{\partial\phi_3}{\partial x}$$

$$=\left[-\left(\frac{\bar{A}_{11}\omega^2}{\bar{A}_{22}}\right)u_2-\left(\frac{\rho \mathrm{h}^2\Omega^2\omega^2}{\bar{A}_{22}}\right)u_2+\left(\frac{2\iota h\Omega\omega^2\,\sqrt{\bar{A}_{11}\rho}}{\bar{A}_{22}}\right)u_1\right], \tag{12}$$

$$\left(\bar{B}_{66}\frac{\partial^2}{\partial x^2}+\bar{B}_{44}\frac{\partial^2}{\partial y^2}-h^2\chi\right)\frac{K_1}{\bar{A}_{11}h^2}\phi_3+K_1\frac{\partial u_1}{\partial y}+K_2\frac{\partial u_2}{\partial x}=-\frac{jK_1\omega^2}{h^2}\phi_3. \tag{13}$$

We define the Fourier transform w. r. t. x as

$$\{\tilde{u}_i(\xi,y,\omega),\tilde{\phi}_3(\xi,y,\omega)\}=\int_{-\infty}^{\infty}\{u_i(x,y,\omega),\ \phi_3(x,y,\omega)\}e^{\iota\xi x}dx. \qquad i=1,2 \tag{14}$$

Applying the Fourier transform defined by (14) on (11)-(13), yield

$$\widetilde{u_1''}=a_{11}\tilde{u}_1+a_{12}\tilde{u}_2+b_{12}\widetilde{u_2'}+b_{13}\widetilde{\phi_3'}, \tag{15}$$

$$\widetilde{u_2''}=a_{21}\tilde{u}_1+a_{22}\tilde{u}_2+a_{23}\tilde{\phi}_3+b_{21}\widetilde{u_1'} \tag{16}$$

$$\widetilde{\phi_3''}=a_{32}\tilde{u}_2+a_{33}\tilde{\phi}_3+b_{31}\widetilde{u_1'}\,, \tag{17}$$

where

$$a_{11}=\frac{\bar{A}_{11}(\xi^2-\omega^2)-\rho\Omega^2\omega^2h^2}{\bar{A}_{88}},\quad a_{12}=-\left(\frac{2\iota h\Omega\omega^2\,\sqrt{\bar{A}_{11}\rho}}{\bar{A}_{88}}\right),\quad b_{12}=\frac{\iota\xi(\bar{A}_{12}+\bar{A}_{78})}{\bar{A}_{88}},$$

$$b_{13}=\frac{K_1^2}{\bar{A}_{11}\bar{A}_{88}},\quad a_{21}=\left(\frac{2\iota h\Omega\omega^2\,\sqrt{\bar{A}_{11}\rho}}{\bar{A}_{22}}\right),\quad a_{22}=\frac{\bar{A}_{77}\xi^2-\bar{A}_{11}\omega^2-\rho\Omega^2\omega^2h^2}{\bar{A}_{22}},$$

$$a_{23}=-\frac{\iota\xi K_1K_2}{\bar{A}_{11}\bar{A}_{22}},\quad b_{21}=\frac{\iota\xi(\bar{A}_{12}+\bar{A}_{78})}{\bar{A}_{22}},\quad a_{32}=\frac{\iota\xi\bar{A}_{11}K_2h^2}{\bar{B}_{44}K_1},$$

$$a_{33} = \frac{-j\omega^2 \bar{A}_{11} + \xi^2 \bar{B}_{66} + h^2\chi}{\bar{B}_{44}}, \quad b_{31} = -\frac{\bar{A}_{11}h^2}{\bar{B}_{44}},$$

and

$$\bar{Z} = Z(1 + \iota Q_i), \qquad Z = A_{11}, A_{22}, A_{77}, A_{88}, A_{12}, A_{78}, B_{44}, B_{66},$$

$$(Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8) = \omega\left(\frac{A_{11}^{\nu}}{A_{11}}, \frac{A_{22}^{\nu}}{A_{22}}, \frac{A_{77}^{\nu}}{A_{77}}, \frac{A_{88}^{\nu}}{A_{88}}, \frac{A_{12}^{\nu}}{A_{12}}, \frac{A_{78}^{\nu}}{A_{78}}, \frac{B_{44}^{\nu}}{B_{44}}, \frac{B_{66}^{\nu}}{B_{66}}\right).$$

The system of equations (15)-(17) can be written as

$$\frac{d}{dy}W(\xi, y, \omega) = A(\xi, \omega)W(\xi, y, \omega), \tag{18}$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, D = \frac{d}{dy}, \quad U = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & b_{12} & b_{13} \\ b_{21} & 0 & 0 \\ b_{31} & 0 & 0 \end{bmatrix}.$$

Assuming the solution of (18) of the form

$$W(\xi, y, \omega) = X(\xi, \omega)e^{qy}, \tag{19}$$

so that

$$A(\xi, \omega)W(\xi, y, \omega) = qW(\xi, y, \omega), \tag{20}$$

which leads to eigen value problem. The characteristic equation for the problem is given by

$$\det[A - qI] = 0, \tag{21}$$

after simplification, we get

$$q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0, \tag{22}$$

where

$$\lambda_1 = a_{11} + a_{22} + a_{33} + b_{12}b_{21} + b_{13}b_{31},$$

$$\lambda_2 = a_{11}a_{22} + a_{22}a_{33} + a_{11}a_{33} - a_{12}a_{21}$$

$$-a_{23}a_{32} + b_{12}(b_{21}a_{33} - a_{23}b_{31}) + b_{13}(a_{22}b_{31} - b_{21}a_{32}),$$

$$\lambda_3 = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}a_{21}a_{33}.$$

The equation (22) is a cubic equation in q^2, the roots of this equation are $\pm q_i, \ i = 1, 2, 3$.

The eigen vector $X(\xi, \omega)$ corresponding to the eigen values q_i can be determined by solving the system of equation

$$[A - qI] \, X(\xi, \omega) = 0. \qquad (23)$$

We assume that X_1, X_2, X_3, X_4, X_5 and X_6 are the components of eigen vector $X(\xi, \omega)$.

The set of eigen vector $X_i(\xi, \omega), \ (i = 1, 2, 3, 4, 5, 6)$ can be written as

$$X_i(\xi, \omega) = \begin{bmatrix} X_{i1}(\xi, \omega) \\ X_{i2}(\xi, \omega) \end{bmatrix},$$

where

$$X_{i1}(\xi, \omega) = \begin{bmatrix} a_i \\ b_i \\ 1 \end{bmatrix}, \quad X_{i2}(\xi, \omega) = \begin{bmatrix} a_i q_i \\ b_i q_i \\ q_i \end{bmatrix}, \quad q = q_i; \ i = 1,2,3,$$

$$X_{j1}(\xi, \omega) = \begin{bmatrix} a_j \\ b_j \\ 1 \end{bmatrix}, \quad X_{j2}(\xi, \omega) = \begin{bmatrix} -a_j q_i \\ -b_j q_i \\ -q_i \end{bmatrix}, \quad j = i + 3, \ q = -q_i; \ i = 1,2,3,$$

$$a_i = [q_i^3 b_{12} + q_i^2 a_{12} + q_i(b_{13}a_{32} - b_{12}a_{33}) - a_{12}a_{33}]/\Delta_i,$$

$$b_i = [q_i^4 - q_i^2(b_{13}b_{31} + a_{11} + a_{33}) + a_{11}a_{33}]/\Delta_i,$$

$$\Delta_i = q_i^2(b_{12}b_{31} + a_{32}) + q_i a_{12}b_{31} - a_{32}a_{11},$$

$$a_j = [-q_i^3 b_{12} + q_i^2 a_{12} - q_i(b_{13}a_{32} - b_{12}a_{33}) - a_{12}a_{33}]/\Delta_j,$$

$$b_j = [q_i^4 - q_i^2(b_{13}b_{31} + a_{11} + a_{33}) + a_{11}a_{33}]/\Delta_j,$$

$$\Delta_j = q_i^2(b_{12}b_{31} + a_{32}) - q_i a_{12}b_{31} - a_{32}a_{11}.$$

The solution of equation (18) satisfies the radiation condition for $y \geq 0$ (medium I) is of the form

$$W(\xi, y, \omega) = \sum_{i=1}^{3} L_{i+3} X_{i+3}(\xi, \omega) exp(-q_i y), \tag{24}$$

where L_{i+3} $(i = 1, 2, 3)$ are arbitrary constants.

The equation (24) represents the solution of the problem in the plain strain case of micropolar orthotropic elasticity by employing the eigen-value approach and therefore can be applied to a broad class of problems in the domains of Laplace and Fourier transforms.

Boundary Conditions

We take the concentrated harmonic normal force acting in the direction of the y-axis at the origin of the Cartesian coordinate system in an orthotropic micropolar viscoelastic medium along with the vanishing of tangential stress and tangential couple stress. Mathematically, these can be written at $(y = 0)$ as

$$t_{22}(x, 0, t\,) = -F_0 \delta(x) e^{\iota \omega t}, \tag{25}$$

$$t_{21}(x, 0, t\,) = 0, \tag{26}$$

$$m_{23}(x, 0, t\,) = 0. \tag{27}$$

Using the non-dimensional variables by (9) in the boundary conditions (25)-(27) along with $F_0' = \frac{F_0}{A_{11}}$, we obtain the boundary conditions in the non-dimensional form. Applying the Fourier transform defined by (14) on non dimensional boundary conditions and with the aid of (6)-(10), (14) and (24), we obtain the expression of displacements, microrotation and stresses in the transformed domain as

$$\left(\tilde{u}_1, \tilde{u}_2, \tilde{\phi}_3, \tilde{t}_{21}, \tilde{t}_{22}, \tilde{m}_{23}\right) = \frac{1}{\Delta} \sum_{j=i+3}^{3} \left(a_j, b_j, 1, A_j, B_j, C_i\right) \Delta_i e^{-q_i y}, \quad i = 1,2,3. \tag{28}$$

where

$$A_j = \frac{-\iota \xi a_j \bar{A}_{12} - b_j \bar{A}_{22} q_i}{\bar{A}_{11}}, \quad j = i + 3, \quad i = 1,2,3$$

$$B_j = \frac{-\iota\xi b_j \bar{A}_{78} - a_j \bar{A}_{88} q_i}{\bar{A}_{11}} + \frac{K_1}{\bar{A}_{11}^2}(\bar{A}_{88} - \bar{A}_{78}), \quad j = i+3, \quad i = 1,2,3$$

$$C_i = \frac{-K_1 \bar{B}_{44}}{\bar{A}_{11}\bar{B}_{66}} q_i, \qquad i = 1,2,3$$

$$\Delta = A_4(B_5C_3 - C_2B_6) + A_5(B_6C_1 - C_3B_4) + A_6(B_4C_2 - C_1B_5),$$

$$\Delta_1 = \frac{F_0(C_2B_6 - B_5C_3)}{\Delta},$$

$$\Delta_2 = \frac{F_0(B_4C_3 - C_1B_6)}{\Delta},$$

$$\Delta_3 = \frac{F_0(C_1B_5 - B_4C_2)}{\Delta}.$$

Thus, we obtain the expressions of displacement components, microrotation and stress components in the Fourier transformed domain in a rotating orthotropic micropolar viscoelastic medium. To get the displacement, microrotation and stress components in the physical domain, we require to invert Fourier integrals.

Particular Case

(i) Neglecting the effect of viscosity i.e. $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8 \to 0$, we obtain the corresponding expressions of displacements, microrotation and stresses for rotating orthotropic micropolar elastic solid.

(ii) Neglecting the effect of rotation i.e. $\Omega \to 0$, the corresponding expressions of displacements, microrotation and stresses for orthotropic micropolar viscoelastic solid are given by

$$\left(\tilde{u}_1, \tilde{u}_2, \tilde{\phi}_3, \tilde{t}_{21}, \tilde{t}_{22}, \tilde{m}_{23}\right) = \frac{1}{\Delta}\sum_{i=1}^{3}(-a_i q_i, b_i, 1, A_i, B_i, C_i)\, \Delta_i e^{-q_i y}, \tag{29}$$

where

$$A_i = \frac{\iota\xi a_i \bar{A}_{12} - b_i \bar{A}_{22} q_i}{\bar{A}_{11}}, \qquad i = 1,2,3$$

$$B_i = \frac{-\iota\xi b_i \bar{A}_{78} + a_i \bar{A}_{88} q_i^2}{\bar{A}_{11}} + \frac{K_1}{\bar{A}_{11}^2}(\bar{A}_{88} - \bar{A}_{78}), \qquad i = 1,2,3$$

$$C_i = \frac{-K_1\bar{B}_{44}}{\bar{A}_{11}\bar{B}_{66}} q_i, \quad i = 1,2,3$$

$$\Delta = A_1(B_2C_3 - C_2B_3) + A_2(B_3C_1 - C_3B_1) + A_3(B_1C_2 - C_1B_2),$$

$$\Delta_1 = \frac{F_0(C_2B_3 - B_2C_3)}{\Delta},$$

$$\Delta_2 = \frac{F_0(C_3B_1 - B_3C_1)}{\Delta},$$

$$\Delta_3 = \frac{F_0(C_1B_2 - B_1C_2)}{\Delta}.$$

Inversion of Transform

The transformed displacement and stresses are function of y and the parameter of Fourier transform ξ, hence are of the form $f(\xi, y, t)$. To get the function $f(x, y, t)$ in the physical domain, we invert the Fourier transform using

$$f(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\iota\xi x}\,\tilde{f}(\xi,y,t)d\xi,$$

$$= \frac{1}{\pi}\int_{0}^{\infty}\{\cos(\xi x)\,f_e -$$

$$\iota\sin(\xi x)f_o\}\,d\xi, \qquad (30)$$

where f_e and f_o are the even and the odd parts of the function $\tilde{f}(\xi, y, t)$ respectively.

Numerical Results

For numerical computations, we take the values of relevant parameters for an orthotropic micropolar viscoelastic solid:

$\bar{A}'_{11} = 13.97 \times 10^{10} dyne/cm^2$, $\bar{A}'_{22} = 13.75 \times 10^{10} dyne/cm^2$,

$\bar{A}'_{77} = 3.0 \times 10^{10} dyne/cm^2$, $\bar{A}'_{88} = 3.2 \times 10^{10} dyne/cm^2$,

$\bar{A}'_{12} = 8.13 \times 10^{10} dyne/cm^2$, $\bar{A}'_{78} = 2.2 \times 10^{10} dyne/cm^2$,

$\bar{B}'_{44} = 0.056 \times 10^{10} dyne$, $\bar{B}'_{66} = 0.057 \times 10^{10}, h = 0.01cm$.

$Q_1 = 1.0$, $Q_2 = 0.5$, $Q_3 = 0.1$, $Q_4 = 1.5, Q_5 = 1.5$, $Q_6 = 1.0$,

$Q_7 = 0.5$, $Q_8 = 1.0$,

For the comparison with a microplar isotropic solid, following Gauthier (1982), we take the following values for the Aluminum epoxy composite as

$$\rho = 2.19 gm/cm^2,\ \lambda = 7.59 \times 10^{10} dyne/cm^2,\ \mu = 1.89 \times 10^{10} dyne/cm^2,$$

$$K = 0.0149 \times 10^{10} dyne/cm^2,\ \gamma = 0.0268 \times 10^{10} dyne/cm^2,\ j = 0.00196 cm^2.$$

Discussion

The comparison of the values of displacements, microrotation and stresses in case of orthotropic micropolar viscoelastic solid with rotation (MVR), orthotropic micropolar viscoelastic solid without rotation (MVW) and orthotropic micropolar solid with rotation (MR) are shown in figures 1-6 respectively. In all these figures, (MVR), (MR) and (MVW) corresponding to solid line (——), small dash line (- - - - -) and dash line with centred symbol (– · – · –) respectively.

Figure 1 shows that the value of u_1 initially increases sharply for MVW as compared to MVR and MR for $0 \le x \le 0.7$ and then oscillates about the origin for the whole range. For the range $0 \le x \le 2.5$, the behavior of MVR and MR is similar. For MVW and MR, the value of u_1 is opposite for $2.8 \le x \le 7.6$, and similar for $7.6 \le x \le 10$.

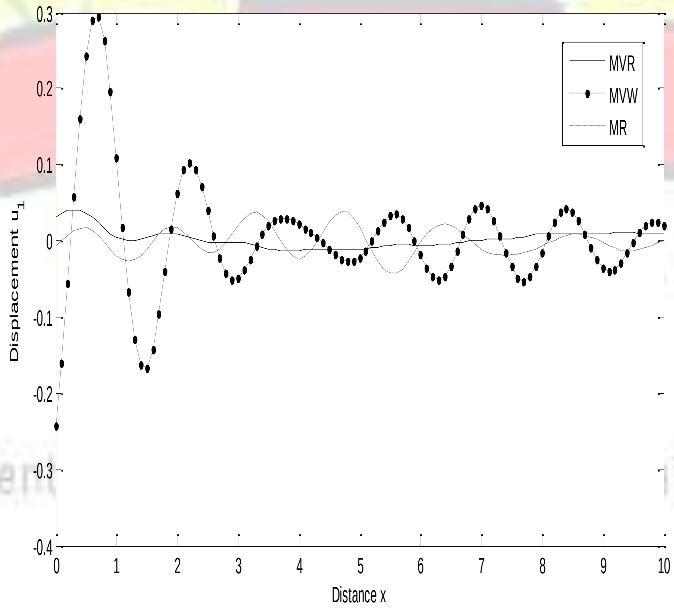

Figure 1 Variations of displacement u_1

Figure 2 represents that the value of normal displacement u_2 starts with a rapid decrease for MVR and MR whereas for MVW, its value sharply increase and then oscillates with x. The behavior of u_2 for MVW and MR is opposite to each other for $0 \leq x \leq 2.2,\ 3.6 \leq x \leq 6,\ 7.4 \leq x \leq 8.8$ and similar for $2.2 \leq x \leq 3.6,\ 6 \leq x \leq 7.4,\ 8.8 \leq x \leq 10$. The value of u_2 for MVR oscillates with large amplitude and approaches to zero.

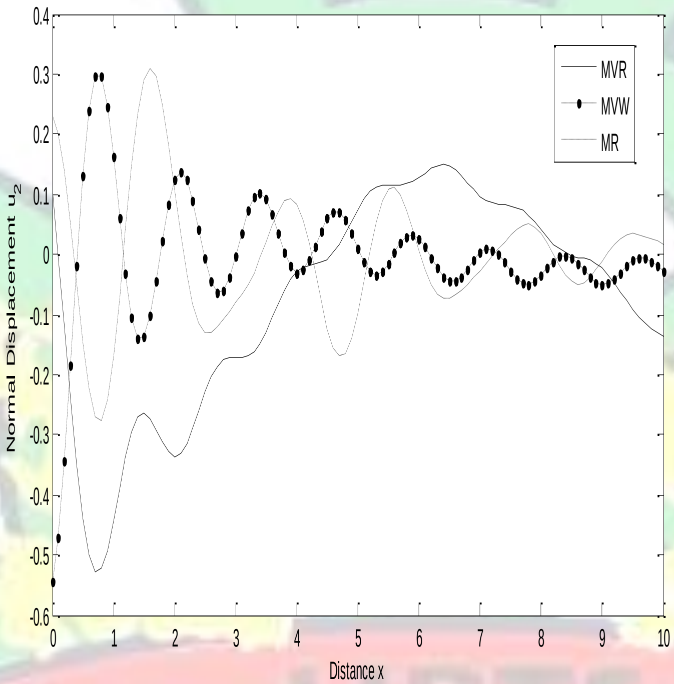

Figure 2. Variations of normal displacement u_2

Figure 3 indicated that initially, the value of microrotation ϕ_3 for MVR decreases for $0 \leq x \leq 3.5$ and subsequently increases for $3.5 \leq x \leq 7$ and then again decreasing for $7 \leq x \leq 10$. The value of ϕ_3 for MVW initially increasing for $0 \leq x \leq 0.3$, sharply decreasing for $0.3 \leq x \leq 1$ and then oscillates for $1 \leq x \leq 10$. The behavior of ϕ_3 for MVW and MR is opposite for $0 \leq x \leq 1.5, 3.3 \leq x \leq 5.2$ and similar for $1.8 \leq x \leq 3.3, 5.8 \leq x \leq 6.3$.

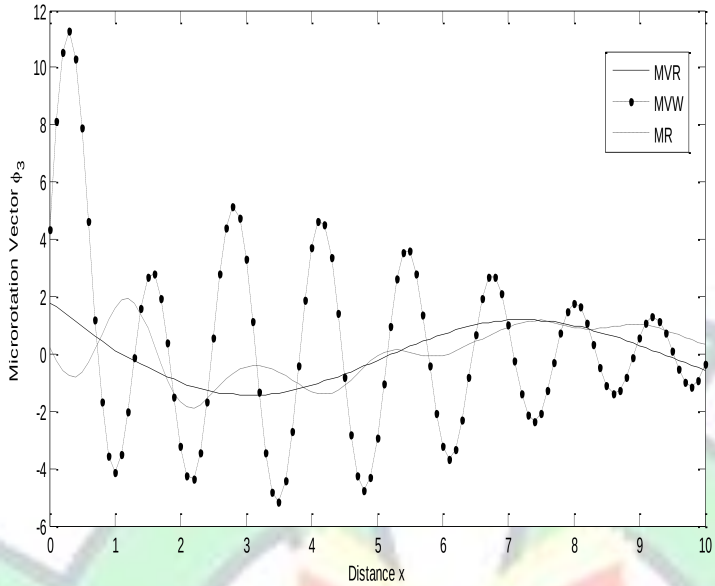

Figure 3. Variations of microrotation vector ϕ_3

It is evident from figure 4 that the value of t_{22} increases for MR for the range $0 \leq x \leq 0.6$, decreases for $0.6 \leq x \leq 1.2$, and then oscillates with x. For MVR and MVW, the value of t_{22} starts with initial increases and then oscillates with large amplitude over the entire range. The oscillation behavior of MVR and MVW is similar for $0 \leq x \leq 10$ and their magnitude values are different.

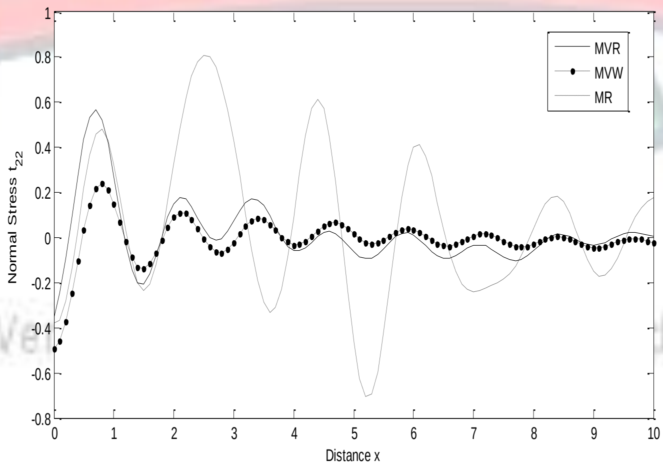

Figure 4. Variations of normal stress t_{22}

Figure 5 depicts that the value of t_{21} for MVR and MR increases sharply for $0 \leq x \leq 0.3$ and then oscillates for $0.3 \leq x \leq 10$ with x about the origin. For MVW, its value oscillates for $0 \leq x \leq 7.6$ with increasing amplitude and for $7.6 \leq x \leq 10$ with constant amplitude. The behavior of t_{21} for MVR and MVW for $0 \leq x \leq 0.2, 0.8 \leq x \leq 3.4$ is similar and opposite for $4.3 \leq x \leq 7.2$.

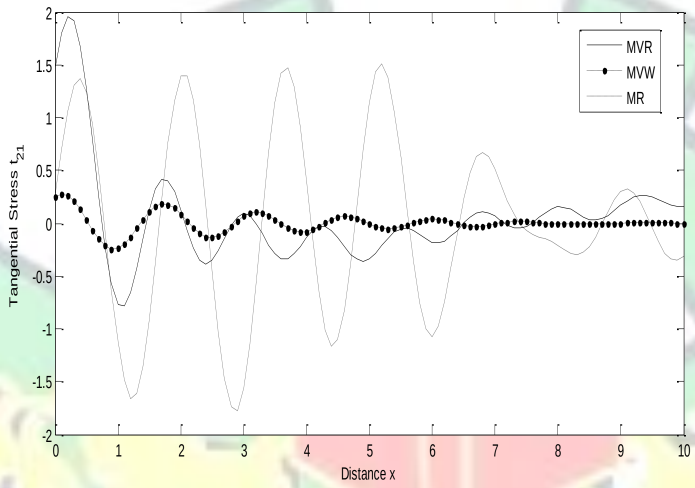

Figure 5. Variations of tangential stress t_{21}

Figure 6 illustrates that the value of m_{23} for MVR, initially increases for $0 \leq x \leq 1$, decreases for $1 \leq x \leq 2.6$ and then oscillates with large amplitude for $2.6 \leq x \leq 10$. For MR, its value initially constant for $0 \leq x \leq 0.8$ and then oscillate with different amplitude for $0.8 \leq x \leq 10$. For MVW, its value increases for $0 \leq x \leq 0.3$, decreases for $0.3 \leq x \leq 1.2$ and then starts oscillate with x.

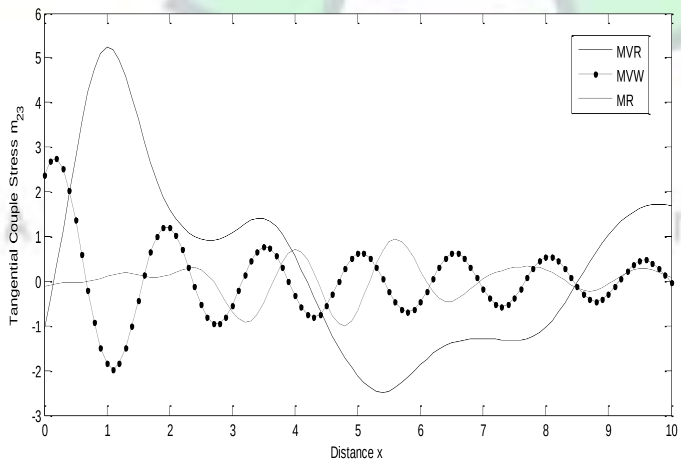

Figure 6. Variations of tangential couple stress

Conclusion

The deformation problem in orthotropic micropolar viscoelastic half space under the effect of rotation has been investigated in frequency domain by applying Fourier transform and eigen value approach. All the physical quantities are significantly affected by viscosity and rotation. The values of normal stress and tangential stress for MVW lie in a short range as compared to the values for MVR and MR. It is concluded that the variation in tangential stress and tangential couple stress is similar in the absence of rotation and also similarity holds for displacement components and microrotation component, although, their magnitude values are different. The viscous effect yields uniformly oscillatory variation and bahaviour of normal stress and tangential stress with different amplitude.

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Some Advanced Topics In Operations Research

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Abstract

In this paper we discussed about Information theory, Forecasting and Decision Analysis. In information theory we discussed about coding, decoding, channel and measure of information(Entropy). We discussed Hoffman's method and The Shan- non Fano Method to determine optimal binary coding. We also discussed about some channel (Discrete memoryless channel, Symmetric channel, Useless channel and capacity of Discrete memoryless channel).

In forcasting discussed about time series and regression analysis.

In decision analysis we discussed an example solved by Bayes' decision rule.

Introduction

Information theory deals with transmitting information from one place to other. There is a source which generates the information and source encoder encodes it into a sequence of binary digits and this information is transmitted through a channel. The channel encoder and decoder allow the binary data to reproduced the binary data. The main question in information theory is how to measure the randomness or uncertainty of a random process represented by a random variable. The entropy function which is defined in terms of the probability distribution of a random variable is a good measure of uncertainty associated with a random process.

Entropy: the Measure of Uncertainty

Entropy is measure of uncertainty. The measure of information is given by

$$h(p) = -\log p$$

where p is probability.

The expected value of information is defined as entropy function. Let X be a random variable with sample space $S = x_1, x_2, x_3, ..., x_M$. The entropy H(x) is is defined by

$$H(X) = -\sum_{k=1}^{M} p_k \log p_k$$

where the base of logarithm is arbitrary and $p_k = p(X = x_k)$, $k = 1$ to M

Properties of Entropy

- If all the M outcomes are equally probable then entropy is maximum.
- If $Z = (X, Y)$ and X, Y are independent then

$$H(Z) = H(X) + H(Y)$$

- Grouping property of $H(X)$. Let the sample space of X be decomposed into two mutually exclusive groups S_1, S_2. We define random variables Y, Z having sample space S_1, S_2 then

$$H(X) = -[P(S_1)\log P(S_1) + P(S_2)\log P(S_2)] + [P(S_1)H(Y) + P(S_2)H(Z)]$$

- H(X) is continuous in each and every independent variable p_k in the interval $[0, 1]$.

Conditional Entropy

Consider two dimensional discrete random variable (X,Y) with joint pdf $p(x_i, y_j)$.

The conditional entropy of X given Y is defined as

$$H(X|Y) = -\sum_{i=1}^{M}\sum_{j=1}^{N} p(x_i, y_j) \log p(x_i|y_j)$$

where $p(x_i|y_j)$ is conditional probability of x_i for given y_j.

Source Coding

Theorem 1.3.1. *Crafts inequality: Let the source alphabets be a_1, a_2, ..., a_k and let n_1, n_2, ., n_k be the lengths in a prefix-free code of corresponding code words and let M be the size of encoder. Then*

$$\sum_{i=1}^{k} M^{-n_i} \leq 1$$

Converse of this theorem is also true.[1]

Theorem 1.3.2. *Let the source alphabets be a_1, a_2, ..., a_k with probabilities p_1, p_2, ...p_k*

and $\overline{n}$ the average length of a uniquely decodable code with alphabets $x_1, x_2, ..., x_M$. Then

$$n \geq -\frac{H(S)}{\log M}$$

where $H(S)$ is entropy
of source.proof:
Consider

$$H(S) - \bar{n} \ln M = -\sum_{i=1}^{k} p_i \ln p_i - \ln M \sum_{i=1}^{k} n_i p_i$$

$$= \sum p_i \ln \frac{M^{-n_i}}{p_i}$$

Since $\ln x \leq x - 1$ *use it in above we get*

$$H(S) - \bar{n} \ln M \leq \sum_{i=1}^{k} p_i \left(\frac{M^{-n_i}}{p_i} - 1 \right)$$

$$= \sum M^{-n_i} - 1 \leq 0$$

using crafts inequality.

$$\bar{n} \geq \frac{H(S)}{\log M}$$

Theorem 1.3.3. *Let the source alphabets be* $a_1, a_2, ..., a_k$ *with probabilities* $p_1, p_2, ..., p_k$ *respectively. It is possible to construct a prefix-free code with alphabets* $x_1, x_2, ..., x_M$

such that

$$\bar{n} \leq H(S)$$

$$\overline{\log M} + 1$$

where H(S) is entropy of sample space.

proof: Let n_i = *length* of the code word for a_i,

$i = 1$ to kwe define $n_i = [-log_M\, p_i]$ $i = 1$ to k

where $[\alpha]$ = Smallest integer greater than or equal to α.

Hence $n_i \geq -log_M\, p_i$

that is $M^{-n_i} \leq p_i$ property of log

hence $\sum M^{-n_i} \leq \sum p_i = 1$ hence the Crafts inequality is satisfied. So it is possible to construct a prefix-free code with code length n_i.

$$n_i < -log_M\, p_i + 1$$

$$n_i p_i < -p_i log_M\, p_i + p_i$$

$$\sum n_i p_i < -\sum p_i log_M\, p_i + 1$$

$$H(S)$$

$$\bar{n} < + 1 logM$$

Unique Decodability and Prefix Condition

A uniquely decodable code in which no code word is prefix of any other code word, is called a prefix-free code. The source has alphabets $S = a_1, a_2, ..., a_k$ with proba- bilities $\{p_1, p_2, ...p_k\}$ respectively. The encoder is with alphabet $X = x_1, x_2, ..., x_M$. The encoder encodes the sequence of source letters by assigning a predetermined se- quence of x_is to each source letter. Let the code words for source letters a_i, $i = 1 to k$ be $a_i = x_{i1}, x_{i2}....x_{in_i}$ wherex_{ij_i} s one of the encoder alphabets x_i and n_i is the length of the code word for a_i. The average length of above code is given by $\bar{n}$.

Optimal Binary Coding

We have two methods to find minimum average length $\bar{n}$ that is

1. Hoffman's method and
2. The Shannon Fano Method. These methods are based on given theorem:

Theorem 1.3.4. *Let us assume that $p_1 \geq p_2 \geq ... \geq p_k$ and word lengths of optimal prefix free code be $n_1, n_2, ..., n_k$ respectively. Thus optimal $\bar{n}$ is $\sum n_i p_i$ then this code can be so arranged that $n_1 \leq n_2... \leq n_k$ and remains optimal.[1]*

Theorem 1.3.5. *For a prefix free code satisfying $p_1 \geq p_2 \geq ... \geq p_k$ and $n_1 \leq n_2 \leq$*

$... \leq n_k$ the following holds:

(a) $n_k = n_{k-1}$

(b) *The code word $X_{k-1}$and X_k for a_{k-1} and a_k differ only in last digit.[1]*

Example: Consider the source S with 6 alphabets $a_1, ...a_6$ having probabilities respectively 0.45, 0.25, 0.10, 0.05, 0.05, 0.04. Find the optimal binary codes by Hoff- man's method and Shannon Fano method.

By Hoffman's method:-

Assign 1 and 0 to last two entries in each column. This is shown as 1| and 0|

alphabets	1	2	3	4	5
*a*1	0.45	0.45	0.45	0.45	1|0.49
*a*2	0.25	0.25	0.25	0.25	0|0.45
*a*3	0.10	0.10	1|0.14	10|0.24	
*a*4	0.05	1|0.09	0|0.10		
*a*5	1|0.05	0|0.05			
*a*6	0|0.04				

We take according coding as

$$x_{K-1} = x^{J}_{K-1}\,1$$

$$x_{K} = x^{J}_{K-1}\quad 0$$

alphabets	1	2	3	4	5	optimal coding
*a*1	0|0.45	0|0.45	0|0.45	0|0.45	1|0.49	0
*a*2	11|0.25	11|0.25	11|0.25	11|0.25	0|0.45	11
*a*3	100|0.10	100|0.10	101|0.14	10|0.24		100
*a*4	1010|0.05	1011|0.09	100|0.10			1010
*a*5	10111|0.05	1010|0.05				1011
*a*6	10110|0.04					10110

so optimum code length is

$$\overline{n} = 1 \times 0.45 + 2 \times 0.25 + 3 \times 0.10 + 4 \times 0.05 + 5 \times 0.05 + 5 \times 0.04 = 1.90$$

Shannon Fano method:- devide the set of alphabets into two disjoint subsets of nearly equal probabilities. This gives the branching into two nodes. Attach digits 0 and 1 to these nodes.

so optimum code length is

$$\overline{n} = 1 \times 0.45 + 2 \times 0.25 + 3 \times 0.10 + 4 \times 0.05 + 5 \times 0.05 + 5 \times 0.04 = 1.90$$

Theorem 1.3.6. *Let (X,Y) be a two dimensional random variable. Then* $H(X|Y) \leq$

$H(X)$ *equality holds iff X and Y are independent random variables.[1]*

Theorem 1.3.7. *Let* $Z = (X, Y)$ *be a two dimensional random variable with joint pdf* $p(x_i, y_i)$. *Then*

$$H(Z) = H(Y|X) + H(X) = H(X|Y) + H(Y)$$

Some Definitions

Average Mutual Information

Let (X,Y) be a two dimensional discrete random variable, then the average mutual information I(X,Y) is defined by[1]

$$I(X, Y) = H(X) - H(X|Y)$$

Discrete Memoryless Channel (DMC)

A discrete channel is called memoryless if each letter in the output sequence is statistically dependent only on the letter in the corresponding position in the input sequence. A conditional probability distribution $q(y_j|x_k)$ is called channel matrix.[1]

Symmetric Channel

A DMC is called symmetric iff all the probabilities of a channel matrix in every row is same set of elements and also in every column the probabilities is the same set ofelements.[1]

Useless Channel

[1] A channel is called useless if and only if X and Y are independent. That is

$$H(X|Y) = H(X)$$

Capacity of a DMC

The maximum value of I(X,Y) is defined as channel capacity C. Thus $C = max I(X, Y)$.[1]Capacity of a useless channel is zero.

Forecasting

Forecasting is an essential tool for successful inventory decision. It is also essential in several decision making situations. The statistical tools used for forecasting is time series and regression analysis.

Time series summarizes the changes in random variable of interest as a function of time. Regression analysis is highly useful technique for developing a quantitative relationship between a dependent variable and one or more independent variables.

Time Series

Let Y_t be the random variable observed at time t then $Y_t = A + e_t$

where A is the constant level and e_t is the random variable such that $E(e_t) = 0$ and

$V(e_t) = \sigma^2$

where $E(e_t)$ is the expected value of e_t and $V(e_t)$ is the variance of e_t.We have three forecasting procedure:

1. Average Forecasting Procedure: The values of Y_t at time $t = 1, 2, \ldots n$ is known.

   Then the Forecast F_{n+1} of Y_{n+1} is defined as

$$F_{n+1} = \sum_{i=1}^{n} y_i$$

2. Moving average forecasting procedure: In this procedure we do not use veryold data. It needs data for last m periods.
   The Forecast for the period $n + 1$ is

$$F_{n+1} = \sum_{i=n-m+1}^{n} \frac{y_i}{m}$$

   where $m < n$.

3. Exponential smoothing procedure: In this method we give the largest weight to most recent data, next recent data less weight and so on. The process is given by:

$$F_{n+1} = \alpha y_n + \alpha(1-\alpha)y_{n-1} + \alpha(1-\alpha)^2 y_{n-2} + \quad , 0 < \alpha < 1$$

so

$$F_{n+1} = \alpha y_n + (1-\alpha)F_n, 0 < \alpha < 1$$

Regression Analysis

Regression analysis is highly useful technique for developing a quantitative relation- ship between a dependent variable and one or more independent variables.

Simple Linear Regression

The sample is

$$X : x_1, x_2 ..., x_n$$

$$Y : y_1, y_2 ..., y_n$$

where X denotes the independent variable and Y the dependent variable. The value of x_i is y_i , $i = 1$ to n. We will fit a line to the given data. So we find the best fits to the available data is the line

$$L : E(Y) = \alpha + \beta x$$

The constants α and β are determined so that the line L is best fit to the sample data. We have to find the measure of best fit. We have $Y = \alpha + \beta x + \varepsilon$ where α and β are constants. ε is a random variable such that the expected value $E(\varepsilon)$ is zero and its variance is σ^2.

$(Y - E(Y))$ represents the error. So we take the measure of total error to be

$$(Y_1 - E(Y_1)) + (Y_2 - E(Y_2)) + \quad + (Y_n - E(Y_n))$$

Least Square Method

The given sample

$$x : x_1, x_2..., x_n$$

$$Y : Y_1, Y_2..., Y_n$$

The constants α and β are determined so that the sum of squares of errors SSEdefined by

$$SSE = (Y_1 - E(Y_1))^2 + (Y_2 - E(Y_2))^2 + \quad + (Y_n - E(Y_n))^2$$

is minimized. Substituting the value of $E(Y) = \alpha + \beta x$, we get

$$SSE = \sum_{1}^{n} \{y_i - \alpha - \beta x_i\}^2$$

The random sample is $(y_1, x_1, z_1), (y_2, x_2, z_2), ...(y_n, x_n, z_n)$. We fit the following lin-ear equation to above sample

$$E(Y) = \alpha + \beta x + \gamma z$$

The above is called multiple linear regression of Y on x and z. The constants α, β and γ are determined so that $E(Y)$ is best fit to the sample data.

Nonlinear Regression

The relation $E(Y) = \alpha e^{\beta x}$ is called nonlinear regression.

The constants α, β and γ are determined so that $E(Y)$ is best fit to the sample data.

Decision Analysis

In decision analysis, the decision is to be made under uncertainty. Here at each stage, instead of deterministic payoff, one has expected payoff. The Bayes' decision rule which takes the optimum of expected payoff's at each stage of decision making under uncertainty is used. Many times the decision is made with some experimentation and some times without.

Bayes' Rule

Suppose that $A_1, A_2, \quad A_n$. are mutually exclusive events whose union is the sample space S. Then if A is any event, we have the following theorem:

Example

An automobile company has advertised for the appointment of trainee engineers in different disciplines. First, the applications received are thoroughly scrutinized and good ones are called to the main centre of the company for two written tests followed by a viva. The written exams are of 2hr duration each . The first exam E_1 tests the general scientific background and the second exam E_2 tests the basics of the disciplines. Earlier all the condidates were required to appear in both E_1 and E_2 , but only those who clear both were allowed to appear in the viva for final selection. To minimize the cost the company has decided to change the procedure. The two exams can be conducted in order E_1E_2 or E_2E_1. A candidate has to pass both the exams and viva for final selection. A candidate is rejected if fails in either E_1 or E_2 and is not allowed to appear in other. Only those who clear both exams are allowed in the viva. The expenses per candidate in E_1, E_2 and viva are respectively Rs.500, Rs. 700 and Rs. 1000. Find in which order the exams E_1 and E_2 should be conducted to minimize the total expense per candidate.

The following data for 100 candidates is available from previous selections, whenboth exams were allowed.

data for 100 candidates		
	PassE_1	Fail E_1
PassE_2	40	20
Fail E_2	30	10

The above is a minimization problem, so in Bayes' decision rule the minimummust be taken.

The solutions shown in the figure given below: sheetal/probwe8.png sheetal/probwe8.pdf

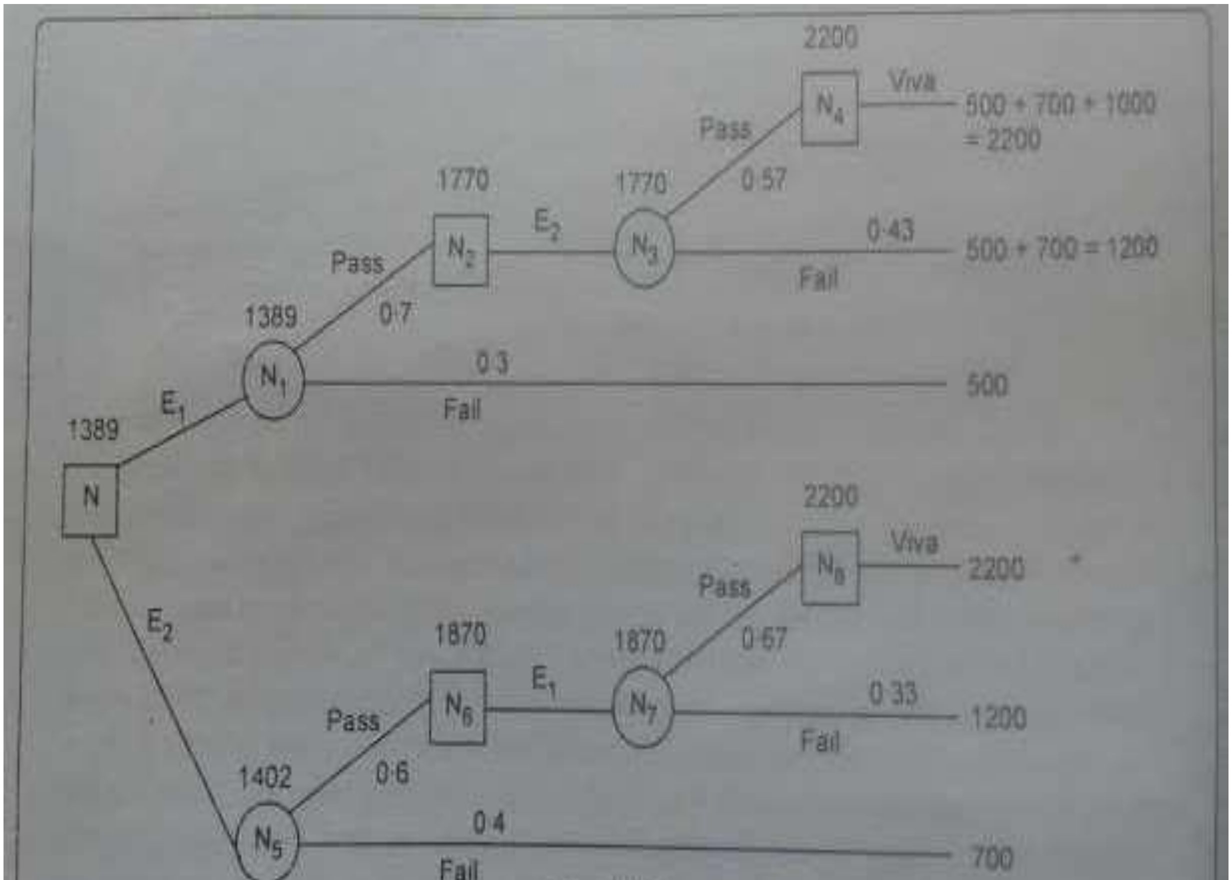

Fig. 17.1.3

Observe that the expense occurs whether a candidate pass or fail.

The expected payoff at node N_3 is 2200 × 0.57 + 1200 × 0.43 = 1770

The minimum expected expense is Rs. 1389. This occurs if examination E_1 is conducted first.

Conclusion

In this paper work, we studied Information theory, Forecasting and Decision analysis.

Information Theory has found applications in many other areas, including statistical inference, natural language processing, cryptography, neurobiology, the evolution and function of molecular codes.

Forecasting is the process of making predictions of the future based on past and present data and most commonly by analysis of trends. Forecasting foreign exchange movements is typically achieved through a combination of chart and fundamental analysis.

From decision analysis we can take decision and find maximum benifit.

It would be challenging to find any new result which will provide interesting problems for future research, which is one of the objective of this report

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RPGT: The Technique For Behavioural Analysis Of Industries

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Abstract

*This paper discusses the technique known as Regenerative Point Graphical Technique **(RPGT)**, which is used for Behavioral Analysis of various industries having Single Unit and multiple Unit connected in series and parallel configuration.The analysis of various Industrial systems using the 'Regenerative Point Graphical Technique**(RPGT)**' is for determining the parameters such as the Mean Time to System Failure(MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement etc. (under steady state conditions). The complete process is Mathematically formulated using the system has a single unit with priority repair and various parameters such as the Mean Time to System Failure (MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement are evaluated for further discussing the profitability of industry under discussion.*

Key words: Reliability, Availability, Priority Maintainance, Primary Circuit, Secondary Circuit, Tertiary Circuit, Base-State, Regenerative Point Graphical Technique **(RPGT)**.

Introduction The researchers have discussed the reliability and availability of many stochastic systems and process industries by using very cumbersome and time-consuming techniques. The general formulae in the closed form are not yet developed to determine the key parameters for a semi Markov renewal process like mean time to system failure (MTSF), availability of the system and busy periods of the servers doing different jobs, the number of server's visits and the number of replacements of the components/sub-systems. Tuteja, R.K.[6-7], Malik, S.C.[1-5] and many others, have analyzed and discussed various systems under steady state conditions, using the following formulae of the regenerative point technique to find the key parameters of a stochastic system:

e) MTSF $= \lim_{s \to 0} \frac{1-\phi_0^{\sim}(s)}{s}$

f) Availability $= \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s.A_0^*(s)$

g) Busy period of the server $= \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} s.B_0^*(s)$

h) Expected number of server's visits/replacements $= \lim_{t \to \infty} \frac{V_0(t)}{t} = \lim_{s \to 0} s.V_0^*(s)$

Regenerative Points The epochs of time $\{t_n\}$ where a given renewal process {N(t), t≥0} probabilistically starts afresh, forgetting (independent of) past history,are called regenerative points, i.e. {N(t), t≥ t_n} is a probabilistic replica of {N(t), t≥0}. The epochs of time at which the renewals occur, are called regenerative points and the event which is renewed is called regenerative state of the system.

Many of the problems, under steady state conditions, can be solved without write any state equations. This can be done for a Semi-Markov renewal process which changes its state according to a finite, ir-reducible and aperiodic Markov chain, by using the probabilistic model.For this, the following steps are essential:

i) The components which constitute the system and whose individual reliability can be evaluated are identified. These components are called units or sub-systems of the system.

ii) Complete description of the system is necessary to prepare a mathematical/statistical model of the system.

iii) The logistic manner or configuration, in which units are connected in the simplest way to form the system, is represented by a block/circuit diagram.

iv) The next step is to establish the conditions for the successful operation of the system. It is studied and decided as how the components should function to serve the intended purpose. The necessary assumptions regarding the types of failures and the nature of repair policy are made.

v) The state transition diagram is prepared correctly and exhaustively.

Regenerative points/states and non regenerative states, good states, down states, partially failed/reduced states, totally failed are correctly determined, keeping in view the status of functioning of all the units of the system and these are well marked in the transition diagram. The state are numbered as 0,1,2,…..

vi) The simple paths from the initial state called 0-state (the state at time t = 0) to the other regenerative states along with all the k-cycles which are formed along the different paths are tracked down/noted.

vii) The p.d.f. of first passage times, waiting times and reliability functions are written.

viii) The steady-state probabilities, mean sojourn times, un-conditional times and waiting-times for different states are calculated.

ix) An appropriation probabilistic model (1) to (4) is used to find (under steady state conditions) MTSF, availability, different busy periods, different types of visits and replacements.

x) The values of these vital parameters are then calculated for different values of the important variables of the system. The data so obtained are expressed in the form of tables and graphs.

xi) From the tables and the graphs, behaviour analysis of the system is carried out regarding the life (MTSF), availability and other fundamental key factors of the system. Profit analysis is done by defining the profit function as the function of the key parameters of the system.

Directed Path

A path in a directed graph, is a sequence of edges such that the terminal node of any edge in the sequence is the initial node of the edge, if any, appearing next in the sequence. The path is said to traverse through the nodes(states) appearing in the sequence originating from the initial node(state) of the first edge and ending in the terminal node(state) of the last edge in the sequence of the path. In other words, a directed path is an ordered sequence of states in the state transition diagram of the system.

Primary Circuit, Secondary circuit, Tertiary Circuits At A Vertex

Primary Circuit (CL1)

A simple directed circuit at a given vertex 'k_1' in the transition diagram of a stochastic system is defined as a primary circuit at a given vertex 'k_1'. A primary circuit at the given vertex 'k_1' is called of dimension/level-1 at the given vertex 'k_1'.

Secondary Circuit(CL2)

A simple directed circuit with terminals at an interior vertex 'k_2' of a given primary k_1-circuit is defined as a secondary circuit at a given vertex 'k_1', provided $k_1 \notin k_2$-cycle. A secondary circuit at the vertex 'k_1' may be called a circuit of dimension/level-2 at the vertex'k_1'

Tertiary Circuit(CL3)

A simple directed circuit with terminals at an interior vertex 'k_3' of a given secondary k_2-circuit at the vertex 'k_1' is defined as a tertiary circuit at the given vertex 'k_1', provided k_1, $k_2 \notin k_3$-cycle. A tertiary circuit at the vertex 'k_1' may be called a circuit of dimension/level-3 at the vertex 'k_1'.

A simple directed circuit with terminals at an interior vertex k_n of a k_{n-1}-circuit of dimension 'n-1' at the vertex 'k_1', is defined as a circuit of dimension 'n' at the vertex 'k_1', provided $k_1,k_2,\ldots,k_{n-1} \notin k_n$-cycle where k_i is the terminal vertex of the 'i-th' dimensional circuit at the vertex 'k_1'. There may be none, one or more than one circuits of a given dimension/level at a given vertex, in the transition diagram of the system. If there does not exist a circuit of a given dimension/level at a given vertex, then there exists no circuit of higher dimension at that vertex.

Gupta, Singh and Vanita [8] defined different types of circuits / cycles like primary, secondary and tertiary circuits which are located in the transition diagram of the system and introduced the concept of base state of the system for determining the key parameters of the system using RPGT. Using RPGT Jindal [9] discussed behavior and availability analysis of industrial systems. Gupta and Singh [11] presented a new approach for availability analysis, behavior and profit analysis of process industries. Goel and Singh [10] discussed the availability analysis of the standby complex system having imperfect switch. Nidhi, Goel & Garg [12] discussed Availability Analysis of a Soap Industry Using Regenerative Point Graphical

Technique (RPGT).

Mean Time To System Failure (MTSF)

The mean time to system failure is the statistical average time for which the system operative before any failure(s) of the system. The term MTSF is used when the system undergoes either preventive and corrective maintenance actions. Mean time to system failure (under steady state conditions) of the system is given by $\sum_i V_{0,i}.\mu_i$ where i is an un-failed regenerative state in the state-transition diagram of the system. The mean time to system failure is

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(0\xrightarrow{s_r(sff)}i\right)\right\}.\mu_i}{\prod_{k_1\neq 0}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(o\xrightarrow{s_r(sff)}0\right)\right\}}{\prod_{k_2\neq 0}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right] \quad \ldots\ldots (1)$$

Steady State Availability Of The System

It is defined as the proportion of time that the system is operational when the time-interval is very-very large and the corrective, preventive maintenance down times and the waiting times are included.

$$A_0 = \frac{MTBM}{MTBM + MDT}$$

Where, MTBM = mean time between maintenance; MDT(mean down time) = statistical mean of the down times caused due to breakdowns, including supply down time, administrative down time.

The steady state availability of a system is

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(0\xrightarrow{s_r}j\right)\right\}f_j.\mu_j}{\prod_{k_1\neq 0}\left\{1-V_{k_1,k_1}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(0\xrightarrow{s_r}i\right)\right\}.\mu_i^1}{\prod_{k_2\neq 0}\left\{1-V_{k_2,k_2}\right\}}\right\}\right] \quad \ldots\ldots\ldots(2)$$

Busy Period of The Server

Busy period of the server (under steady state conditions) doing a given job is defined by

$$B_0 = \frac{MTTR}{MTBM + MDT}$$

Where, MTTR = mean time to repair; MTBM = mean time between maintenance; MDT(mean down time) = statistical mean of the down times caused due to breakdowns, including supply down time, administrative down time.(MDT is replaced by M or MTTR as per the real situation to which the stochastic process is subjected during its operation).

The busy period of the server (under steady state conditions) doing a given job is given by

$$B_0 = \left[\sum_j V_{0,j}.\eta_j\right] \div \left[\sum_i V_{0,i}.\mu_i^1\right]$$

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(0\xrightarrow{s_r}j)\}.\eta_j}{\prod_{k_1\neq 0}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(0\xrightarrow{s_r}i)\}.\mu_i^1}{\prod_{k_2\neq 0}\{1-V_{k_2,k_2}\}}\right\}\right] \qquad \text{..........(3)}$$

Number Of Server's Visits/ Number Of Replacements

The expected number of visits of the server/replacements is defined by

$$V_0 = \left[\sum_j V_{0,j}.\delta_j\right] \div \left[\sum_i V_{0,i}.\mu_i^1\right]$$

Where $\delta_j = 1$ if the visit of the server for the given job/replacement is afresh at the regenerative state i, otherwise $\delta_j = 0$.

The expected number of visits of the server/replacements is given by:

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(0\xrightarrow{s_r}j)\}}{\prod_{k_1\neq 0}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(0\xrightarrow{s_r}i)\}.\mu_i^1}{\prod_{k_2\neq 0}\{1-V_{k_2,k_2}\}}\right\}\right] \qquad \text{............(4)}$$

Base-State of the System

The calculation work can be economically minimized and simplified by locating a regenerative state (preferably an un-failed regenerative state) in the transition diagram which is associated with a largest number of primary circuits and the least/minimum number of secondary, tertiary circuits and circuits of higher dimensions at that regenerative state. Such a state is defined as a '**base-state'** of the system.

Regenerative Point Graphical Technique(Using Base-State 'ξ')

Mean Time To System Failure: The mean time to system's failure being a positional measure, therefore, it depends upon the initial state of the system from which it is measured. MTSF is measured w.r.t. the un-failed initial state 'ξ'(at t=0).

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} i\right)\right\}.\mu_i}{\prod_{k_1\neq\xi}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} \xi\right)\right\}}{\prod_{k_2\neq\xi}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right] \quad \ldots (5)$$

Total fraction of time for which the system is available

$$A_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\} f_j.\mu_j}{\prod_{k_1\neq\xi}\left\{1-V_{k_1,k_1}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}.\mu_i^1}{\prod_{k_2\neq\xi}\left\{1-V_{k_2,k_2}\right\}}\right\}\right] \quad \ldots (6)$$

The busy period of the Server doing any given job

$$B_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\}.\eta_j}{\prod_{k_1\neq\xi}\left\{1-V_{k_1,k_1}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}.\mu_i^1}{\prod_{k_2\neq\xi}\left\{1-V_{k_2,k_2}\right\}}\right\}\right] \quad \ldots (7)$$

The number of the Server's visits/Replacements

$$V_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\}}{\prod_{k_1\neq\xi}\left\{1-V_{k_1,k_1}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}.\mu_i^1}{\prod_{k_2\neq\xi}\left\{1-V_{k_2,k_2}\right\}}\right\}\right] \quad \ldots (8)$$

Here, We take an example of the Dairy Plant, which is divided into two unit 'A' and 'B' in which unit 'A' is milk producing unit and other subsidiary products like milk water, Cream, Ghee etc are produced by unit 'B'. As milk is the main demand of market so unit 'A' is the main unit of system, so we try to keep unit 'A' in working as far as possible, therefore we give priority in repair to unit 'A' over unit 'B'. The system is in working state if at least one unit is in working state and fails when both units fail. Nothing can fail when the system is in failed state. When both units are working the system is good otherwise it may be working in reduced state or failed state. If any unit of the system fails the system works in reduced capacity and the failed unit is immediately put under repair. Repairs are perfect i.e. repair does not damage to any part of the system. The repaired unit works like a new one. Further if both 'A' and 'B' units of system are in failed system, then repair to unit 'A' is given priority in repair over 'B'. The distributions of the failure times and repair times are exponential and general respectively and also different for units 'A' and 'B'. These are also assumed to be independent of each other.

A transition diagram of the systems is drawn to find the primary circuit, secondary circuit, tertiary circuit and 'base state'. The system is discussed for steady-state conditions, Using Regenerative Point Graphical Technique various parameters of the system are evaluated. Expression for profit function of the system is also given. Some special cases are discussed to depict the behavior of the system. Some particular cases, tables and graphs followed by discussion are given.

Assumptions and Notations

The system consists of two non – identical units 'A' and 'B'. 'A' is main unit and 'B' unit is subsidiary.

- A single repair facility is available for both units 'A' and 'B'.
- The distributions of failure times and repair times are exponential and general respectively and also different for units 'A' and 'B'. Failures and repairs are statistically independent.
- Repair is perfect i.e. it does not damage any units during repair.
- Repaired units to be good.
- When both units fail then the system is in failed state.
- Nothing can fail when the system is in failed state. After the failure of any one unit the system works in reduced state.
- When both units 'A' and 'B' are in failed state then repairman repair unit 'A' on priority basis.
- The system is discussed for steady-state conditions.
- Upon failure, if main unit 'A' is under repair and unit 'B' also fails it joins the queue of the failed unit.
- Both the units cannot fail simultaneously.

λ / λ_1 : Constant failure rate of units A/ of unit B.

g(t)/G(t) / $\bar{G}$(t) : Probability density function/ Cumulative distribution function / complement of the repair – time of unit A.

h(t)/H(t) / $\bar{H}$(t) : Probability density function / Cumulative distribution function / complement of the repair time of the unit B.

A/a : Main unit in the operative state / failed state.

B/b : Operative state / failed state.

Transition Diagram

Considering the above assumptions and notations the Diagram of the system is

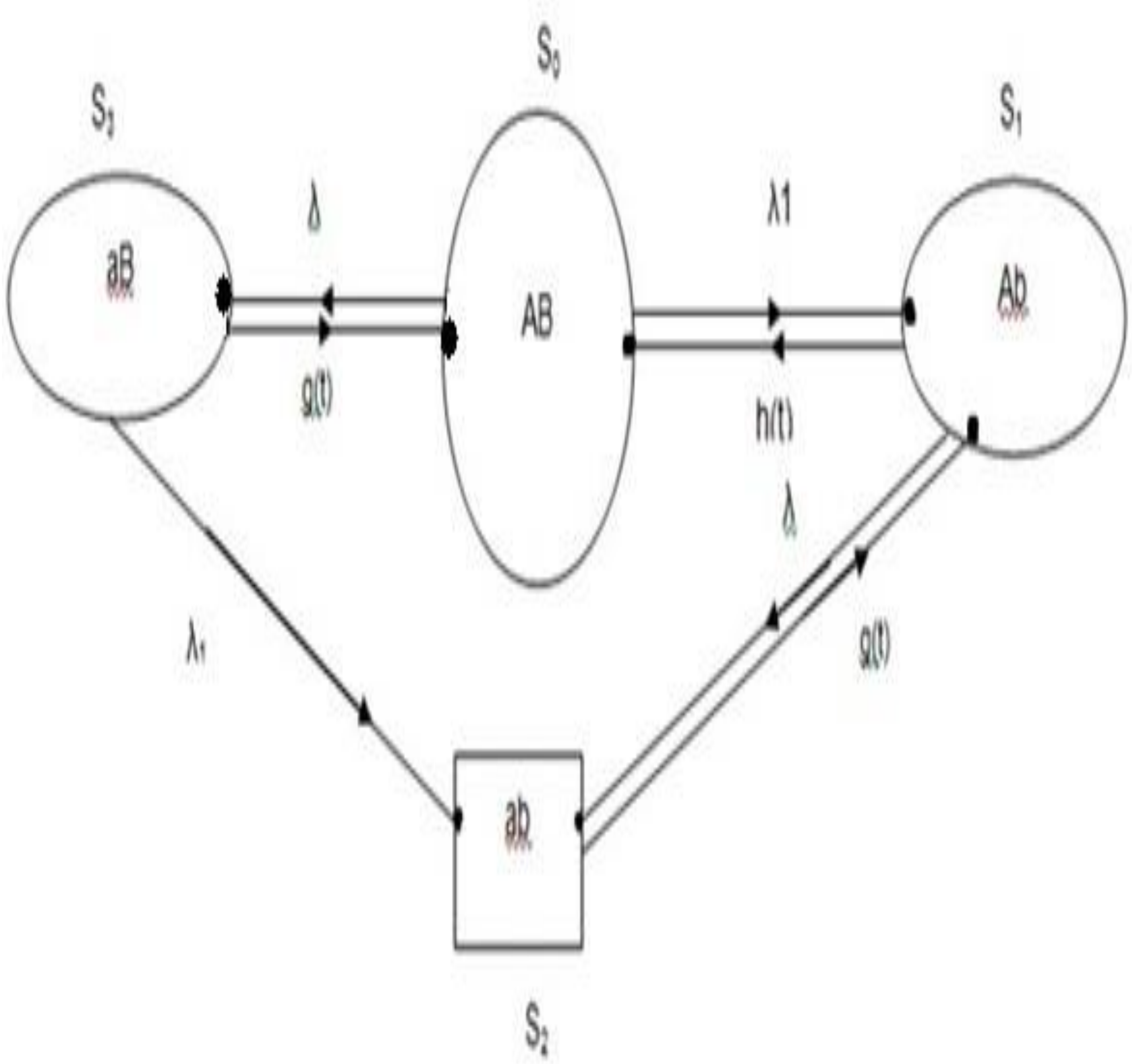

Figure – 1

State	Symbol	Model
Regenerative State/Point		0-3
Up-state		0
Failed State		2
Reduced State		1,3

Table 1

The states of the system with respect to the above figure.

S_0 = AB S_1 = Ab

S_2 = ab S_3 = aB

States S_0, S_1, S_2, and S_3 are regenerative states.

The possible transitions between states along with transition time c. d. f's are shown in Fig. 1.

Possible simple paths from state 'i' to reachable state 'j' are given in table 2.

Simple Paths from State ' i ' to the Reachable State ' j ': P0

Vertex	j = 0	j = 1	j = 2	j = 3
0	(0,1,0),(0,3,0),(0,3,2,1,0)	(0,1),(0,3,2,1)	(0,1,2),(0,3,2)	(0,3)
1	(1,0)	(1,0,1),(1,2,1),(1,0,3,2,	(1,2),(1,0,3,2)	(1,0,3)
2	(2,1,0)	(2,1)	(2,1,2),(2,1,0,3,	(2,1,0,3)
3	(3,0),(3,2,1,0)	(3,0,1),(3,2,1)	(3,2),(3,0,1,2)	(3,0,3),(3,2,1,0,3)

Table 2

Possible Primary, Secondary, Tertiary circuits at a vertex 'i' are given in table 3.

Primary, Secondary, Tertiary Circuits at a Vertex

Vertex i	Primary (CL1)	Secondary (CL2)	Tertiary (CL3)
0	(0,1,0) (0,3,0) (0,3,2,1,0)	(1,2,1) Nil (2,1,2)	Nil Nil Nil
1	(1,0,1), (1,2,1), (1,0,3,2,1)	(0,3,0) Nil (0,3,0)	Nil Nil Nil
2	(2,1,2), (2,1,0,3,2)	(1,0,1) (1,0,1) (0,3,0)	(0,3,0) (0,3,0) Nil
3	(3,2,1,0,3),	(2,1,2)	(1,0,1)

	(3,0,3)	(1,0,1) (0,1,0)	Nil (1,2,1)

Table 3

Analysis table of Primary, Secondary & Tertiary circuits to determine the base state is given in table 4

Analysis table of Primary, Secondary and Tertiary Circuits to find base state.

Vertex i	**No. of Primary Circuits**	**No. of Secondary Circuits**	**No. of Tertiary Circuits**
0	3	2	0
1	3	2	0
2	2	3	2
3	2	3	2

Table 4

From above table we conclude that we can take ‘0’ or ‘1’ as base state. We take ‘0’ as base state.

Primary, Secondary and Tertiary circuits w.r. to simple path (‘0’ as base state) are given in table 5

Vertex j	**0 $\xrightarrow{S_R}$ j):P0**	**P1**	**P2**	**P3**
1	0 $\xrightarrow{S_1}$ 1):(0,1) (0 $\xrightarrow{S_2}$ 1):	(1,2,1) (2,1,2)	Nil	Nil
2	0 $\xrightarrow{S_1}$ 2):(0,1,2) (0 $\xrightarrow{S_2}$ 2) : (0,3,2)	(1,2,1) (2,1,2)	Nil	Nil
3	0 $\xrightarrow{S_1}$ 3): (0,3)	Nil	Nil	Nil

Table 5

Transition Probability and the Mean sojourn times.

$q_{m,n}(t)$**:** The probability density function (p.d.f.) of the first passage time from a regenerative state 'm' to a regenerative state 'n' or to a failed state 'n' without visiting any other regenerative state in time interval (0,t].

$p_{m,n}$**:** The steady state transition probability from a regenerative state 'm' to a regenerative state 'n' without visiting any other regenerative state. $p_{m,n} = q^*_{m,n}(0)$; where * denotes Laplace transformation.

$q_{m,n}(t)$	$p_{m,n}=q_{m,n}^*(0)$
$q_{0,1(t)}= \lambda_1 e^{-(\lambda+\lambda 1)t}$ $q_{0,3(t)}= \lambda e^{-(\lambda+\lambda 1)t}$	$p_{0,1}= \lambda_1/\lambda+\lambda_1$ $p_{0,3}= \lambda/\lambda+\lambda_1$
$q_{1,0(t)}=h(t)\ e^{-(\lambda)t}$ $q_{1,2(t)}= \lambda e^{-(\lambda)t}\ (\bar{H}(t))$	$p_{1,0}=h^*(\lambda)$ $p_{1,2}= 1-h^*(\lambda)$
$q_{2,1}(t)= g(t)$	$p_{2,1}=g^*(0)$
$q_{3,0}(t)= g(t)\ e^{-(\lambda 1)t}$ $q_{3,2}(t)= \lambda_1 e^{-(\lambda 1)t}\bar{G}(t)$	$p_{3,0}=g_3^*(\lambda_1)$ $p_{3,2}=1-g^*(\lambda_1)$

Table 6

$p_{0,1} + p_{0,3} = \lambda_1/ \lambda+ \lambda_1 + \lambda/ \lambda+ \lambda_1 = 1$ $\quad$ $p_{1,0} + p_{1,2} = h^*(\lambda)+1-h^*(\lambda) = 1$

$p_{2,1} = 1;\ p_{3,0} + p_{3,2} = 1$

Hence it is verified that for each state total state probability is 1.

Mean Sojourn Times

$R_j(t)$: The reliability of the system at time t, given that the system is in regenerative state 'j'.

μ_j: The mean sojourn time spent in state 'j', before visiting any other states;

$R_j(t)$	$\mu_j=R_j^*(0)$
$R_0(t) = e^{-(\lambda+\lambda 1)t}$	$\mu_0=1/\lambda_1+ \lambda$
$R_1(t) = e^{-\lambda t}\ (\bar{H}(t))$	$\mu_1=1-h^*(\lambda)/ \lambda$
$R_2(t) = \bar{G}(t)$	$\mu_2= - g^{*\prime}(o)$
$R_3(t) = e^{-\lambda_1 t}\ (\bar{G}(t))$	$\mu_3= 1 - g^*(\lambda_1)/ \lambda_1$

Table 7

Evaluation of Parameters

The MTSF and all the key statistical parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using ‘0’ as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state ‘0’ are:

$V_{0,0}$ = (0,1,0)/ [1-(1,2,1)] + (0,3,0) + (0,3,2,1,0) / [1-(2,1,2)] = $p_{0,1}\ p_{1,0} / (1 – p_{1,2}\ p_{2,1}) + p_{0,3}\ p_{3,0} + p_{0,3}\ p_{3,2}\ p_{2,1}\ p_{1,0} / (1 – p_{2,1}\ p_{1,2})$

$= p_{0,1}\ p_{1,0} / (1 – p_{1,2}) + p_{0,3}\ p_{3,0} + p_{0,3}\ p_{3,2}\ p_{1,0} / (1 – p_{1,2})$

$V_{0,1}$ = (0,1)/ [1-(1,2,1)] + (0,3,2,1) / [1- (2,1,2)] = $p_{0,1} / (1 – p_{1,2}\ p_{2,1}) + p_{0,3}\ p_{3,2}\ p_{2,1} / (1 – p_{2,1}\ p_{1,2})$

$= p_{0,1} + p_{0,3}\ p_{3,2}\ p_{2,1} / (1-p_{1,2}\ p_{2,1}) = p_{0,1} – p_{0,1}\ p_{0,3}\ p_{3,2} / (1 – p_{1,2})$

$= (1 – \lambda / \lambda + \lambda_1 (g^*(\lambda_1)\}h^*(\lambda)$

$V_{0,2}$ = (0,1,2)/ [(1-(1,2,1)] + (0,3,2) / [(1-(2,1,2)] = $p_{0,1}\ p_{1,2} / (1 – p_{1,2}\ p_{2,1} + p_{0,3}\ p_{3,2}) / (1 – p_{2,1}\ p_{1,2})$

$= p_{0,1}\ p_{1,2} / (1-p_{1,2}) + p_{0,3}\ p_{3,2} / (1 – p_{1,2}) = \lambda_1/ \lambda+ \lambda_1 (1 – \lambda+ \lambda_1 + \lambda/ \lambda + \lambda_1 [(1 – g^* (\lambda_1)]$

$= p_{0,1}\ p_{1,2} + p_{0,3}\ p_{3,2} / (1-p_{1,2}) = \lambda_1/ \lambda+ \lambda_1 - \lambda_1/ \lambda+ \lambda_1 h^* (\lambda) + \lambda/ \lambda+ \lambda_1 – \lambda/ \lambda+ \lambda_1 g^* (\lambda_1)$

$= \{1 - \lambda_1 / \lambda+ \lambda_1 h^*(\lambda) – \lambda/ \lambda+ \lambda_1 g^* (\lambda)\} / h^* (\lambda)$

$V_{0,3}$ = (0,3) = $p_{0,3} = \lambda/ \lambda+ \lambda_1$

MTSF(T_0):

From Fig.1, the regenerative un-failed states to which the system can transit(initial state ‘0’), before entering any failed state are: i = 0,1,3 taking ‘ξ’ = ‘0’,

$$\mathrm{MTSF}(T_0) = \left[\sum_{i,s_r} \left\{ \frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} i\right)\right\} \mu_i}{\prod_{m_1 \neq \xi}\{1 - V_{\overline{m_1,m_1}}\}} \right\}\right] \div \left[1 - \sum_{s_r} \left\{ \frac{\left\{pr\left(\xi \xrightarrow{s_r(sff)} \xi\right)\right\}}{\prod_{m_2 \neq \xi}\{1 - V_{\overline{m_2,m_2}}\}} \right\}\right] \text{--------} \quad (5)$$

T_0 = [(0,0) μ_0 +(0,1) μ_1 + (0,3) μ_3] /[1-(0,1,0) + (0,3,0)]

Availability of the System (A_0):

From figure 1 the regenerative states at which the system is available are j = 0,1,3, and regenerative states are i = 0-3 taking ‘ξ’ = ‘0’,

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\} f_j . \mu_j}{\prod_{m_1 \neq \xi}\{1 - V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\} . \mu_i^1}{\prod_{m_2 \neq \xi}\{1 - V_{m_2,m_2}\}}\right\}\right] \quad \text{-------- (6)}$$

$$A_0 = \left[\sum_j V_{\xi,j} . f_j . \mu_j\right] \div \left[\sum_i V_{\xi,i} . \mu_i^1\right]$$

$A_0 = V_{0,0}\ f_0\ \mu_0 + V_{0,1}\ f_1\ \mu_1 + V_{0,3}\ f_3\ \mu_3\ / V_{0,0}\ \mu_0 + V_{0,1}\ \mu_1 + V_{0,2}\ \mu_2 + V_{0,3}\ \mu_3$

(where $f_j = 1$, $\mu_j^1 = \mu_j$ for all j)

$A_0 = f_0\ \mu_0 + V_{0,1}\ f_1\ \mu_1 + p_{0,3}\ f_3\ \mu_3\ / \mu_0 + V_{0,1}\ \mu_1 + V_{0,2}\ \mu_2 + p_{0,3}\ \mu_3$

Busy Period of the Server:

From Fig 1, the regenerative states where Server is busy while doing repairs are: j = 1, 2, 3 and the regenerative states are: i = 0, 1, 2, 3 taking 'ξ' = '0'

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\} . \eta_j}{\prod_{m_1 \neq \xi}\{1 - V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\} . \mu_i^1}{\prod_{m_2 \neq \xi}\{1 - V_{m_2,m_2}\}}\right\}\right] \quad \text{-------- (7)}$$

$$B_0 = \left[\sum_j V_{\xi,j} . \eta_j\right] \div \left[\sum_i V_{\xi,i} . \mu_i^1\right]$$

$= V_{0,1}\eta_1 + V_{0,2}\eta_2 + V_{0,3}\eta_3\ /\ V_{0,0}\ \mu_0^{\ 1} + V_{0,1}\ \mu_1^1 + V_{0,2}\ \mu_2^1 + V_{0,3}\ \mu_3^1$

$B_0 = V_{0,1}\eta_1 + V_{0,2}\ \eta_2 + p_{0,3}\eta_3\ /\ \mu_0 + V_{0,1}\ \mu_1 + V_{0,2}\ \mu_2 + V_{0,3}\ \mu_3$

(where $\eta_j = \mu_j$, $\mu_j^1 = \mu_j$ for all j)

Expected Number of Server's Visits:

From Fig. 1, the Regenerative States where the Server visits (afresh) for repairs of the system are: j=3 and the Regenerative States are: i = 0,1,2,3 taking 'ξ' = '0', the Expected number of Server Visits per unit time is given by

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{m_1 \neq \xi}\{1 - V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\} . \mu_i^1}{\prod_{m_2 \neq \xi}\{1 - V_{m_2,m_2}\}}\right\}\right] \quad \text{-------- (8)}$$

$V_0 = \sum_j V\xi,j \div \sum_i V\xi,i.\mu_i^1$ **(where $\mu_j^1 = \mu_j$ for all j)**

$= V_{0,3}\ /\ V_{0,0}\ \mu^1{}_0 + V_{0,1}\ \mu^1{}_1 + V_{0,2}\ \mu^1{}_2 + V_{0,3}\ \mu^1{}_3$

Profit Function:

The profit analysis of the system can be done by using the profit function:

The profit P per unit time is as $P = K_1\ A_0 - K_2\ B_0 - K_3\ V_0$

Where K_1 = Revenue per unit of time the system is available.

K_2 = Cost per unit time the server remains busy for the repairs.

K_3 = Cost per visit of the server.

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'Z' Test: Analytical Tool used in Nutritional Status of Pre-Schoolers (3-5 years) of Sirsa (Haryana) and Nagpur (Maharashtra) Districts

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Abstract

The present study was conducted on pre-school children (3 to 5 years) of Sirsa district (Haryana) and Nagpur district(Maharashtra) to study their nutritional status. Most of the respondents in the present study, suffered from under nourishment due to less intake of various food stuff i.e. cereals, pulses, green leafy vegetables, fruits, milk & milk products, fats & oils and sugar & jaggery etc. in both the States. But boys of the 3 to 4 years age group were consuming adequate amount of roots & tubers in Haryana. The girls of 4 to 5 years age group were consuming better amount of other vegetables than the boys in Haryana. Non-significant differences ($P<0.05$) occurred between boys-boys and girls-girls of both the states as compared to their RDI's in the intake of cereals, pulses, roots & tubers, other vegetables, milk & milk products and fats & oils. While significant differences ($P<0.01$) were witnessed amongst boys-boys and girls-girls intake of green leafy vegetables when compared with RDI in both the age groups in Haryana and Maharashtra. Fruit intake, when compared with its RDI i.e. 100 g/day showed significant differences between boys-boys (3 to 4 years) of both the states. On the other Hand, girls-girls (4 to 5 years) of both the States had significant differences ($P<0.05$) when compared to its RDI.

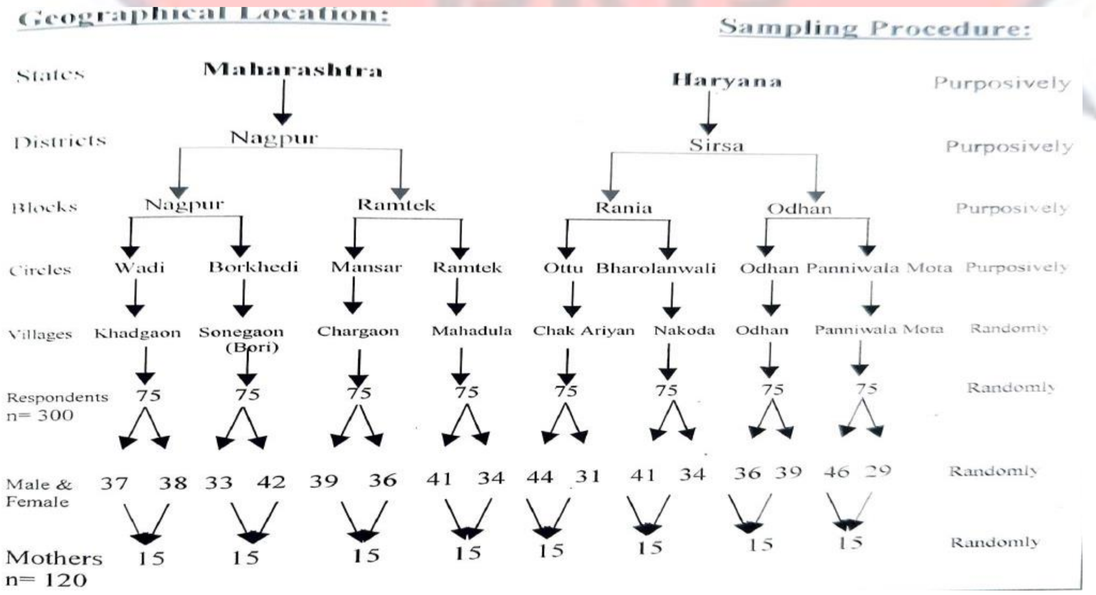
Introduction

Nutritional status is an evaluation or assessment of how well the needs of the body for essential nutrients are being met, that's why the evaluation of nutritional status of pre-school children is important for ensuring improved physical and mental development and optimum functional performance. Pre-school children constitute about 15 per cent of our total population and account for 40 per cent of total deaths in India are notoriously fraught with the risk of protein mal-nutrition. Over the past three decades, a slow but definite decline was observed in the underfive and infant mortality rates due to improvement in the socio-economic status of the general population. Nutritional status of any population is considered as vital determining factor for its bio-social development including both physical and mental wellbeing.

Available evidences indicate that severe mal-nutrition experienced during critical phase of life (usually in infancy and early childhood) affects not only expression of genetic potential for physical growth and development and personality. It is recognised that rapid physical, mental and emotional growth of children takes place in early childhood. To obtain optimum nutritional status, proper nutrition is of great importance.

In view of national growth and development, it becomes essential to maintain physical power of pre-school children at an optional level. For this purpose, monitoring the health and growth rate of pre-school children is one of the most simple, reliable and important parameter. Although few studies for estimating the nutritional status in pre-schoolers have been conducted in some states of India but no systematic comparative study seems to have been carried out in different districts of Haryana and Maharashtra to estimate the nutritional status and to correlate this prevalence with dietary and other factors. Keeping all these perspectives into consider action, the present study was conducted.

METHODOLOGY



Results & Discussion

Table 1: Mean daily food intake of 3 to 4 years old pre-school children (Boys and Girls)

BOYS							
Sr. No.	Foodstuffs	Daily mean food intake (g)					'Z' Value Boys-Boys
		RDI (g)	3-4 years		% RDI		
			Haryana (n=144)	Maharashtra (n=120)	Boys Hry	Boys Mah	
i)	Cereals	120	109.90±32.1	112.18±36.7	91.58	93.48	0.1253^{NS}
ii)	Pulses	30	17.70±10.5	18.90±19.7	59.00	63.00	0.8062^{NS}
iii)	Roots & tubers	50	52.23± 34.2	45.58± 14.8	104.46	97.16	0.0125^{NS}
iv)	Other Vegetables	50	25.54±46.1	16.28±34.2	51.08	32.56	0.2721^{NS}
v)	Green leafy vegetables	50	3.87±5.4	2.41±7.5	7.74	4.82	18.1016**
vi)	Fruits	100	14.10±14.6	15.41±23.6	14.10	15.41	3.7475**
vii)	Milk & Milk Products	500	365.00±140.7	110.31±119.8	73.00	22.06	0.1220^{NS}
viii)	Fats & Oils	20	16.44 ± 9.6	12.56 ± 6.5	82.20	62.80	1.1816^{NS}
ix)	Sugar & Juggery	25	21.31 ± 7.0	20.30 ± 7.3	85.24	81.20	1.3166^{NS}
GIRLS							
			(N=27)	(N=25)	Girls Hry	Girls Mah	Girls-Girls
i)	Cereals	120	106.16±36.7	109.87±36.1	88.47	91.56	0.0485^{NS}
ii)	Pulses	30	16.70±10.7	17.69± 9.8	55.67	58.97	0.7948^{NS}
iii)	Roots & tubers	50	36.55± 120.5	38.31±32.5	73.10	76.62	0.0057^{NS}
iv)	Other Vegetables	50	34.23±31.2	18.53±43.5	68.46	37.06	0.1662^{NS}

v)	Green leafy vegetables	50	1.29±6.7	1.93±4.1	2.58	3.86	10.9656**
vi)	Fruits	100	10.50±25.4	14.13±13.8	10.50	14.13	1.4391^{NS}
vii)	Milk & Milk Products	500	350.31±124.7	102.00±135.6	70.06	20.40	1.1677^{NS}
viii)	Fats & Oils	20	17.64 ± 7.8	10.45 ± 8.3	88.20	52.25	0.9262^{NS}
ix)	Sugar & Juggery	25	22.23 ± 6.3	18.41 ± 8.5	88.92	73.64	0.7192^{NS}

Values are mean ± SD.

'Z' Values showing comparison of intake and RDI of both Boys-boys as well as girls-girls of both the States.

**Significant at 1% level *Significant at 5% level NS = Non-significant

RDI = Recommended Dietary Intake (ICMR, 1989)

Table 2 : Mean daily food intake of 4 to 5 years old pre school children (Boys and Girls)

BOYS							
Sr. No.	Foodstuffs	Daily mean food intake (g)					'Z' Value Boys-Boys
		RDI (g)	4-5 years		% RDI		
			Haryana (n=23)	Maharashtra (n=23)	Boys Hry	Boys Mah	
i)	Cereals	210	153.92±37.7	181.16±31.5	73.29	86.27	0.2175^{NS}
ii)	Pulses	45	20.61±94.4	37.53±41.6	45.80	83.40	0.0219^{NS}
iii)	Roots & tubers	100	43.20± 32.5	57.68±39.2	43.20	57.68	0.2565^{NS}
iv)	Other Vegetables	50	35.49±58.9	20.05±43.7	70.98	40.10	0.0640^{NS}
v)	Green leafy	50	1.98±5.1	4.97±5.4	3.96	9.94	11.2464**

	vegetables						
vi)	Fruits	100	13.06±23.1	20.41±17.3	13.06	20.41	1.4548^{NS}
vii)	Milk & Milk Products	500	398.84±117.1	50.49±132.8	79.77	10.10	0.0475^{NS}
viii)	Fats & Oils	25	20.34 ± 9.5	18.36 ±12.7	81.36	73.44	0.3202^{NS}
ix)	Sugar & Juggery	30	24.75 ± 6.4	19.90 ± 7.1	82.50	66.33	1.1175^{NS}
				GIRLS			
			(N=106)	(N=125)	Girls Hry	Girls Mah	Girls-Girls
i)	Cereals	210	146.17±34.6	169.97±36.4	69.60	80.94	0.5504^{NS}
ii)	Pulses	45	28.65±13.4	28.63±93.6	63.66	63.22	0.0516^{NS}
iii)	Roots & tubers	100	40.86± 37.0	61.25±34.7	40.86	61.25	0.5287^{NS}
iv)	Other Vegetables	50	50.50±42.4	19.63±75.3	101.00	39.26	0.0596^{NS}
v)	Green leafy vegetables	50	1.99±4.2	3.89±7.4	3.98	7.78	18.4880**
vi)	Fruits	100	14.14±18.2	15.03±29.3	14.14	15.03	2.1040*
vii)	Milk & Milk Products	500	379.38±128.7	41.91±144.6	75.88	8.38	0.0977^{NS}
viii)	Fats & Oils	25	20.37± 10.3	14.41 ± 8.6	81.48	57.64	1.2242^{NS}
ix)	Sugar & Juggery	30	23.10 ± 7.2	17.85 ± 7.2	77.00	59.50	2.5228*

Values are mean ± SD.

‘Z’ Values showing comparisaon of intake and RDI of both Boys-boys as well as girls-girls of both the States.

**Significant at 1% level *Significant at 5% level NS = Non-significant

RDI = Recommended Dietary Intake (ICMR, 1989)

Mean daily intake of cereals of 3 to 4 years boys was 109.90 and 112.18 g/day, which was 91.58 and 93.48 per cent of RDI (Table 1). Girls of the same age group also had almost similar cereal intake i.e. 88.47 and 91.56 per cent of RDI in Haryana and Maharashtra, respectively. Mean daily intake of cereals of children 4 to 5 years age group was also lower than RDI (Table 2). Boys were taking 153.92 and 181.16 g of cereals/day in Haryana and Maharashtra. This intake was 73.29 and 86.27 per cent of RDI in both the States, respectively. Girls (4 to 5 Years) were still taking fewer amounts of cereals i.e. 146.17 and 169.97 g/day, which was 69.60 and 80.94 per cent of RDI in Haryana and Maharashtra, respectively.

The Mean intake of pulses in the boys and girls of 3 to 4 years was (17.70 and 18.90) and (16.70 and 17.69) g/day, respectively in Haryana and Maharashtra which was lower than RDI in both the sexes (Table 3). The intake of pulses by boys-boys and girls-girls of this age group was not significantly ($P<0.05$) different when compared to RDI. Girls were taking little better amount of pulses in their diets than the boys in Haryana. At the same time, intake of pulses by girls (4 to 5 years) was lower than RDI. Non-significant ($P<0.05$) differences were observed while comparing pulses intake of boys-boys and girls-girls of both the States.

The consumption of roots & tubers among boys and girls of 3 to 4 years was (52.23 and 48.58) and (36.55 and 38.31) g/day, respectively which was (104.46 and 97.16) and (73.10 and 76.62) per cent of RDI in Haryana and Maharashtra, respectively. No significant ($P<0.05$) difference was observed in the intake of roots & tubers between boys-boys and girls-girls of this age group. Intake of roots & tubers in Haryana and Maharashtra State by boys of 4 to 5 years was (43.20 and 57.68) g/day (43.20 and 57.68 % of RDI). Girls of 4 to 5 Years age group were consuming on an average 40.86 amd 61.25 g/day of roots & tubers in Haryana and Maharashtra, respectively. This intake was only 40.86 amd 61.25 per cent of RDI. Intake of roots & tubers by this age group was found to be lower than RDI but no significant ($P<0.05$) difference was observed in the intake of roots & tubers was observed between boys-boys and girls-girls of both the States.

The mean intake of these vegetables by boys and girls of 3 to 4 years was 25.54 and 16.28 g/day (51.08 and 32.56% of RDI) and 34.23 and 18.53 g/day 68.46 and 37.06 % of RDI, respectively in Haryana and Maharashtra. Both the boys and girls were consuming fewer amounts of other vegetables in their daily diet. Non-significant differences ($P<0.05$) were obtained among boys-boys and girls-girls other vegetables intake. Similarly, intake of other vegetables by boys and girls of 4 to 5 years was less i.e. only (35.49 and 20.05) and (50.50 and 19.63) g/day, respectively in Haryana and Maharashtra. The intake of these vegetables by boys was (70.98 and 40.10) per cent of RDI while by girls, intake was (101 and 39.26) per cent of RDI (Table 2) in Haryana and Maharashtra, respectively. No significant ($P<0.05$) differences were observed in the intake of other vegetables, between the same sex of 4 to 5 years age group in both the States.

The intake of green leafy vegetables by boys-boys and girls-girls of this age group was significantly ($P<0.01$) different when compared to standard value.

Daily mean intake of fruits by boys (3 to 4 years) was only 14.10 and 15.41 g/day, which was 14.10 and 15.41 per cent of RDI in Haryana and Maharashtra, respectively. Girls of this age group were taking still less amount of fruits in their diet than boys (3 to 4 years) i.e. their daily mean intake was only 10.50 and 14.13 g/day which was 10.50 and 14.13 per cent of RDI, respectively in Haryana and Maharashtra (Table 1). Significant ($P<0.01$) differences were observe between boys-boys but in case of girls-girls differences were not significant ($P<0.05$). In the age group of 4 to 5 years, the mean intake of fruits was (13.06 and 20.41) and (14.14 and 15.03) g/day among boys and girls, respectively in Haryana and Maharashtra. Their consumption of fruits was only 13.06 and 20.41 (boys) and 14.14 and 15.03 (girls) per cent of RDI in Haryana and Maharashtra, respectively (Table 2). A non-significant ($P<0.05$) difference was observed for intake of fruits between boys-boys but significant ($P<0.05$) differences were observed between girls-girls of 4 to 5 years age group.

Among children of 3 to 4 years, the daily mean intake of milk & milk products of boys was 365 and 110.31 g/day, which was 73 and 22.06 per cent of RDI in Haryana and Maharashtra, respectively. As compared to boys, girls were taking fewer amounts of milk & milk products in their daily diet; their daily milk consumption was 350.31 and 102 g, which was only 70.06 and 20.40 per cent of RDI, respectively in Haryana and Maharashtra. There was no significant ($P<0.05$) difference between the daily mean intake of milk & milk products of boys-boys and girls-girls of this age group (Table 1).

Mean intake of milk & milk products in age group of 4 to 5 years by boys and girls was (398.84 and 50.49) and 379.38 and 41.91) g/day, which was (79.73 and 10.10) and (75.88 and 8.38) per cent of RDI, respectively in Haryana and Maharashtra. Daily mean intake of milk & milk products of (4 to 5 years) boys-boys and girls-girls did not differ significantly (P<0.05). However, when comparison of daily intake of milk & milk products was made between the age groups, it was found that both boys and girls of 4 to 5 years were consuming more amount of milk & milk products as compared to boys and girls of 3 to 4 years in Haryana. While the case was reverse in Maharashtra (Table 2)

Table 1 depicts that daily mean consumption of fats & oils in boys of 3 to 4 years was 16.44 and 12.56 g/day, which was 82.20 and 62.80 per cent of RDI in Haryana and Maharashtra, respectively. Mean intake of fats & oils among girls was 17.64 and 10.45 g/day, which was 88.2 and 52.25 per cent of RDI in Haryana and Maharashtra, respectively. These girls were taking a little higher amount of fats & oils in Haryana than that of boys. Non-significant (P<0.05) differences were seen between boys-boys and girls-girls fat intake. Daily mean intake of fats & oils among boys and girls of 4 to 5 years was (20.34 and 18.36) and (20.37 and 14.41) g/day, respectively which was less than RDI (57.64 to 81.48% of RDI) in both the sexes (Table….) in Haryana and Maharashtra, respectively. In this age group, both boys and girls were taking almost similar amount of fats & oils in daily diets in Haryana. Between both the age groups, intake of fats & oils by pre-school children of 4 to 5 years was higher than that of pre-school children aged 3 to 5 years in Maharashtra. Desi ghee and mustard oil were mostly preferred as a cooking medium among pre-schoolers' families in Haryana (Table 1 and 2). Non-significant differences (P<0.05) were observed in the intake of fats & oils between boys-boys and girls-girls of both the age group.

All respondents in different food preparations consumed sugar. The mean intake of sugar & jaggery observed among boys and girls (3 to 4 years) was (21.31 and 20.30) and (22.23 and 18.41) g/day which was (85.24 and 81.20) and (88.92 and 73.64) per cent of RDI respectively in Haryana and Maharashtra. Non-significant (P<0.05) difference was observed in sugar intakes between boys-boys and girls-girls in both the States. Mean intake of sugar & jaggery of 4 to 5 years pre-school children was (24.75 and 19.90) g/day in boys and (23.10 and 17.85) g/day in girls against the recommended daily allowance of 30 g/day (NIN, 2000). Hence, both the boys and girls were taking fewer amounts of sugar & jaggery in their daily diets. Non-significant (P<0.05) difference was noticed in sugar intake of boys-boys in both the States but significant (P<0.05) difference was observed in girls-girls sugar & jaggery intake. Almost ¾ of the requirement of sugar & jaggery as recommended by ICMR (NIN, 2020) was met in case of both boys and girls (Table 2). The values reported in the studies are similar with the references listed below.

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Utilizing Business Mathematics to Address Financial Challenges

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Abstract

Business mathematics plays a crucial role in solving financial problems by providing a set of mathematical tools and techniques to analysis and understand various financial transactions and decisions. These tools and techniques help in making informed decisions and finding solutions to various financial problems in a systematic and efficient manner. This paper provides an overview of the different methods used for project appraisal and selection, including the estimation of present values, decision making with benefit cost, APR, NPV, IRR, and payback period. These methods are used to evaluate the financial viability of a project and to determine the most suitable investment strategy.

Keywords: Financial Problems, Investment Decisions, Risk Management, Return on Investment and Net Present Value (NPV)

Introduction

Business mathematics plays a crucial role in project appraisal and selection are essential activities that determine the success of a project. Project appraisal involves evaluating the feasibility of a project, while project selection involves choosing the best investment strategy for the project. The primary goal of project appraisal and selection is to determine the financial viability of a project and to ensure that the project generates a return on investment.

There are different methods used for project appraisal and selection, including the estimation of present values, decision making with benefit cost, APR, NPV, IRR, and payback period. These methods evaluate the financial benefits of a project, including the cost of the investment and the return on investment.

Investment Decisions

Investment decisions are an important aspect of financial planning, and business mathematics plays a critical role in analysing and evaluating investment opportunities. It helps in determining the return on investment (ROI) and the rate of return (ROR) on different investment options. The net present value (NPV) and internal rate of return (IRR) methods are commonly used in India to evaluate investment opportunities.

Risk Management

Business mathematics also helps in managing and mitigating financial risks e.g., the use of statistical methods like regression analysis and Monte Carlo simulation helps in predicting the potential losses and returns from a given investment, which can help in managing risk. Additionally, financial derivatives like futures and options can be used to hedge against potential losses in the market.

Capital Budgeting

Capital budgeting is an important aspect of financial management in India, and business mathematics plays an essential role in this process. It helps in analysing the costs and benefits of different investment options and determining which options are more profitable and feasible. Methods such as net present value (NPV) and internal rate of return (IRR) are commonly used in capital budgeting in India.

Financial Planning

Business mathematics also helps in financial planning in India by providing a set of tools and techniques to help individuals and organizations plan and manage their financial resources. This includes methods like budgeting, saving and investment planning, retirement planning, and risk management.

Loan Management

Business mathematics also plays a crucial role in loan management. It helps in determining the interest rate and repayment schedule for loans and mortgages, and helps in evaluating the feasibility of different loan options.

Financial Modelling

Business mathematics is also used in financial modelling, which involves building mathematical models to represent and analysis financial systems and processes. Financial models can help in predicting future financial performance, identifying trends and patterns in financial data, and analysing the impact of different financial decisions.

Decision Making with Benefit Cost

The Benefit Cost Ratio (BCR) is a method of project appraisal that compares the expected benefits of a project to its costs. It is calculated by dividing the present value of the project benefits by the present value of its costs. A BCR of 1 or higher indicates that the benefits outweigh the costs, while a BCR of less than 1 indicates that the costs outweigh the benefits e.g., suppose a project costs Rs.100,000 and is expected to generate benefits of Rs. 150,000 over the next five years. The present value of the project benefits and costs is calculated to be Rs. 120,000 and Rs. 90,000, respectively. The BCR of the project is calculated as follows:

BCR = Present value of benefits / Present value of costs = Rs. 120,000 / Rs. 90,000 = 1.33

The BCR of 1.33 indicates that the benefits of the project outweigh the costs, making it a viable investment option.

Annual Percentage Rate

The annual percentage rate (APR) is a method used to calculate the effective interest rate of a loan. The APR takes into account the interest rate, the loan fees, and the time period of the loan. The APR is used to evaluate the cost of financing a project and to determine the most suitable loan for the project.

The APR is the interest rate charged on a loan or investment over a year. It is a method of project appraisal that determines the interest rate required to break even on an investment. The APR considers the time value of money and the risk associated with the project e.g., suppose an investor invests Rs. 10,000 in a project that generates Rs. 1,500 annually for the next five years. The present value of the project benefits is calculated to be Rs. 6,521, while the present value of the investment is Rs. 10,000. The APR of the project is calculated as follows:

APR = ((1 + I) ^n - 1) / n x 100% where I is the interest rate, n is the number of years, and ^ is the exponent operator.

Net Present Value

The net present value (NPV) is a method used to evaluate the financial viability of a project. The NPV is calculated by subtracting the present value of the cash outflows from the present value of the cash inflows. If the NPV is positive, the project is considered financially viable, and if the NPV is negative, the project is not financially viable.

Internal Rate of Return

The internal rate of return (IRR) is a method used to evaluate the potential return on investment of a project. The IRR is the interest rate at which the present value of the cash inflows is equal to the present value of the cash outflows. If the IRR is greater than the cost of capital, the project is considered financially viable.

Payback Period

The payback period is a method used to evaluate the time it takes for a project to generate enough cash inflows to cover the initial investment. The payback period is calculated by dividing the initial investment by the annual cash inflows. If the payback period is short, the project is considered financially viable.

In conclusion, business mathematics plays a crucial role in solving financial problems in India by providing a set of mathematical tools and techniques to analysis and understand various financial transactions and decisions. These tools and techniques help in making informed decisions and finding solutions to various financial problems in a systematic and efficient manner.

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